PROCEEDINGS OF SCIENCE

Determination of hybrid charmonium meson masses

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We report initial results from our study of the masses and decay constants of the lightest multiplet of charmonium-like hybrid mesons. We obtain precise measurements of the 1^{-+} state through the use of a variational basis and a large number of configurations at three lattice spacings. We use staggered fermion operators using configurations generated with the HISQ action with 2+1+1 dynamical flavours. The mixing of the vector hybrid with the J/ψ is examined and a preliminary bound on the vector hybrid decay constant is presented.

The 38th International Symposium on Lattice Field Theory, LATTICE2021 26th-30th July, 2021
Zoom/Gather@Massachusetts Institute of Technology

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PoS(LATTICE2021)240

1. Introduction

Since 2003 a series of resonances have been observed in the charmonium and bottomonium sectors that do not correspond neatly with Quark model states [1, 2]. Many competing non-Quark model explanations for these so-called XYZ states have been proposed, including (but not limited to) gluonic hadrons, bound states of more than 3 quarks, and hadronic molecules [3–5]. A hybrid meson is a meson *with an excited gluonic component* [6]. Although hybrid mesons (and the other states we've noted) are exotic in the sense that they fall outside the well-tested Quark model, nothing in principle prevents $\bar{q}qg$ states in QCD. Hybrid mesons have been studied on the lattice for many decades [7–12]. Relevant experimental searches include the upcoming PANDA experiment at FAIR [13], which will search for evidence of gluonic excitations in the hadron spectrum, and GlueX at the Jefferson Lab [14, 15]. Accurate lattice QCD calculations of the properties of the hybrid mesons in charmonium may encourage the LHCb collaboration to search for them.

In this paper we present initial results for the mass of the 1^{-+} and 1^{--} hybrid mesons in charmonium, based on lattice QCD calculations using the staggered formalism. The advantages of using staggered fermions is that the lattice spacing errors are small and the HPQCD collaboration has done many comparison between lattice calculations and the experimental properties of mesons in the charmonium system [16]. There are some disadvantages of using staggered fermions over other formulations of the Dirac operator. For example, because the staggered correlators have a contribution from the parity partner, the hybrid mesons are the first excited state, but in Wilson like formulations the hybrid meson is the ground state. The initial goal of this project was to check whether it was possible to get accurate results for the mass of hybrid mesons made from charm quarks using staggered fermions.

Although at this stage in the project we focus on single meson operators, we will eventually need to include two meson operators. There are only a few exploratory calculations of hadronic decays using staggered fermions [17, 18]. For completeness, we briefly discuss the possible hadronic decays of hybrid charm mesons. There are possible string breaking-like decays where the hybrid meson can decay into pairs such as $\overline{D}D$, or two P-wave D mesons, or a P-wave D and a S-wave D, as well as similar channels with charm-strange mesons. There is a selection rule in the heavy quark limit [19] (and some quark models [20]) that a hybrid meson can not decay into two S-wave mesons. It is possible for the hybrid meson to decay to a standard charmonium meson with a light meson and there is a lattice QCD calculation in the heavy quark limit [21].

2. Interpolating operators for hybrid mesons

Since hybrids have an excited gluonic component the operators which we would expect to couple to them include the field strength tensor, F_{ij}^{ab}, whose components are the chromoelectric and chromomagnetic fields ($\frac{1}{2}\epsilon_{ijk}F_{jk} = B_i$). These are contracted, over the colour indices, with the usual fermion bilinears (so that the quark-antiquark pair is a colour octet).

As we are using staggered fermions we replace the γ matrices with phases. These phases, along with the F_{ij}^{ab}, determine the quantum numbers of the operators. In the staggered basis the operators also have a taste assignment. Taste breaking effects should drop out in the continuum limit.

We started by considering the lightest hybrid multiplet, as determined by the HadSpec collaboration in [12],

$$\begin{aligned}
1^{-+} &: \epsilon_{ijk}\bar{\psi}\gamma_j B_k \psi && \longrightarrow && \gamma_i \otimes \gamma_i : \bar{\chi}\epsilon_{ijk}(-1)^{x_j} B_k \chi && (1)\\
1^{--}_H &: \bar{\psi}\gamma_5 B_i \psi && \longrightarrow && \gamma_5 \otimes \gamma_5 : \bar{\chi} B_i \chi && (2)\\
0^{-+}_H &: \bar{\psi}\gamma_i B_i \psi && \longrightarrow && \gamma_i \otimes \gamma_i : \bar{\chi}(-1)^{x_i} B_i \chi && (3)\\
2^{-+}_H &: |\epsilon_{ijk}|\bar{\psi}\gamma_j B_k \psi && \longrightarrow && \gamma_i \otimes \gamma_i : \bar{\chi}|\epsilon_{ijk}|(-1)^{x_j} B_k \chi \quad . && (4)
\end{aligned}$$

Note that we have suppressed the colour indices, labelled the states that can mix with conventional charmonium by a H subscript, and given the ‘spin ⊗ taste’ assignment for the operators formed from staggered fields χ on the right. We have neglected disentangling states due to the reduced cubic symmetry of the lattice, such as 1^{-+} with 4^{-+} [22].

The majority of the results from lattice QCD for the masses of hybrid mesons have used clover or Wilson fermions. There has been one calculation by the MILC collaboration [9] using staggered fermions, which computed the mass of the 1^{-+} hybrid meson with light and strange quarks. The previous MILC calculation used the taste singlet non-local ρ when constructing hybrid operators to minimize taste breaking [9]. We use the local ρ operator because taste breaking is less of an issue with charm quarks and local operators can be less noisy than non-local operators.

3. Simulation Details

We use the Highly Improved Staggered Quark (HISQ) action [23] with 2+1+1 sea quarks. The gauge configurations were provided courtesy of the MILC Collaboration [24, 25]. The lattice spacing is set using w_0 with the value calculated by HPQCD [26]. In table 1 the ensembles used are listed. We used the MILC code to do the calculations. We modified the existing hybrid operators in the MILC code and tested the correlators against our own implementation in the Grid library [27].

name	size	a (fm)	m_l/m_s	$M_\pi L$	M_π (MeV)	am_c^{sea}	am_c^{val}	configurations
very coarse	32^3x48	0.15088(79)	1/27	3.30	131.0(1)	0.8447	0.863	1505
coarse	32^3x64	0.12225(65)	1/10	4.29	216.9(2)	0.628	0.650	1000
fine	32^3x96	0.09023(48)	1/5	4.50	312.7(6)	0.440	0.450	1008

Table 1: The lattice ensembles used. Both the fine and coarse lattices are at heavier than physical pion masses, while the very coarse lattice is at the physical point. The tuned valence charm quark masses are from [28]

The correlators of hybrid meson operators are typically very noisy. We therefore employ a series of well-established techniques to reduce this noise. We average over multiple time sources and polarisations per configuration. We do variational smearing with local and covariant smearing applied to the quarks. DeTar and Lee report on using variational smearing with staggered fermions [29]. We apply APE smearing on the gauge links in the field strength tensor and in the Gaussian smearing on the quarks.

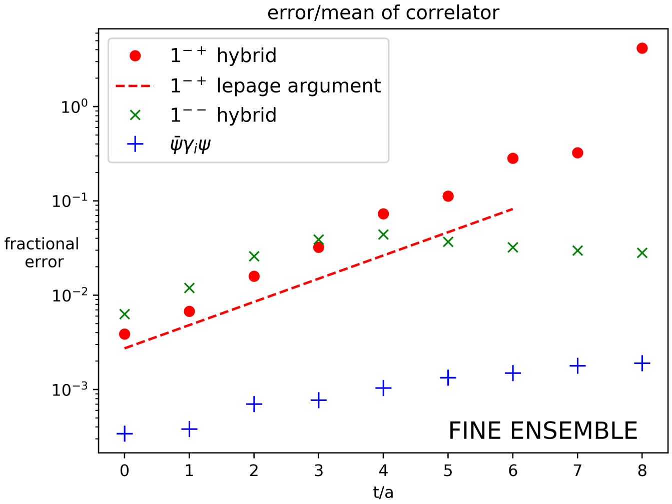

Figure 1: The error/mean of correlators against time for 1^{-+}, 1^{--} hybrid and conventional vector operators on the fine ensemble. We include the predicted fractional error from an argument by Lepage, using the fitted hybrid and pseudoscalar masses.

4. Results for the mass of the 1^{-+} hybrid meson

In figure 1 we plot fractional errors for the 1^{-+} hybrid correlator, 1^{--} hybrid correlator, and for comparison the J/ψ correlator with charm quarks on the fine ensemble. The hybrid correlators are considerably more noisy than the conventional vector operator, $\bar{\psi}\gamma_i\psi$, with the signal dying by $t = 8$ at the latest. In figure 1 we also compare the fractional error of the 1^{-+} hybrid correlator with the estimate from Lepage [30], which says it should be proportional to $\exp\big(-(M_H - M_{\eta_c})t\big)$.

We use Lepage's python library `corrfitter`, which implements a suite of functions to fit correlators and matrices of correlators within a Bayesian framework [31, 32]. In the 1^{-+} analysis our fitting procedure is as follows. We use a 2 operator basis, with and without smearing, to produce a 2×2 matrix of correlators. We choose a time t_0 and use it to generate priors by diagonalising the correlator matrix with the eigenvectors associated to the solution of a Generalized Eigenvalue Problem (GEVP) at t_0 and $t_0 + 1$. The staggered formalism requires a parity partner (PP) state, which oscillates in time, to be included in the fit model. For the 1^{-+} channel the quantum numbers of the parity partner are 1^{++}. We vary the fit range until the goodness of fit parameters indicate an acceptable fit. Our preliminary results are in table 2.

Ensemble	# tsrc	t_0	range	svdcut	χ^2 per dof	Q	Mass (GeV)	PP Mass (GeV)
very coarse	16	1	1-6	3×10^{-7}	1.1	0.31	4.29(11)	3.89(25)
coarse	16	1	1-5	3×10^{-5}	1.3	0.22	4.575(66)	3.58(28)
fine	16	2	2-8	7×10^{-4}	0.93	0.55	4.23(18)	3.35(48)

Table 2: Summary of 1^{-+} fits. We define a good fit as having $\chi^2/\text{dof} \sim 1$ and a Q value > 0.1. As discussed in the text 'PP' is the Parity Partner state.

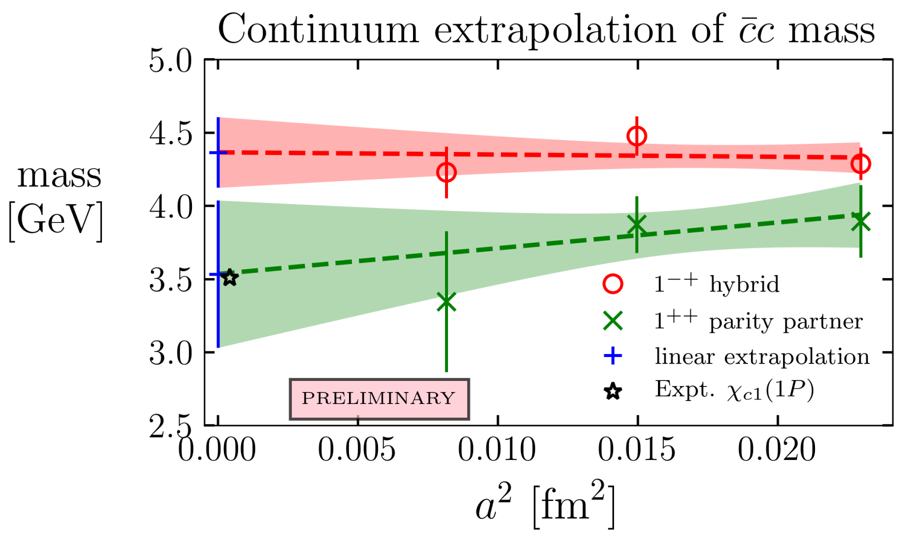

Figure 2: Continuum extrapolation of the mass of the 1^{-+} hybrid charmonium meson and the parity partner state. The extrapolated PP mass agrees with experiment albeit with a large uncertainty.

In figure 2 we show a continuum extrapolation of the mass of the charmonium 1^{-+} and parity partner 1^{++} state using our results at three lattice spacings. We only use a linear dependence on a^2 to do the continnum extrapolation and neglect any light quark mass dependence or finite volume effects in this preliminary result. The continuum limit of the mass of the parity partner state agrees with the mass of the $\chi_c(1P)$, which is an important cross-check. In figure 3 we plot our results with the results from the HadSpec collaboration [12] and Bali et al. [33]. We also include some of the physical decay thresholds in figure 3, as briefly discussed in section 1.

5. Results for the properties of the 1^{--} hybrid meson

There are many unexplained resonances with $J^{PC} = 1^{--}$, where charmonium hybrid mesons may occur. See the review by Brambilla et al. [5]. For example it has been speculated that the $\psi(4230)$ is a hybrid meson, but confirmation will require accurate lattice QCD calculations.

To examine the 1^{--} hybrid state we perform a GEVP analysis with a 4 operator basis: standard vector operator, the hybrid operator, and their smeared and local counterparts. Here we assume the hybrid state is the *second* excited state, after the J/ψ and $\psi(2S)$. We therefore expect the extraction of its properties to be more difficult than in the 1^{-+} case. Our initial results are shown in table 3 and a comparison of our 1^{--} hybrid mass to other groups is given in figure 4.

Brambilla et al. [5] review the importance of the decay constant in probing the properties of 1^{--} hybrid mesons. The decay constants can constrain the leptonic decay width. The decay constant, f_H, is defined through a matrix element,

$$\langle 0|\hat{V}_i|H\rangle = f_H M_H \epsilon_i \quad , \tag{5}$$

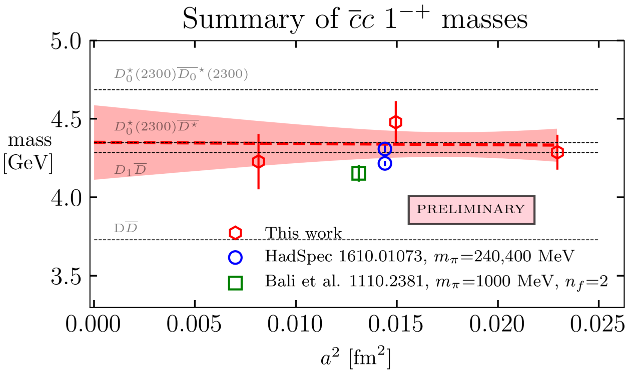

Figure 3: Summary plot for the 1^{-+} mass as a function of the square of the lattice spacing, including some of the previous determinations of the mass by other groups [12, 33]

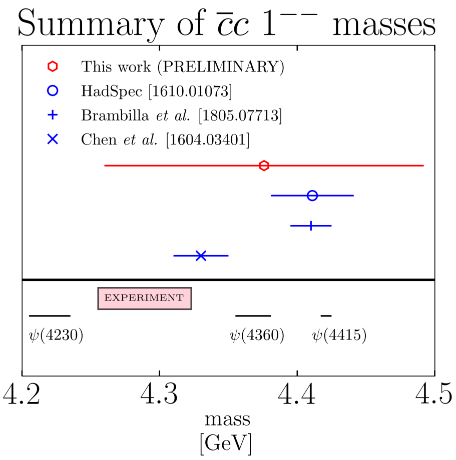

Figure 4: Comparison of our determination of the 1^{--} hybrid mass at 0.09 fm from a 4-by-4 fit to the results of three other groups, and three resonances the PDG lists as 'estabished'[12, 34, 35].

state	fit range	χ^2 per dof	Q	mass [GeV]	amplitude	f [MeV]	Γ_{ee} [keV]	comment
J/ψ $\psi(2S)$ $h_c(1P)$	6-24	0.88	0.72	3.097(17) 3.781(28) 3.512(29)	0.16441(26) 0.1860(78) 0.0578(75)	417.5(2.3) 428(18) –	5.836(36) 5.01(42) –	concurrent 2x2 fit with only vector ops
Hybrid	1-6	0.9	0.58	4.33(16)	0.086(15)	–	–	2x2 fit w/only hybrid ops
J/ψ Hybrid	2-6	0.81	0.82	3.110(18) 4.38(12)	0.1701(27) 0.065(18)	431.1(7.2) 9(167)	6.20(20) 0.002(67)	concurrent 4x4 fit with hybrid and vector ops

Table 3: Fit results for the conventional charmonia and the hybrid state on the fine ensemble. The $h_c(1P)$ meson is the parity partner state. The masses and decay constants from the 2-by-2 fits agree well with experiment. The 4-by-4 and 2-by-2 masses are consistent, though there is a slight tension between the the J/ψ amplitudes (and therefore the decay constants too). The leptonic decay widths are also shown in the penultimate column. The hybrid leptonic width is small, consistent with zero, and has a large uncertainty, stemming from the sizeable uncertainty in the amplitude.

where $\hat{\mathcal{V}}$ is the vector current operator. The local vector operator was used so we use the Z_V renormalization factor from [36]. As a check we computed the decay constant of the J/ψ meson (see [28] for a systematic study of the leptonic decay constants of the J/ψ meson).

From this we can compute the leptonic width of our hybrid vector charmonium state using

$$\Gamma(V_{c\bar{c}} \hookrightarrow e^+e^-) = \frac{16\pi}{27}\alpha_{\text{QED}}^2 \frac{f_H^2}{M_H} \quad , \tag{6}$$

where α_{QED} is the electromagnetic coupling at the charm quark mass. Our amplitude for the vector operator into the 1^{--} hybrid state is very small. From this amplitude we obtain an upper bound of 70 eV. A previous calculation of this leptonic width bounded it from above at 40 eV [35].

6. Summary and outlook

We have presented a first continuum extrapolation of the mass of the 1^{-+} charmonium hybrid meson from unquenched lattice QCD. Additionally, preliminary results for the mass and leptonic decay constant of the 1^{--} hybrid meson at a single lattice spacing have been reported. We are currently computing correlators for 1^{-+} operators with charm quarks on the coarse and fine ensembles with physical pion masses. Also we are refining our analysis of the 4-by-4 variational analysis of the 1^{--} mesons. Results at a finer lattice spacing ($\sim$ 0.06 fm) would improve the quality of our continuum extrapolation. We plan to compute the mass of the 1^{-+} hybrid meson with bottom quarks [37] using the extrapolation method with the HISQ action [38].

7. Acknowledgements

This work was supported by the UK Science and Technology Facilities Council. The calculations used the DiRAC Data Analytic system at the University of Cambridge, operated by the University of Cambridge High Performance Computing Service on behalf of the STFC DiRAC HPC Facility (www.dirac.ac.uk). This is funded by BIS National e-infrastructure and STFC capital grants and STFC DiRAC operations grants.

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