PROCEEDINGS OF SCIENCE

Light meson physics and scale setting from a mixed action with Wilson twisted mass valence quarks

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We consider a mixed action approach where valence Wilson twisted mass (Wtm) fermions at maximal twist are combined with CLS ensembles consisting of $N_f = 2+1$ flavours of $\mathrm{O}(a)$-improved Wilson sea quarks. We present an update of the results of the matching of valence and sea quarks, and of the subsequent continuum-limit scaling studies of light-quark observables. A scale setting procedure combining the results from the $\mathrm{O}(a)$-improved Wilson setup to the ones from the valence Wtm regularisation is discussed.

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1. Introduction

A central aspect of present-day particle physics is related to the understanding of Flavor Physics. The parameters that characterize the masses and mixings of elementary particles, and, in particular, of quarks, constitute the largest share of fundamental parameters of the Standard Model (SM). A theoretical challenge in the study of flavor physics within the quark sector arises from the fact that the relevant electroweak interactions take place in a hadronic environment. As a result, our ability to produce reliable predictions for the amplitudes of relevant processes, both within the SM and in candidate extensions of the latter, often relies on a proper control of low-energy strong interaction physics through lattice QCD simulations.

In the following we will describe a setup [1–5] whose main objective is to achieve precise lattice QCD computations of heavy-quark observables based on the following features: (*i*) the ability to perform lattice simulations at very fine lattice spacings through the use of open boundary conditions in time [6] and (*ii*) the adoption of a mixed-action strategy where the valence sector exploits the twisted mass QCD (tmQCD) regularization while the sea sector is based on CLS ensembles with O(a)-improved Wilson quarks [7]. An advantage of this setup is the automatic removal of the leading lattice artifacts proportional to the twisted mass when working at maximal twist [2, 8]. This property is particularly relevant for lattice calculations in the heavy-quark sector.

In the present work we report an update on the use of this lattice formulation in the light ($\ell = u, d$) and strange s quark sectors. This is a necessary step for other ongoing studies in the heavy-quark sector as it involves the matching of the valence quark masses to the $N_f = 2+1$ flavours of sea quarks, along with the tuning of the tmQCD formulation to maximal twist. Furthermore, the computation of light-quark observables permits an independent analysis of the gradient flow scale t_0 through a scale setting procedure, as well as a determination of the masses of light quarks.

2. Lattice Formulation

The gauge configuration ensembles used in this study, see Table 1, were generated by the CLS initiative using the Lüscher-Weisz tree-level improved gauge action and $N_f = 2+1$ flavours of O(a)-improved Wilson quarks. They lie along a line of constant trace of the bare quark mass matrix,

$$\mathrm{tr} M_\mathrm{q} = 2 m_{q,\ell} + m_{q,\mathrm{s}} = \mathrm{const}\,, \tag{1}$$

where $m_{\mathrm{q,f}} = m_{0,\mathrm{f}} - m_{\mathrm{cr}}$. In this way, cut-off effects proportional to $\mathrm{tr} M_q$ are kept constant when varying the quark masses. In practice, it is beneficial to depart from the relation in eq. (1), defined in terms of bare quark masses, towards a renormalized chiral trajectory in terms of following dimensionless quantities,

$$\phi_2 = 8 t_0\, m_\pi^2\,, \qquad \phi_K = 8 t_0\, m_K^2\,, \qquad \phi_4 = 8 t_0 \left(m_K^2 + \frac{1}{2} m_\pi^2 \right) = \phi_K + \frac{1}{2}\phi_2\,, \tag{2}$$

depending on the pion and kaon masses, m_π and m_K, respectively, in units of the gradient flow scale t_0. In particular, a constant value of ϕ_4 corresponds to keeping the trace of the renormalized quark mass matrix fixed at lowest order in the chiral expansion. This mass corrections in the u, d, s

β	a [fm]	Id.	N_s	N_t	m_π [MeV]	m_k [MeV]	$m_\pi L$
3.40	0.086	H101	32	96	420	420	5.8
		H102	32	96	350	440	4.9
		H105	32	96	280	460	3.9
3.46	0.076	H400	32	96	420	420	5.2
		H401	32	96	550	550	7.3
		H402	32	96	450	450	5.7
3.55	0.064	N202	48	128	420	420	6.5
		N203	48	128	340	440	5.4
		N200	48	128	280	460	4.4
		D200	64	128	200	480	4.2
3.70	0.050	N300	48	128	420	420	5.1
		J303	64	192	260	470	4.1

Table 1: CLS ensembles considered in this work. The parameters N_s and N_t refer to the spatial and the time lattice size, respectively. Approximate values for the pion and the kaon masses, in MeV, are given in the fifth and sixth columns.

quark masses required for the shift to a renormalized chiral trajectory can be obtained through a low-order Taylor expansion [9].

In the valence sector, a chirally rotated mass term [8, 10, 11], $\mu_q = \{\mu_\ell, \mu_s, \mu_c\}$, is added to the Wilson operator as follows,

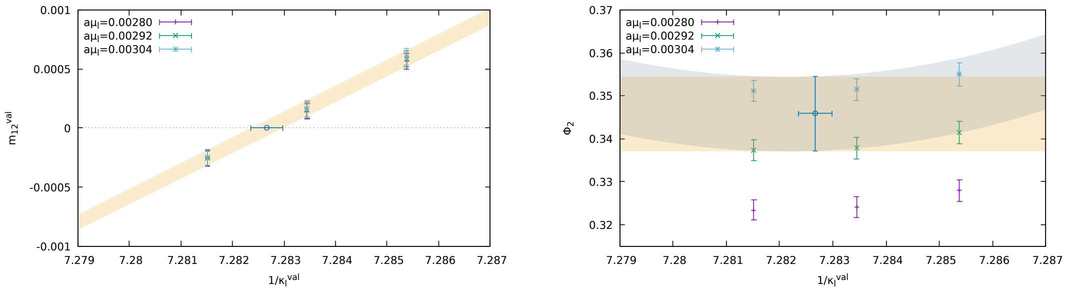
$$\frac{1}{2}\sum_{\mu=0}^{3}\{\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - a\nabla_\mu^*\nabla_\mu\} + \frac{i}{4}ac_{\mathrm{SW}}\sum_{\mu,\nu=0}^{3}\sigma_{\mu\nu}\widehat{F}_{\mu\nu} + m_{0,q} + i\gamma_5\,\mu_q\,. \tag{3}$$

The inclusion of the Sheikholeslami-Wohlert term guarantees that sea and valence regularizations share the same set of renormalization constants.

3. Matching of Valence and Sea Quark Masses

Maximal twist is achieved, ensemble by ensemble, by tuning the valence current quark mass m_{12}^{val} to zero, where the mass of the two degenerate light flavours 1 and 2 corresponds at the physical point to the average (u, d) quark mass. In the following, flavor 3 will refer to the heavier fermion, corresponding to the strange quark at the physical point. The matching of the quark masses is achieved by imposing that ϕ_2 and ϕ_4, defined in eq. (2), are equal in the sea and valence sectors. To enforce that the matching is obtained through small interpolations, we determine ϕ_2^{val}, ϕ_4^{val} and m_{12}^{val} for a set of values of the valence bare parameters, $m_{0,q}$ and μ_q, appearing in eq. (3), in the neighborhood of the matching point. An illustration of the matching of the quark masses and of the tuning to maximal twist through a combined fit of the mass dependence of ϕ_2^{val} and m_{12}^{val} is shown in Fig. 1.

As an alternative to the use of ϕ_2 and ϕ_4, the matching can also be performed by imposing that the sea and valence renormalized quark masses coincide. A comparison of the two matching procedures is shown in Fig 2(a) on symmetric point ensembles where, $m_\pi = m_K = 420\,$MeV.



(a) (b)

Figure 1: (a) Tuning to maximal twist by linearly interpolating to zero the valence current quark mass as a function of $1/2\kappa_\ell^{\rm val} = am_{0,\ell} + 4$. (b) Dependence of $\phi_2 = 8t_0\, m_\pi^2$ on $1/2\kappa_\ell^{\rm val}$ in the vecinity of maximal twist. The horizontal yellow band shows the result from the Wilson formulation. Data points are fitted to the expected parabolic behavior of $m_\pi^2\big|_{\rm v}$ in the neighborhood of maximal twist (gray band). The empty circles correspond to the matching point obtained from a combined fit of the data points in the two figures, for the case of the ensemble H105 at the coarsest lattice spacing (see Table 1).

We confirm the expectation that the difference of the values of ϕ_4 coming from the two matching procedures is reduced when decreasing the lattice spacing.

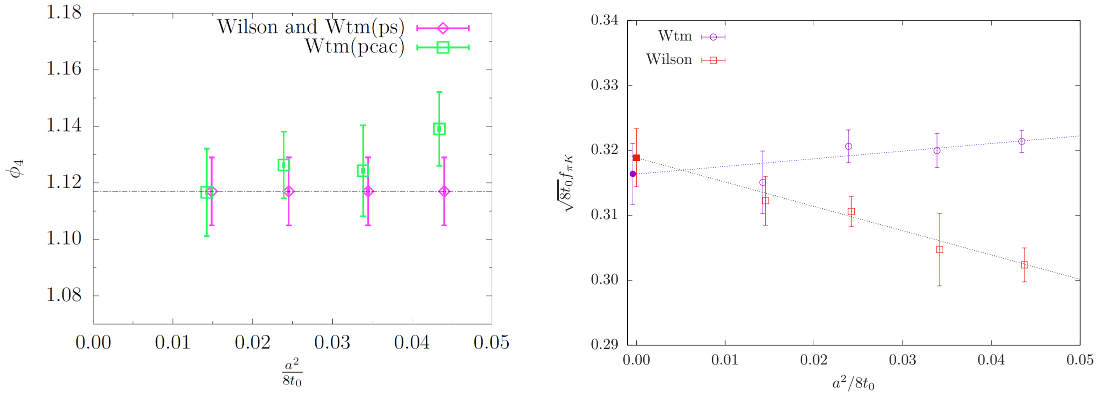
4. Scale Setting

Following Ref. [9], we carry out a scale setting procedure that uses the pion and kaon masses and decay constants as physical input [12, 13]. More specifically, the linear combination $f_{\pi K}$ of the pion and kaon decay constants,

$$f_{\pi K} = \frac{2}{3}\left(\frac{1}{2} f_\pi + f_K\right), \tag{4}$$

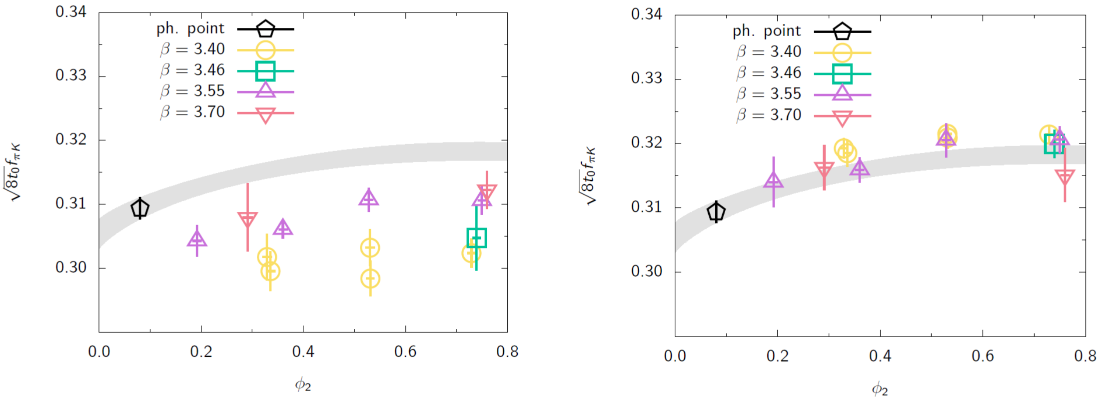
is considered given that at NLO in SU(3) chiral perturbation theory (ChPT), it remains constant along the tr$M$ chiral trajectory up to logarithmic corrections. The continuum-limit scaling of $f_{\pi K}$ in units of t_0 – based on the Wilson and the Wilson twisted mass (Wtm) formulations – is shown in Fig 2(a). The fact that consistent continuum results for $f_{\pi K}$ are obtained from these two regularizations, opens the door to explore the possibility of enforcing their equality to further constrain the scale setting procedure.

Examples of the light quark mass dependence of $\sqrt{8t_0} f_{\pi K}$ and of the ratio f_K/f_π for the Wilson and Wtm regularizations are shown in Figs. 3 and 4. The gray bands show the result of a combined chiral-continuum extrapolation of $\sqrt{8t_0} f_{\pi K}$ and f_K/f_π using the corresponding SU(3) ChPT expressions and an O(a^2) term to parameterize the lattice spacing dependence. In order to estimate the systematic uncertainties, we consider various alternative analyses that, for instance, include functional forms based on a Taylor expansion on ϕ^2, cuts on the maximal pion mass values to be considered, or additional terms to probe mass-dependence cutoff effects [14]. The various models are then weighted by their value of the corresponding Akaike Information Criteria [15].



(a) (b)

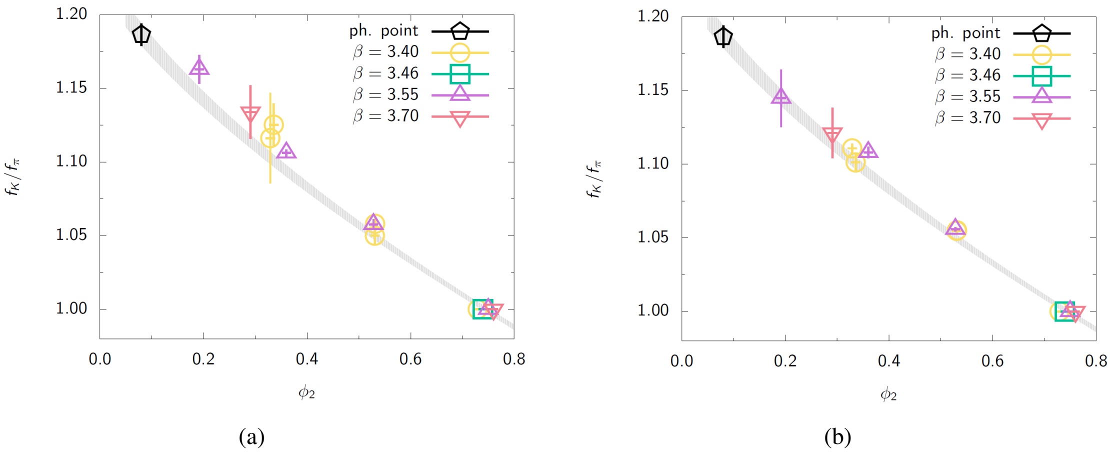
Figure 2: (a) Measurements of ϕ_4, defined in eqs. (2), on symmetric point ensembles with four different values of the lattice spacing. Blue diamond data points refer to the case of the Wilson and the fully twisted tmQCD mixed action matched through the pseudoscalar meson masses to $\phi_4 = 1.117(12)$. The yellow squares refer to a similar case but using a matching of the sea and valence by means of the renormalized current quark masses. (b) Continuum limit scaling of $f_{\pi K}$, defined in eq. (4), in units of t_0 using symmetric point ensembles. Red square data points refer to the Wilson case while blue circles indicate the fully twisted tmQCD mixed action matched through pseudoscalar masses to $\phi_4 = 1.117(12)$.



(a) (b)

Figure 3: Light quark mass dependence of $f_{\pi K}$, defined in eq. (4), in units of t_0 for (a) the Wilson and (b) the Wilson twisted mass regularizations.

This analysis is carried out separately for the Wilson and the Wilson twisted mass regularizations, using the matching procedures based, on the one hand, on ϕ_2 and ϕ_4, or, on the other hand, on the renormalized quark masses. As mentioned above, the study is furthermore repeated by imposing the equality of the Wilson and Wtm results in the continuum limit. Preliminary results for the scale setting procedure based on the ensembles in Table 1, indicate the stability of the physical value of t_0 under the above variations of the considered analysis strategies.



(a) (b)

Figure 4: Light quark mass dependence of the ratio of the kaon and pion decay constants for (a) the Wilson and (b) the Wilson twisted mass regularizations.

5. Light and Strange Quark Masses

The calculations of the light and strange quark masses – from the Wilson and the Wtm regularizations – use ALPHA collaboration results for the quark mass renormalisation and renormalisation group running [16] determined in the Schrödinger functional scheme.

When considering the Wilson case, the renormalized current quark mass is employed. In the Wtm case, we denote $m_{rs} = (\mu_r + \mu_s)/2$, where r and s are two distinct flavours. Following Ref. [17], we consider the quantity,

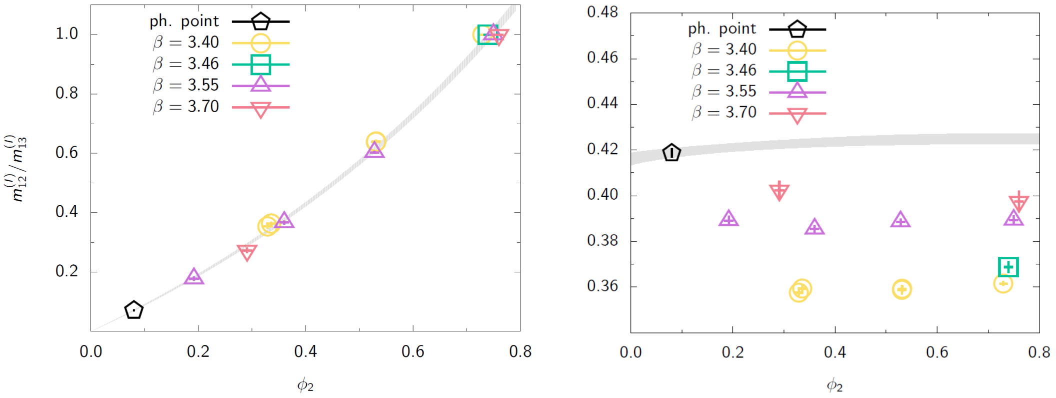
$$R_{\pi K} = \sqrt{8t_0}\left(2\frac{m_{13}^R}{\phi_K} + \frac{m_{12}^R}{\phi_2}\right) , \tag{5}$$

which in ChPT is expected to mildly depend on the light-quark mass.

The mass dependence of the ratio of quark masses m_{12}/m_{13} and the $R_{\pi K}$ are shown in Fig. 5. The gray bands show the results on a combined fit of these two quantities based on an SU(3) NLO ChPT fit including $\mathrm{O}(a^2)$ discretisation effects. A strategy similar to that outlined in the previous section has been considered to estimate the systematic uncertainties. Preliminary results for the light and strange quark mass are in good agreement with the determinations reported in Ref. [17], which are also based on CLS $N_f = 2 + 1$ ensembles.

6. Conclusions

We have reported an update on the study of a mixed action approach based on Wilson twisted mass valence quarks on CLS ensembles with $\mathrm{O}(a)$-improved sea quarks. We observe a promising evidence of the effectiveness of combining the Wilson and Wilson twisted mass regularizations in a scale setting procedure based on physical input for the pion and kaon decay constants. An extension of the analysis is planned to include physical point ensembles and a finer lattice spacing. We refer



(a) (b)

Figure 5: Light quark mass dependence of (a) the ratio of quark masses m_{12}/m_{13} and of (b) $R_{\pi K}$ defined in eq. (5). In both panels, the Wilson twisted mass is being considered.

to Refs. [18, 19] for an account on the application of this mixed action approach to the study of leptonic and semileptonic decays of charmed mesons.

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