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Distribution Amplitudes of K^* and ϕ from Lattice QCD (Lattice Parton Collaboration (LPC))

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In this work, we present the lattice QCD calculation of the distribution amplitudes of longitudinally and transversely polarized vector mesons K^* and ϕ using large momentum effective theory. This investigation was carried out on three ensembles with 2+1+1 flavors of highly improved staggered quarks (HISQ) action, at physical pion mass and $a \approx \{0.06, 0.09, 0.12\}$ fm lattice spacings, choosing three different hadron momenta $\{1.29, 1.72, 2.15\}$ GeV. The bare correlations are renormalized by a hybrid scheme proposed recently. The finial results are derived after the extrapolation to the continuum and infinite momentum limit. The finial results will provide crucial *ab initio* theory inputs for analyzing pertinent exclusive processes.

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1. Introduction

Searching for new physics beyond the Standard Model (SM) is a primary goal of high energy physics nowadays. A unique possibility of doing so is to investigate processes which are highly suppressed in the SM, such as flavor-changing neutral processes in B meson decays. The recent Belle and LHCb experimental analyses [1, 2]on processes $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ have revealed notable tensions between SM predictions and experimental data. To solve these tensions, various new physics interpretations have been proposed. However, without reliable knowledge of LCDAs of K^* and ϕ, the predictions on physical observables are untrustworthy and the new physics interpretations can not be confirmed.

In the low recoil region (high q^2), the $B \to K^*$ and $B_s \to \phi$ form factors can be directly calculated on the lattice (see in Refs. [3, 4]), however these decays at large recoil are also of experimental interests. For instance the P_5' anomaly has attracted many theoretical and experimental attentions [5, 6]. In the latter kinematics region, decay amplitudes are split into short-distance hard kernels and long-distance universal inputs. The universal inputs that enter include the light-cone distribution amplitudes (LCDAs) of the vector mesons K^*, ϕ, which to the leading-twist accuracy, specify the longitudinal momentum distribution amongst the valence quark and antiquark in the meson. While the hard scattering kernel is perturbatively calculable, the LCDAs can only be extracted from nonperturbative methods or from fits to relevant data. A reliable knowledge of LCDAs is essential in making predictions on physical observables, and in particular the transition form factors at large recoil can be typically affected by $O(10\%)$ by the non-asymptotic terms of LCDAs in light-cone sum rules approach [7, 8]. Most of the available analyses to data have made use of estimates based on QCD sum rules [9] or Dyson-Schwinger equation [10], but a first-principle description of LCDAs for the vector (K^*, ϕ) meson is still missing.

Lattice QCD provides an ideal ab *initio* tool to access nonperturbative quantities in strong interaction. Though the lowest moments of LCDAs can be studied by operator expansion, the entire distribution has not been viable until the proposal of large momentum effective theory (LaMET) [11, 12]. In this Letter, we present the first lattice calculation of LCDAs for vector mesons K^*, ϕ using large momentum effective theory. We find that the longitudinal distribution amplitudes tend to be close to the asymptotic form, but the transverse ones deviate rather significantly from the asymptotic form. This finding is different with QCD sum rules estimates.

2. Numerical Setup

The numerical simulation are based on ensembles with $N_f = 2+1+1$ flavors of highly improved staggered quarks (HISQ) action as shown in Table 1, generated by MILC collaboration [13], at physical pion mass and three lattice spacings $\{0.06, 0.09, 0.12\}$ fm, choosing three hadron momenta $P_z = \{1.29, 1.72, 2.15\}$ GeV . The smearing transformation of hyperubi(HYP) fat link [14] is taken to improve the signal-to-noise ratio of simulation. The momentum smeared 2-2-2 grid source are used in calculation, which allows to obtain the even momenta in unit of $2\pi/L$ with ~ 8 times of the statistics. We also repeat the calculation at 8, 6, 4 time slices at three ensembles, fold the data in the two directions of time and further reversed the non local separation direction to double the statistics.

Table 1: Information on the simulation setup. The light and strange quark mass(both valence and sea quark) of the clover action are tuned such that m_π=140 MeV and m_{η_s}=670 MeV.

Ensemble	a(fm)	$L^3 \times T$	$c_{\rm SW}$	$m_{u/d}$	m_s
a12m130	0.12	48× 64	1.05088	-0.0785	-0.0191
a09m130	0.09	64× 96	1.04239	-0.0580	-0.0174
a06m130	0.06	96×192	1.03493	-0.0439	-0.0191

In total, it is equivalent to having $570 \times 8 \times 8 \times 2 \times 2$, $730 \times 8 \times 6 \times 2 \times 2$ and $970 \times 8 \times 4 \times 2 \times 2$ measurements at three ensembles of 0.06, 0.09 and 0.12 fm.

3. LCDAs in LaMET

The leading-twist LCDAs of longitudinally and transversely polarized vector mesons can be defined as:

$$\int d\xi^- e^{-ixp^+\xi^-} \langle 0|\bar{\psi}_1(0)\not{n}_+ U(0,\xi^-)\psi_2(\xi^-)|V\rangle = f_V n_+ \cdot \epsilon \Phi_{V,L}(x), \tag{1}$$
$$\int d\xi^- e^{-ixp^+\xi^-} \langle 0|\bar{\psi}_1(0)\sigma^{+\mu_\perp} U(0,\xi^-)\psi_2(\xi^-)|V\rangle = f_V^T[\epsilon^+ p^{\mu_\perp} - \epsilon^{\mu_\perp} p^+]\Phi_{V,T}(x),$$

where $U(0,\xi^-) = P\exp\left[ig_s \int_{\xi_-}^0 dsn_+ \cdot A(sn_+)\right]$ is the gauge-link along the minus direction on lightcone, n_+ is the unit vector along the plus direction on lightcone, ϵ is the polarization vector of the vector meson, f_V and f_V^T are the decay constants. According to LaMET, the above LCDAs can be derived by the bare equal-time correlations calculated on lattice:

$$\langle 0|\bar{\psi}_1(0)\gamma^t U(0,z\hat{z})\psi_2(z\hat{z})|V\rangle = H_{V,L}(z)\epsilon^t f_V, \tag{2}$$
$$\langle 0|\bar{\psi}_1(0)\sigma_{\nu\rho} U(0,z\hat{z})\psi_2(z\hat{z})|V\rangle = H_{V,T}(z) f_V^T[\epsilon_\nu p_\rho - \epsilon_\rho p_\nu],$$

where the Lorentz indices are chosen as $\{\nu,\rho\} = z, y$, $U(0,z\hat{z})$ is the gauge-link along the z direction. The bare matrix elements $H_{V,\{L,T\}}(z)$ should be renormalized nonperturbatively by an appropriate scheme. In this work, we choose the hybrid scheme [15] proposed recently which can renormalize the factor without introducing extra nonperturbative effects at large z region and distort the IR property of the bare correlations. This scheme works as follows: At short distance $|z| \le z_S$, we choose the RI/MOM scheme which can avoid certain discretization effects at this region,

$$Z(z,a) = \frac{1}{12}\mathrm{Tr}\Big[\big\langle S(p)\big\rangle^{-1} \times \big\langle S(p|z)\big\rangle \gamma_z\gamma_5 \times \prod_n U_z(n\hat{z})S(p|0)\big\langle S(p)\big\rangle^{-1}\gamma_z\gamma_5\Big]_{\substack{p^2=-\mu_R^2,\\ p_z=0}}. \tag{3}$$

At the $|z| > z_S$ region, we apply a gauge-link mass subtraction scheme:

$$H_V^R(z,a,P_z) = \frac{H_V(z,a,P_z)}{Z(z,a)}\theta(z_S - |z|) + H_V(z,a,P_z)e^{-\delta m(\tilde{\mu})z} Z_{\rm hybrid}(z_S,a)\theta(|z| - z_S), \tag{4}$$

where $\tilde{\mu}$ represents the UV and IR intrinsic scale dependence of the gauge-link, and the mass counterterm $\delta m(\tilde{\mu})$ can be extracted by fitting the RI/MOM renormalization factor. The $Z_{\rm hybrid}$

denotes the endpoint renormalization constant which can be determined by imposing a continuity condition at $z = z_S$,

$$Z_{\rm hybrid}(z_S, a) = e^{\delta m(\bar{\mu}) z_S} / Z(z_S, a)\,. \tag{5}$$

The z_S should be chosen at perturbative region, in this work we have chosen 0.24 fm and 0.36 fm and taken their difference as a systematic error. At large z region where the lattice data signal can not reach, we adopt a physics-based extrapolation:

$$H_{V,\{L,T\}}(z, P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right] e^{-\lambda/\lambda_0}, \tag{6}$$

where the exponential decay term comes from the finite momentum of the finite correlation length, and algebraic terms account for the law behavior of the LCDAs at end point region (momentum fraction $x \to 0/1$). The $\lambda = zP_z$ is the quasi light-cone distance, and the other parameters $c_{1,2}, a, b, \lambda_0$ are fitted by the lattice results. This extrapolation will have a reasonable control of uncertainties at large z region without introducing a truncation which will cause unphysical oscillations in momentum space, but with the price of altering the endpoints of LCDAs.

The quasi DAs can be obtained by Fourier transforming the $H_{V,\{L,T\}}(z, P_z)$ to momentum space:

$$\tilde{\Phi}_{V,\{L,T\}}(y, P_z) = \int dz e^{-iyP_z z} H^R_{V,\{L,T\}}(z, P_z). \tag{7}$$

The LCDAs and the quasi-DAs can be related by the factorization formular:

$$\tilde{\Phi}_{V,\{L,T\}}(y, P_z, \mu_R) = \int_0^1 dx\, C_{V,\{L,T\}}(x, y, P_z, \mu_R, \mu)\Phi_{V,\{L,T\}}(x, \mu), \tag{8}$$

where the matching kernel $C_{V,\{L,T\}}$ was derived in RI/MOM scheme in Ref.[15], the μ and μ_R are the generic renormalization scale of LCDAs and quasi DAs.

4. Results

After renormalization in hybrid scheme, we make a phase rotation $e^{izPz/2}$ to the renormalized correlation, therefor the imaginary part directly reflects SU(3) flavor asymmetry between the s quark and u/d quarks. The real part (left panel) of and the imaginary part (right panel) of transversely-polarized K^* with the momentum $P_z = 2.15GeV$ are shown in Fig. 1. As shown in the real part, the data of different lattice spacings are consistent with each other, which means the linear divergence are canceled up to the current numerical uncertainty. The right panel has a positive imaginary part which corresponds to the asymmetry peak at $x < 1/2$ in momentum space. After renormalization and extrapolation in large $\lambda = zP_z$ region, the quasi DAs can be obtained by the Fourier transforming in Eq. 7, and the LCDAs can be obtained by an inverse matching based on Eq. 8. Take the transversely polarized K^* correlator at $P_z = 2.15GeV$ and $a = 009 fm$ as an example, the comparison of quasi-DA and corresponding LCDA are given in Fig. 2. As shown in this figure, there is non-zero tail for the quasi-DA in the unphysical region $x > 0$ or $x < 1$, which becomes much better after the perturbative matching.

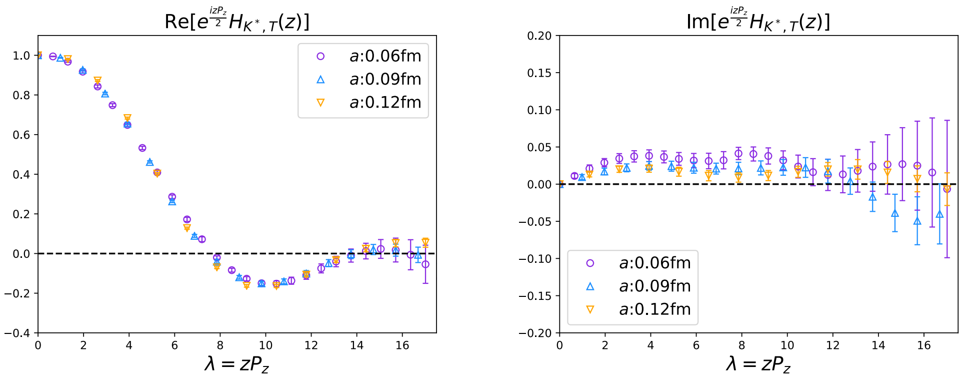

Figure 1: The two-point correlation function for the transversely-polarized K^* in coordinate space. We make a phase rotation by multiplying a factor $e^{izP_z/2}$ with $P_z = 2.15$GeV.

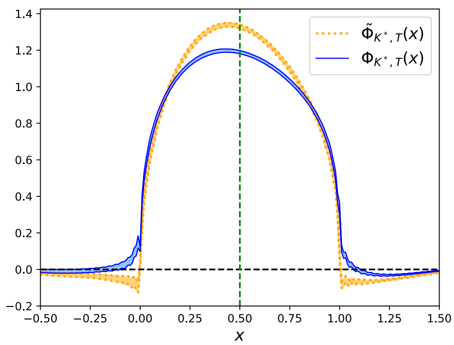

Figure 2: Quasi-DA and LCDA extracted from it for the transversely-polarized K^* using data at $a = 0.09$ fm, $P_z = 2.15$ GeV.

We perform a simple extrapolation to the continuum limit using the results from three different lattice spacings,

$$\psi(a) = \psi(a \to 0) + c_1 a + O(a^2), \tag{9}$$

where the $O(a)$ correction comes from the mixed action of the clover valence fermion in HISQ sea. The three different $P_z = 1.29, 1.72, 2.15 GeV$ results are used for a $P_z \to \infty$ extrapolation with the following form:

$$\psi(P_z) = \psi(P_z \to \infty) + \frac{c_2}{P_z^2} + O\Big(\frac{1}{P_z^4}\Big). \tag{10}$$

After matching from quasi-DAs to LCDAs and performing the above two extrapolations, the final results of K^* and ϕ are shown in Fig. 3 and 4. The endpoint regions of LCDAs are difficult to access precisely in LaMET, a roughly estimate of reliable region can come form the largest attainable λ (the conjugate variable of momentum fraction x). In this work, we have $\lambda_{\max} \approx 14$, therefore the estimated predictable region is $[0.1 < x < 0.9]$. Beyond this region, we plot a shaded area

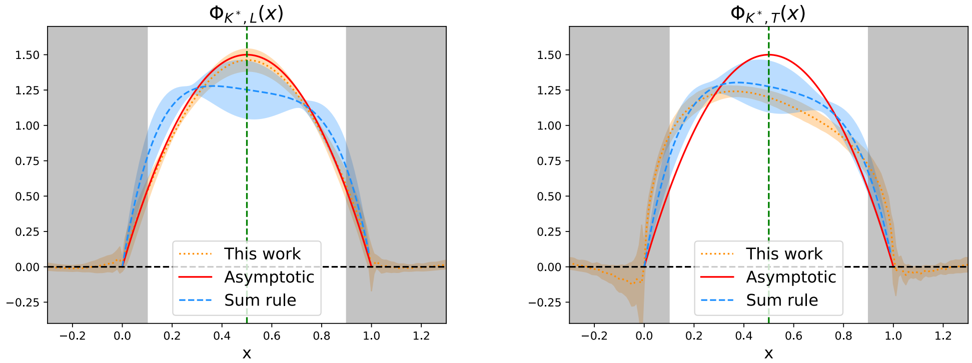

Figure 3: LCDAs for the longitudinally-polarized K^*(upper panel) and transversely-polarized K^*(lower panel). The results are extrapolated to the continuous limit ($a \to 0$) and the infinite momentum limit ($P_z \to \infty$). Regions with $x < 0.1, x > 0.9$ are shaded, as systematic errors in these regions are difficult to estimate.

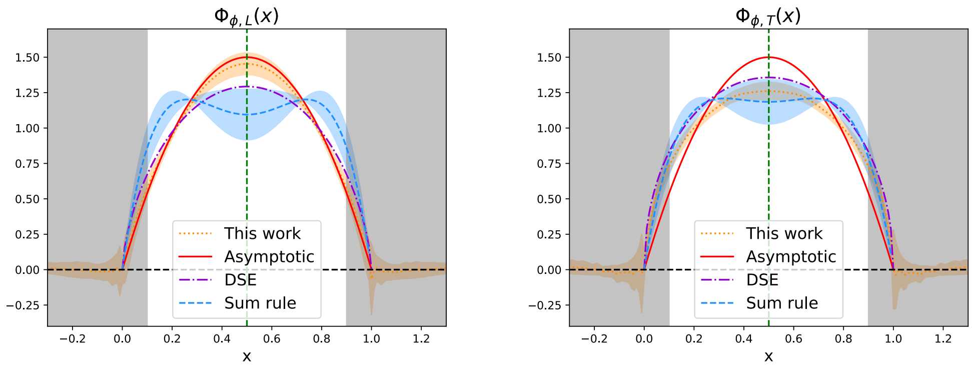

Figure 4: Similar to Fig. 3, but for the ϕ vector meson.

where the systematic errors are difficult to estimate. As a comparison, we also plot the asymptotic form $6x(1-x)$, the results from earlier QCD sum rule [9] and the results from Dyson-Schwinger equations(DSE) [10]. As shown in these figures, the results of longitudinal LCDAs are much similar to the asymptotic form while the transverse LCDAs have a relatively significant deviation from the asymptotic form. The transverse part of K^* LCDAs exhibits a peak at $x < 1/2$ which reflects the SU(3) flavor asymmetry between the s quark and u/d quark.

5. Conclusions

We present the first lattice calculation of LCDAs for vector mesons K^*, ϕ in LaMET using the hybrid renormalization scheme. The continuum and infinite momentum limit are taken based on results from three lattice spacings and momenta at physical pion mass. Comparing with the asymptotic form and QCD sum rule results, the longitudinal LCDAs in our results tend to be close to the asymptotic form while the transverse ones have relatively large deviations. Our final results can provide crucial *abinitio* theory inputs for analyzing relevant exclusive processes.

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