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Towards precision calculation of partonic structure of hadrons from lattice QCD

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Calculating the partonic structure of hadrons from lattice QCD has attracted a lot of interest in the past few years, and now has moved to a stage which calls for precision. Here I discuss some important steps towards such precision calculations.

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1. Introduction

The partonic structure of hadrons plays an important role in understanding experimental data at hadron colliders such as the LHC. In field theory, such structure is characterized by correlation functions along the light-cone, which pose a challenge for calculations from first principles of quantum chromodynamics (QCD). In the past few years, an effective field theory approach–large-momentum effective theory (LaMET)–has been developed to study the partonic structure of hadrons from their physical properties at large momentum [1, 2] (For a recent review see, e.g., Refs. [3, 4]), where the latter can be calculated from systematic approximations to Euclidean QCD such as lattice field theory.

In LaMET applications, one begins with lattice calculations of appropriately chosen spatial correlation functions. Such correlation functions contain both power and logarithmic ultraviolet (UV) divergences which have to be removed by renormalization. After removing all UV divergences, one can take the continuum limit and match the renormalized correlation functions to those in a conventional scheme such as the $\overline{\rm MS}$ scheme. Moreover, a Fourier transform and factorization are also required to convert these renormalized correlation functions to standard parton distributions.

Since its proposal, LaMET has been used in calculating various partonic quantities, including the quark isovector distribution functions, distribution amplitudes, generalized parton distributions, and recently transverse-momentum-dependent distributions, and even higher-twist distributions. For a summary of the relevant work, see, e.g., the reviews [3–5]. However, most of these calculations are either exploratory or using inappropriate extraction strategies, and thus need to be improved to reach an accuracy that matches the accuracy of experimental extractions. This is true, in particular, for those quantities where a huge amount of experimental data has been collected.

In the following, I present some important steps towards precision calculations of the partonic structure of hadrons.

2. Renormalization Strategy of Quasi-LF correlations in LaMET

The first step towards such a goal is to apply an appropriate renormalization strategy. To illustrate the procedure, let us take the collinear quark parton distribution functions (PDFs) as an example. They are defined in terms of the following light-front (LF) correlation function

$$h(\lambda) = \frac{1}{2P^+}\langle P|\overline{\psi}(\lambda n)\Gamma W(\lambda n, 0)\psi(0)|P\rangle\,, \tag{1}$$

through the Fourier transform [6]

$$f(x) = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-ix\lambda}h(\lambda)\,, \tag{2}$$

where Γ is a Dirac matrix, W is a straight lightlike Wilson-line gauge link, and λ is the LF distance. Due to the invariance of the LF under Lorentz boosts along the z-direction, the above correlation function is independent of the external momentum, which is often taken to be zero.

To calculate the collinear PDFs, one starts with lattice calculations of the following quasi-LF correlation

$$\tilde{h}(z, P^z) = \frac{1}{2N}\langle P|\overline{\psi}(z)\Gamma W(z, 0)\psi(0)|P\rangle\,, \tag{3}$$

where $W(z,0)$ is a spacelike straight-line gauge link, and N is a normalization factor depending on the Dirac matrix. On a discrete lattice with spacing a, the nonlocal quark bilinear operator that defines $\tilde{h}(z,P^z)$ can be multiplicatively renormalized as [7–9]

$$\left[\bar{\psi}(z)\Gamma W(z,0)\psi(0)\right]_B = e^{\delta m|z|}Z(a)\left[\bar{\psi}(z)\Gamma W(z,0)\psi(0)\right]_R , \tag{4}$$

up to lattice artifacts [10, 11]. Here the subscript "B" and "R" denote bare and renormalized operators, respectively. In the equation above, there are both z-independent logarithmic and z-dependent linear divergences. The former arises from the renormalization of quark and gluon fields as well as the vertices at the endpoints of the Wilson line, which is included in the factor $Z(a)$, while the latter comes from the Wilson-line self-energy, which is factored into the exponential $e^{\delta m|z|}$ with δm being the "mass correction" .

Given the multiplicative renormalizability, various proposals have been made in the literature [9, 10, 12–18] to renormalize the lattice correlation function $\tilde{h}(z,P^z)$ above, which essentially correspond to dividing by the matrix element of the same correlation operator in a different external state. However, all these renormalization schemes have a common problem that they introduce additional non-perturbative effects at large distance, and thus alter the infrared (IR) properties of the original correlation function [19]. Since renormalization is supposed to remove UV divergences which are perturbative in asymptotically-free theories like QCD, it shall be possible to find a renormalization procedure which does not introduce non-perturbative effects. This is realized in the so-called hybrid scheme [19], where one separates the correlations at short and long distances and renormalizes them separately, and matches both procedures at an intermediate distance z_S. The matching point must lie within $[0,z_{\rm LT}]$ where the leading-twist (LT) approximation for the correlation operator is valid. To be more explicit, the renormalized quasi-LF correlation takes the following form

$$\tilde{h}_R(z,a,P^z) = \frac{\tilde{h}_B(z,a,P^z)}{Z_X(z,a)}\theta(z_S-|z|) + \tilde{h}_B(z,a,P^z)e^{-\delta m|z|}Z_{\rm hybrid}(z_S,a)\theta(|z|-z_S), \tag{5}$$

where we have restored the a dependence in the above equation. At short distance $|z|<z_S$, one can choose, e.g., $Z_X(z,a)=\tilde{h}_B(z,a,P^z=0)$ [15]. At large distance $|z|>z_S$, one chooses the Wilson line mass subtraction scheme which explicitly subtracts linear and logarithmic divergences [7–9, 13] without introducing extra non-perturbative effects.

The mass counterterm δm takes the following form

$$\delta m = m_{-1}(a)/a + m_0 , \tag{6}$$

where m_{-1}(a) is the coefficient of the power divergence and is independent of the specific matrix element, while the a-independent finite term m_0 arises from renormalon effect, etc. [19].

To determine δm, one can fit, for example, the hadron matrix element at large z, where the dominant decay is

$$\tilde{h}(z,a,P^z) \sim \exp(-\delta m|z|) . \tag{7}$$

Of course, the δm fitted this way shall to be independent of P^z. One can, therefore, choose $P^z=0$. In Ref. [20], a somewhat different fit has been performed for hadron matrix elements at various lattice spacings and distances to extract δm.

A convenient way to fix the renormalization factor $Z_{\rm hybrid}$ is to directly matching the renormalized quasi-LF correlations at $z = z_{\rm S}$ from the short and long distance regimes, which is essentially a continuity condition,

$$Z_{\rm hybrid}(z_S, a) e^{-\delta m|z_S|} \tilde{h}_B(z_S, a, P^z) = \frac{\tilde{h}_B(z_S, a, P^z)}{Z_X(z_S, a)}\,, \tag{8}$$

leading to

$$Z_{\rm hybrid}(z_{\rm S}, a) = e^{\delta m|z_S|}/Z_X(z_{\rm S}, a)\,. \tag{9}$$

Of course, one needs to check whether the final result is stable with respect to the choice of z_S.

3. Fourier Transform and Extrapolation to Large Quasi-LF Distance

For finite hadron momentum, lattice calculations of quasi-LF correlations always end up with data at finite $\lambda_{\rm L} = P^z z_{\rm L}$ where $z_{\rm L}$ is usually smaller than the lattice size due to increasing finite volume corrections and worse signal-to-noise ratios at large z. However, to reconstruct the full parton distribution through Fourier transform, we need the correlations at all quasi-LF distances.

At finite momentum, the quasi-LF correlation in general has a finite correlation length (in the $\overline{\rm MS}$ or hybrid scheme) and exhibits an exponential decay at large z. This is similar to the case of density-density [21] or current-current correlation since the quasi-LF correlation can be viewed as the product of two heavy-to-light currents in the auxiliary field formalism [22–25]. As a consequence, its Fourier transform converges fast at finite z or λ, as compared to that of the LF or twist-2 correlation which only decays algebraically at large λ due to the Regge behavior. If $z_{\rm L}$ is large enough such that the quasi-LF correlation falls close to zero, we can do a truncated Fourier transform up to $z_{\rm L}$ to obtain the quasi-PDF, and the resulting systematic uncertainty is negligible compared to other sources.

However, in practical lattice calculations, the choice of z_L is limited by fast-growing errors of quasi-LF correlations. This is particularly true for large hadron momentum. Thus, when we choose a z_L or $\lambda_{\rm L}$ with a target error, the quasi-LF correlation may still have a sizeable nonzero value at that point. In this case, a truncated Fourier transform will lead to an unphysical oscillation and inaccurate small-x result in the quasi-PDF, which is the inversion problem pointed out in Ref. [26]. Several strategies have been adopted in the literature to address this issue, e.g., the Gaussian reweighting method that suppresses the long-range correlations [27], the derivative method [28] which amounts to doing integration-by-parts and ignoring the boundary terms at the truncation point, or the Bayes-Gauss-Fourier transform which reconstructs a continuous form of the quasi-LF correlation over the whole domain by employing Gaussian process regression [29]. However, the assumptions employed in these strategies are not well justified physics wise. In Ref. [19], a physically motivated extrapolation to $\lambda = \infty$ has been proposed for the quasi-LF correlations, based on the knowledge of their asymptotic behavior. Although this extrapolation does not provide a first-principle prediction of the small-x PDF, it helps remove the unphysical oscillation and offers a reasonable way to estimate the systematic uncertainties in this region.

Depending on how large the momentum is, one can use either the exponential or algebraic decay form for the extrapolation [19], which read

$$\tilde{h}(z, P^z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right] e^{-\frac{\lambda}{\lambda_0}},$$
$$\tilde{h}(z, P^z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}, \tag{10}$$

where the terms in the bracket follow from the asymptotic behavior of parton distributions near the endpoints, while the factor $e^{-\lambda/\lambda_0}$ accounts for the finite correlation length of the quasi-LF correlation function, and a, b, c_i, λ_0 are parameters to be determined. This extrapolation strategy has been applied to the study of meson distribution amplitudes [30] and yielded fairly promising results.

4. Factorization and matching

After appropriate renormalization and extrapolation to infinite quasi-LF distance, we are ready to convert the Fourier transformed momentum space distribution

$$\tilde{f}(y, P^z) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda y}\, \tilde{h}_R(z, P^z) \tag{11}$$

to the standard PDF through the factorization formula [1, 31–33]

$$\tilde{f}(y, P^z) = \int_{-1}^{1} dx\, C\Big(\frac{y}{x}, \frac{xP^z}{\mu}\Big) f(x, \mu) + O\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big), \tag{12}$$

where μ is a factorization scale, $\Lambda_{\rm QCD}$ is the hadronic scale. Clearly, the validity of this factorization relies on the smallness of the expansion parameters $\Lambda^2_{\rm QCD}/[y^2(P^z)^2]$ and $\Lambda^2_{\rm QCD}/[(1-y)^2(P^z)^2]$.

The C factor in the above equation can be calculated perturbatively. For the unpolarized isovector quark PDF, this has been done up to the NNLO [18, 34]. However, some preliminary tests [35] seem to indicate that their impact is small.

5. Conclusion

To conclude, we have presented some important steps towards precision calculations of the partonic structure of hadrons, which are expected to play an important role in improving the accuracy of theoretical predictions.

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