A spin-charge flip symmetric fixed point in 2+1d with massless Dirac fermions

Hanqing Liu[1,*]

Department of Physics, Duke University,
Box 90305, Durham, NC 27708, USA

E-mail: hanqing.liu@duke.edu

We study a quantum phase transition of electrons on a two-dimensional square lattice. Our lattice model preserves the full O(4) symmetry of free spin-$\frac{1}{2}$ Dirac fermions on a bipartite lattice. In particular, it not only preserves the usual SO(4) (spin-charge) symmetry like in the half-filling Hubbard model, but also preserves a $\mathbb{Z}_2$ spin-charge flip symmetry. Using sign-problem-free Monte Carlo simulation, we find a second order quantum phase transition from a massless Dirac phase to a massive phase with spontaneously chosen spin order or charge order, which become simultaneously critical at the critical point. We analyze all the possible 4-fermion couplings in the continuum respecting the lattice symmetry, and identify the terms whose effective potential in the broken phase is consistent with the numerical results. Using renormalization group calculations in the continuum, we show the existence of the new spin-charge flip symmetric fixed point and calculate its critical exponents.

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[*]Speaker

1. Introduction

Understanding the mechanism of mass generation in 2 + 1d relativistic fermions is of interest in both condensed matter physics [1–3] and high energy physics [4–6]. One of the most important mechanisms is through spontaneous symmetry breaking driven by strong four-fermion interactions. The study of 2 + 1d relativistic fermions also leads to the idea of deconfined quantum criticality [2, 7–9] and emergent symmetry [10–14].

Due to the non-perturbative nature of this problem, it is important to design lattice models which are amenable to sign-problem-free Monte Carlo simulations. Therefore we study the mass generation in a model of spin-$\frac{1}{2}$ Dirac fermions on a two-dimensional square lattice, which can be simulated efficiently with the fermion bag algorithm [15, 16]. This model is a natural generalization of a 1 + 1d model we studied earlier [17, 18]. The model is not only invariant under the SO(4) symmetry of the Hubbard model at half-filling, but more importantly, it also has an additional $\mathbb{Z}_2$ spin-charge flip symmetry, which combines with the SO(4) symmetry to form an O(4) symmetry [19]. This $\mathbb{Z}_2$ symmetry protects the renormalization group (RG) flow from leaving the spin-charge flip symmetric subspace, allowing us to explore a new fixed point without fine-tuning. By tuning a single coupling, our model undergoes a quantum phase transition from a massless Dirac fermion phase to a massive phase with either an anti-ferromagnetic (spin) order or a superconducting-CDW (charge), and they become simultaneously critical at the spin-charge flip symmetric fixed point, as confirmed by the Monte Carlo simulation. If we add a Hubbard coupling which breaks the $\mathbb{Z}_2$ symmetry, our model will flow to the usual spin or charge fixed points, which can be described by the "chiral Heisenberg university class" [20–28]. This contribution will focus on the continuum analysis of the model, while the numerical results can be found in [29, 30].

This contribution is organized as follows. In Section 2, we write down the lattice Hamiltonian and identify its symmetries, and in Section 3, we map the symmetries of the Hamiltonian to the continuum Lagrangian. Then all the independent four-fermion couplings in the continuum respecting those lattice symmetries are constructed in Section 4. In Section 5, we identify the relevant interactions whose effective potential in the broken phase is consistent with our numerical results. Finally in Section 6, we calculate the β functions, confirm the existence of a new spin-charge flip symmetric fixed point and evaluate the critical exponents in $4 - \varepsilon$ dimension.

2. The lattice Hamiltonian and its symmetries

The lattice model we study can be described by the Hamiltonian

$$H = - \sum_{\langle ij \rangle} \exp \left(\kappa \eta_{ij} \sum_{\alpha} (c_{i\alpha}^{\dagger} c_{j\alpha} + c_{j\alpha}^{\dagger} c_{i\alpha}) \right), \tag{1}$$

where $\langle ij \rangle$ means i and j are nearest neighbor sites on a square lattice, $\alpha = 1, 2$, η_{ij} are phases that create the π-flux, κ is the coupling of the model. If we expand the exponent, H can be written in a more conventional form

$$H \propto - \sum_{\langle ij \rangle} \prod_{\alpha} \left[- t\eta_{ij}(c_{i\alpha}^{\dagger} c_{j\alpha} + c_{j\alpha}^{\dagger} c_{i\alpha}) + V \left(n_{i\alpha} - \frac{1}{2} \right) \left(n_{j\alpha} - \frac{1}{2} \right) - \frac{t^2}{V} \right], \tag{2}$$

where $V/t = 2 \tanh \frac{\kappa}{2}$. The original form of H in Eq. (1) makes it clear that each bond of the Hamiltonian is only a function of the free hopping term, while from Eq. (2) we can see that each bond is a product of the $t - V$ Hamiltonian [15, 16]. If we further expand the terms in Eq. (2) into the quadratic, quartic and higher order terms we get

$$H \propto - \sum_{\langle ij \rangle} \left[t\eta_{ij} \sum_\alpha (c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha}) + \frac{V}{2} \left(\sum_\alpha (c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha}) \right)^2 + \text{(6th and 8th orders)} \right]. \quad (3)$$

In this form, we can identify the quadratic and quartic terms to the ones in the model studied in [14] with $V = J/2$ in their notation.

Since each bond in this Hamiltonian is exponential of the free hopping term, it has all the space-time and internal symmetries of the free Hamiltonian: spatial translations by one unit $T_a^{1,2}$, $\mathbb{Z}_4$ rotation symmetry R, parity P, time-reversal Θ and charge conjugation C, an $SU(2)_s \times SU(2)_c$ spin-charge symmetry, or equivalently, $SO(4)$ symmetry, which is manifest in the Majorana Language, and most importantly, a $\mathbb{Z}_2$ spin-charge flip symmetry, or equivalently, charge conjugation on a single layer, which enhances the internal symmetry to $O(4)$.

From the viewpoint of Wilson RG, all interactions respecting the symmetries of the lattice Hamiltonian can be generated in the continuum. Therefore it is important to know how the symmetries of the Hamiltonian are mapped to the continuum. We can understand this by using the free lattice Hamiltonian which we will do next.

3. Embedding lattice symmetries in the continuum

Let us consider free staggered fermions on a square lattice, given by the first term in Eq. (3). Linearizing the dispersion relation of this Hamiltonian near the Fermi points, we get the following continuum Hamiltonian

$$H_0 = - \int d^2x \ i\psi_\alpha^\dagger (\Gamma^1 \partial_1 + \Gamma^2 \partial_2) \psi_\alpha, \quad (4)$$

where $\Gamma^1 = \sigma^3 \otimes \sigma^3$ and $\Gamma^2 = \sigma^3 \otimes \sigma^1$ come from the four corners of the Brillouin zone. Using Grassmann coherent fermion path integral, we can rewrite Eq. (4) as the following Euclidean Lagrangian density

$$\mathcal{L}_0 = -\bar\psi_\alpha \gamma^\mu \partial_\mu \psi_\alpha, \quad (5)$$

where ψ_α is a 4-component Dirac fermion, $\bar\psi_\alpha = \psi_\alpha^\dagger \gamma^0$, $\alpha = 1, 2$, and $\mu = 1, 2, 3$. Here $\gamma^{0,1,2,3,5}$ are five 4×4 Hermitian matrices satisfying the Clifford algebra $\{\gamma^i, \gamma^j\} = 2\delta^{ij} \mathbb{1}_4$. We can choose $\gamma^{0,3,5}$ to be real and $\gamma^{1,2}$ to be imaginary. One basis consistent with $\Gamma^{1,2}$ in Eq. (4) is given by

$$\gamma^0 = \sigma^1 \otimes \mathbb{1}, \ \gamma^1 = \sigma^2 \otimes \sigma^3, \ \gamma^2 = \sigma^2 \otimes \sigma^1, \ \gamma^3 = \sigma^3 \otimes \mathbb{1}, \ \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \sigma^2 \otimes \sigma^2. \quad (6)$$

Space-time transformations on the lattice mix Dirac components in the continuum as follows

$$T_a^{1,2} : \psi \mapsto i\gamma^{3,5}\psi, \ R : \psi \mapsto e^{i\frac{\pi}{4}(i\gamma^1\gamma^2 + i\gamma^3\gamma^5)}\psi, \ P : \psi \mapsto i\gamma^5\gamma^1\psi, \ \Theta : \psi \mapsto \gamma^0 K\psi, \quad (7)$$

where K is the complex conjugation operator. Except for R, all of them act as $\mathbb{Z}_2$ symmetries on fermion bilinears. R acts as a $\mathbb{Z}_4$ symmetry on the lattice, and will be enhanced to an $SO(2)_R$ internal symmetry

$$\tilde{R} : \psi \mapsto e^{i\frac{\theta}{2}(i\gamma^3\gamma^5)}\psi \tag{8}$$

in the continuum. When analyzing the internal symmetries, especially the charge symmetry, it is more convenient to use Majorana representation $\psi_1 = \xi_1 - i\xi_2$, $\psi_2 = \xi_3 - i\xi_4$, and

$$\mathcal{L}_0 = -\xi_a^T \gamma^0 \gamma^\mu \partial_\mu \xi_a, \tag{9}$$

where $a = 1, 2, 3, 4$, and each ξ_a is a four-component Majorana fermion. Clearly Eq. (9) has an $O(4)$ symmetry, which is nothing but the spin-charge symmetry $SO(3)_s \times SO(3)_c$ and the spin-charge flip symmetry $\mathbb{Z}_2^{sc}$ on the lattice. In fact, it turns out that this continuum Lagrangian actually has an $O(8)$ symmetry. Thus the lattice realizes the subgroup $SO(3)_s \times SO(3)_c \times \mathbb{Z}_2^{sc} \times SO(2)_R$ of this $O(8)$ symmetry in the continuum.

4. Interactions respecting the lattice symmetries

In this section, we analyze all the Lorentz-invariant interactions allowed by the lattice symmetries. First, the allowed four-fermion interactions must be singlets under $SO(3)_s \times SO(3)_c \times SO(2)_R$, and they can be constructed from fermion bilinears, including space-time (pseudo-)scalars, i.e., masses, and space-time (pseudo-)vectors, i.e., currents. The fermion bilinears form reducible representations of the symmetry group $SO(3)_s \times SO(3)_c \times SO(2)_R$, and decompose into irreducible representations (irreps) as 36 masses,

$$\mathbf{36} = (\mathbf{3} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{3}) \otimes \mathbf{1} + (\mathbf{3} \otimes \mathbf{3} + \mathbf{1} \otimes \mathbf{1}) \otimes (\mathbf{2} + \mathbf{1}), \tag{10}$$

which agrees with a previous work [1], and 28 currents,

$$\mathbf{28} = (\mathbf{3} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{3}) \otimes (\mathbf{2} + \mathbf{1}) + (\mathbf{3} \otimes \mathbf{3} + \mathbf{1} \otimes \mathbf{1}) \otimes \mathbf{1}. \tag{11}$$

If we also take into account the $\mathbb{Z}_2$ lattice space-time symmetries, no mass terms are invariant under all of them, and therefore our continuum theory cannot have any mass terms. Building singlets from bilinear irreps that transform according to Eqs. (10) and (11), we get 6 Gross-Neveu couplings and 6 Thirring couplings, and these $\mathbb{Z}_2$ lattice symmetries are automatically satisfied, while the spin-charge flip symmetry $\mathbb{Z}_2^{sc}$ flips some of those terms. However, due to the Fierz identity, only 4 of these 12 couplings are independent, and remarkably, they can all be chosen to be Gross-Neveu couplings,

$$\mathcal{L}_s = \frac{g_s^2}{2}|\vec{M}_s|^2, \quad \mathcal{L}_c = \frac{g_c^2}{2}|\vec{M}_c|^2, \quad \mathcal{L}_R = \frac{g_R^2}{2}|\vec{M}_R|^2, \quad \mathcal{L}_{\text{singlet}} = \frac{g_{\text{singlet}}^2}{2}M_{\text{singlet}}^2, \tag{12}$$

where

$$\vec{M}_s = \bar{\psi}_\alpha \vec{\sigma}_{\alpha\beta} \psi_\beta, \quad \vec{M}_c = (\psi_2^T \gamma^0 \psi_1 + \bar{\psi}_1 \gamma^0 \bar{\psi}_2^T, i(\psi_2^T \gamma^0 \psi_1 - \bar{\psi}_1 \gamma^0 \bar{\psi}_2^T), \bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2),$$
$$\vec{M}_R = (\bar{\psi}_\alpha i\gamma^3 \psi_\alpha, \bar{\psi}_\alpha i\gamma^5 \psi_\alpha), \quad M_{\text{singlet}} = \bar{\psi}_\alpha i\gamma^3\gamma^5 \psi_\alpha. \tag{13}$$

For example, in [14], the authors use $\mathcal{L}_R$ built from $\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{2}$ to study a phase transition from Dirac phase to Kekulé valance-bond-solid (VBS) phase.

5. The continuum model and the effective potential

From our Monte Carlo results [29, 30], we see either anti-ferromagnetic (spin) order or superconducting-CDW (charge) order at strong couplings, but no VBS order, i.e., $|\langle \vec{M}_s \rangle| \neq 0$ or $|\langle \vec{M}_c \rangle| \neq 0$, but $|\langle \vec{M}_R \rangle| = 0$. Therefore we believe our model represents a lattice regularization of the Gross-Neveu model with spin and charge couplings given by the Lagrangian density

$$\mathcal{L}_{\text{GN}} = \mathcal{L}_0 + \mathcal{L}_s + \mathcal{L}_c. \tag{14}$$

Furthermore, the $\mathbb{Z}_2^{sc}$ spin-charge flip symmetry of the lattice model imposes the restriction that $g_s^2 = g_c^2$. Adding interactions to the lattice model that breaks the spin-charge flip symmetry, like the Hubbard coupling, would lead to $g_s^2 \neq g_c^2$.

We can confirm the expected symmetry breaking pattern of the above interaction by calculating the one-loop effective potential. In order to do so, we introduce auxiliary scalar fields $\vec{\phi}_s$ and $\vec{\phi}_c$ which transform in the **3**⊗**1**⊗**1** and **1**⊗**3**⊗**1** representations respectively, and rewrite the Lagrangian as,

$$\mathcal{L}_{\text{GN}}^{\text{aux}} = -\bar{\psi}_\alpha \gamma^\mu \partial_\mu \psi_\alpha + \frac{1}{2g_s^2}\phi_s^2 + \frac{1}{2g_c^2}\phi_c^2 + \phi_s(\bar{\psi}^1\psi^1 - \bar{\psi}^2\psi^2) + \phi_c(\bar{\psi}^1\psi^1 + \bar{\psi}^2\psi^2), \tag{15}$$

where we have rotated $\vec{\phi}_s$ and $\vec{\phi}_c$ such that $\phi_s^{1,2} \equiv \phi_c^{1,2} \equiv 0$, and relabeled $\phi_s^3 \to \phi_s$ and $\phi_c^3 \to \phi_c$, because the boson fields will be treated as constants in space-time. By integrating out the quadratic fermions, we get the following effective potential of the ϕ_s and ϕ_c fields [29],

$$\frac{1}{S_d}V_{\text{eff}}[\phi_s, \phi_c]$$
$$= \frac{2\pi}{3}|\phi_c - \phi_s|^3 - \frac{4}{3}(\phi_c - \phi_s)^3 \tan^{-1}\frac{\phi_c - \phi_s}{\Lambda} + \frac{2\pi}{3}|\phi_c + \phi_s|^3 - \frac{4}{3}(\phi_c + \phi_s)^3 \tan^{-1}\frac{\phi_c + \phi_s}{\Lambda}$$
$$- \frac{8}{3}\Lambda(\phi_c^2 + \phi_s^2) - \frac{2}{3}\Lambda^3 \log\left(1 + 2\frac{\phi_c^2 + \phi_s^2}{\Lambda^2} + \frac{(\phi_c^2 - \phi_s^2)^2}{\Lambda^4}\right) + \frac{1}{2S_d g_s^2}\phi_s^2 + \frac{1}{2S_d g_c^2}\phi_c^2, \tag{16}$$

where $S_d = \frac{2}{(4\pi)^{d/2}\Gamma(d/2)}$ is the loop integral factor and here we should set $d = 3$, and Λ is the cutoff.

In Fig. 1 we plot the effective potential in the broken phase at $S_d\Lambda g_s^2 = S_d\Lambda g_c^2 = 5$. The critical values of g_s and g_c are at $S_d\Lambda g_s^2 = S_d\Lambda g_c^2 = \frac{1}{8}$. From the effective potential we see that in the broken phase, we have either spin order or charge order, but not both, and therefore the $\mathbb{Z}_2^{sc}$ spin-charge flip symmetry is also spontaneously broken.

6. RG analysis and critical exponents

In order to understand the RG flow and critical properties of the Lagrangian in Eq. (14), we calculate the usual $4 - \varepsilon$ expansion using the corresponding Gross-Neveu-Yukawa Lagrangian

$$\mathcal{L}_{\text{GNY}} = -\bar{\psi}_\alpha \gamma^\mu \partial_\mu \psi_\alpha + g_s\vec{\phi}_s \cdot \vec{M}_s + g_c\vec{\phi}_c \cdot \vec{M}_c$$
$$+ \sum_{a=s,c}\left(\frac{1}{2}\partial_\mu\vec{\phi}_a \cdot \partial^\mu\vec{\phi}_a + \frac{1}{2}m_a^2\vec{\phi}_a \cdot \vec{\phi}_a + \frac{1}{4!}\lambda_a(\vec{\phi}_a \cdot \vec{\phi}_a)^2\right) + \frac{1}{12}\lambda_{sc}(\vec{\phi}_s \cdot \vec{\phi}_s)(\vec{\phi}_c \cdot \vec{\phi}_c). \tag{17}$$
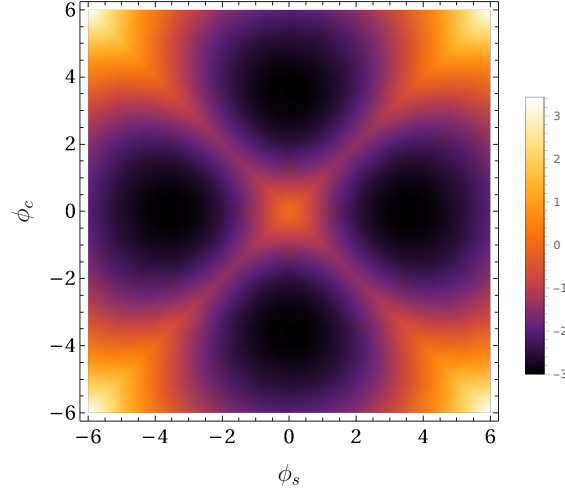
Figure 1: Effective potential in the broken phase at $S_d \Lambda g_s^2 = S_d \Lambda g_c^2 = 5$.

We have calculated the one-loop β functions for the coupling g_s^2, g_c^2, λ_s, λ_c and λ_{sc}, and they are given by [29]

$$\frac{\mathrm{d}g_s^2}{\mathrm{d}\log\mu} = -\varepsilon g_s^2 + S_d\big((2N_f + 1)g_s^4 + 9g_s^2 g_c^2\big), \tag{18}$$

$$\frac{\mathrm{d}\lambda_s}{\mathrm{d}\log\mu} = -\varepsilon\lambda_s + S_d\Big(\frac{11}{6}\lambda_s^2 + \frac{1}{2}\lambda_{sc}^2 + 4N_f g_s^2\lambda_s - 24N_f g_s^4\Big), \tag{19}$$

$$\frac{\mathrm{d}\lambda_{sc}}{\mathrm{d}\log\mu} = -\varepsilon\lambda_{sc} + S_d\Big(\frac{5}{6}(\lambda_s + \lambda_c)\lambda_{sc} + \frac{2}{3}\lambda_{sc}^2 + 2N_f (g_s^2 + g_c^2)\lambda_{sc} - 24N_f g_s^2 g_c^2\Big), \tag{20}$$

where the β functions for g_c^2 and λ_c can be obtained by the spin-charge flip symmetry.

Using these β functions, we plot the RG flows in Fig. 2. From Eq. (18) we see that the Yukawa couplings mix only among themselves, and from Fig. 2a we see that there is a spin-charge flip symmetric fixed point (SC) on the $g_s^2 = g_c^2$ axis, which separates the massless Dirac phase from the broken phase. The $\mathbb{Z}_2^{sc}$ spin-charge flip symmetry prevents the flow from leaving the diagonal $g_s^2 = g_c^2$ axis. Breaking this $\mathbb{Z}_2^{sc}$ symmetry through the Hubbard interaction for example, would drive the flow away from the diagonal axis to the usual spin or charge fixed points, depending on the sign of the Hubbard coupling. Assuming $g_s^2 = g_c^2$ are at SC, the flow diagram of the boson self-interactions in the spin-charge flip symmetric slice is shown in Fig. 2b. There is only one stable fixed point, which can be identified as the SC fixed point.

Evaluating the critical exponents at the spin-charge symmetric fixed point using the $4 - \varepsilon$ expansion, we have [29],

$$\eta = \frac{2}{7}\varepsilon, \quad \eta_\psi = \frac{3}{14}\varepsilon, \quad 1/\nu = 2 - \frac{6}{7}\varepsilon. \tag{21}$$

In the large N_f limit (N_f is the number of values α takes, and in our model $N_f = 2$), we obtain

$$\eta = \varepsilon - \frac{5\varepsilon}{N_f + 5}, \quad \eta_\psi = \frac{3}{2(N_f + 5)}\varepsilon, \quad 1/\nu = 2 - \varepsilon - \frac{3\varepsilon}{N_f + 5}. \tag{22}$$

These exponents are different from those obtained from the Hubbard model [20].

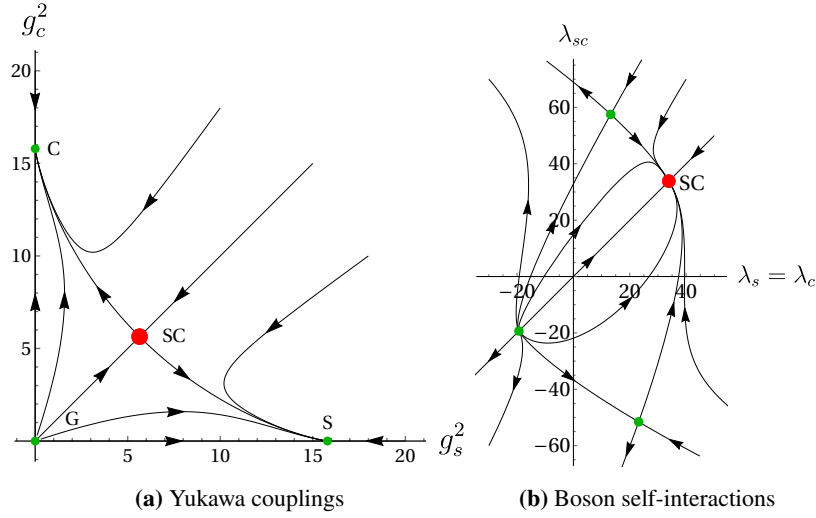**(a)** Yukawa couplings   **(b)** Boson self-interactions

Figure 2: RG flow of the couplings in $4 - \varepsilon$ expansion

7. Conclusions

Our lattice Hamiltonian, which can be solved using the fermion bag approach, has a spin-charge flip symmetry. We have shown that the presence of this additional symmetry leads to a new fixed point that can be reached by tuning a single coupling on the lattice. The fixed point thus describes an interesting phase transition between a massless Dirac fermion phase and a phase featuring spontaneous spin symmetry breaking or charge symmetry breaking, as well as spontaneous spin-charge flip symmetry breaking. Here we uncover the physics of the continuum model by calculating its effective potential and computing the critical exponents using the $4 - \varepsilon$ expansion up to one loop.

Acknowledgments

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