Lattice QCD results for the topological up-quark mass contribution: too small to rescue the $m_u = 0$ solution to the strong CP problem

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A vanishing Yukawa coupling of the up quark could in principle solve the strong CP problem. To render this solution consistent with current algebra results, the up quark must receive an alternative mass contribution that conserves CP symmetry. Such a contribution could be provided by QCD through non-perturbative topological effects, including instantons. In this talk, we present the first direct lattice computation of this topological mass contribution, using gauge configurations generated by the Extended Twisted Mass collaboration. We use the Iwasaki gauge action, Wilson twisted mass fermions at maximal twist, and dynamical up, down, strange and charm quarks. Our result for the topological mass contribution is an order of magnitude too small to account for the phenomenologically required up-quark mass. This rules out the "massless" up-quark solution to the strong CP problem, in accordance with previous results relying on χPT fits to lattice data. The talk is based on Ref. [1], where more details can be found.

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1. Introduction

The strong CP problem is one of the most fundamental open questions of the Standard Model (SM) of particle physics. Its origin is the CP-violating θ-term in the Lagrangian of quantum chromodynamics (QCD) that describes the theory of strong interactions,

$$\mathcal{L}_{\text{QCD}} \supset -\frac{1}{16\pi^2} \theta\, G_{\mu\nu} \tilde{G}^{\mu\nu}. \tag{1}$$

This θ-term originates from the super-selection sectors of the topologically nontrivial QCD vacuum, which are labeled by an angular parameter θ [2, 3]. Here, $G^{\mu\nu}$ denotes the gluon field strength and $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ its Hodge dual. The totally antisymmetric Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ changes sign under parity transformations, which is the reason why the θ-term violates the combined CP symmetry of charge (C) and parity transformations (P).

As $G_{\mu\nu}\tilde{G}^{\mu\nu}$ is a total derivative, one may naively expect that the θ-term disappears after integrating over the Lagrangian in Eq. (1). However, the integral $\int d^4x\, G_{\mu\nu}\tilde{G}^{\mu\nu} \neq 0$ gets nonzero contributions from quantum corrections [4] and instantons [2], which are topologically nontrivial field configurations that describe tunneling between the different QCD vacua. These topologically nontrivial phenomena are known to contribute to the mass of the η' meson [5, 6]. Thus, the same nonperturbative effects that give a large mass to the η' meson are also expected to give rise to strong CP violation via Eq. (1). However, there are strong experimental constraints on CP-violating effects in QCD. These stem from the electric dipole moment d_n of the neutron, which is experimentally excluded down to $|d_n| < 3.0 \times 10^{-13}$ e fm [7–9]. This translates into a strong upper bound on the angle $|\theta| < 1.3 \times 10^{-10}$ due to $d_n \propto m_q \theta$ [10, 11]. Thus, the θ-parameter needs to be strongly fine-tuned, which is the essence of the strong CP problem. The requirement of strong fine-tuning often hints towards the existence of new physics beyond the SM, in particular towards new symmetries.

One possible solution to the strong CP problem is the Peccei-Quinn (PQ) mechanism [12], which relies on the existence of a chiral $U(1)_{\text{PQ}}$ symmetry that is anomalous under the QCD gauge group. In this mechanism, the QCD θ-term gets absorbed by rephasing the axion particle a, which is the pseudo-Goldstone boson of the spontaneously broken $U(1)_{\text{PQ}}$ symmetry [13, 14],

$$a \to a + \text{const.}, \qquad \frac{a}{f_a} \to \frac{a}{f_a} - \theta. \tag{2}$$

Here, f_a is the axion decay constant, and the absorption in Eq. (2) can happen because the $U(1)_{\text{PQ}}$ symmetry is explicitly broken by the Adler-Bell-Jackiw (ABJ) anomaly of QCD [15, 16].

Within the SM, the simplest realization of an anomalous chiral PQ symmetry $U(1)_{\text{PQ}}$ could be achieved if one of the quark flavors, for example the up quark, had no Yukawa coupling to the Higgs doublet. The resulting $U(1)$ symmetry is perturbatively safe and only nonperturbatively broken, which means that it is a true symmetry with regard to 't Hooft's technical naturalness argument [17]. In case of a vanishing up-quark Yukawa coupling to the Higgs doublet, the anomalous chiral PQ symmetry would be an axial $U(1)_u$ symmetry acting on the up quark,

$$u \to e^{i\alpha\gamma_5} u, \qquad \theta \to \theta + \alpha, \tag{3}$$

where we combined the left-handed (u_L) and right-handed (u_R) components of the up quark into a single Dirac fermion u. As before, due to the ABJ anomaly, the vacuum θ-angle can be removed

by performing the chiral transformation (3) and thus becomes unobservable. This scenario is sometimes presented as being fundamentally different from the PQ case, but it actually represents a particular version of the PQ solution: the chiral symmetry (3) is spontaneously broken by the QCD up-quark condensate and the role of the axion is played by the η' meson (see, e.g., [18–20]).

This so-called "massless up-quark solution" would be the simplest solution to the strong CP problem, as it does not require any new particles or new fundamental energy scales. However, chiral perturbation theory (χPT) indicates the need for a nonzero up-quark mass that breaks the chiral symmetry in Eq. (3). In Refs. [21–24], it was proposed that this nonzero up-quark mass could be generated through the same nonperturbative QCD effects that also contribute to the η' mass. This would imply that the up-quark mass in the chiral Lagrangian has two different contributions. First, the perturbative, CP-violating contribution m_u from the Yukawa coupling to the Higgs doublet, which could be easily set to zero by an accidental symmetry [24–28]. Second, the nonperturbative, CP-conserving contribution m_{eff} from topological effects, such as instantons. Crucially, the latter term does not contribute to the neutron electric dipole moment and therefore could explain the observed up-quark mass without spoiling the solution to the strong CP problem [21–24, 29].

Due to the nonperturbative nature of the topological mass contribution m_{eff}, lattice gauge theory is required to determine its magnitude [24, 30, 31]. Previous lattice computations have focused on the CP-violating mass contribution m_u, demonstrating that it is non-vanishing, $m_u(2 \text{ GeV}) = 2.130(41)$ MeV [32–34]. However, these computations relied on fits of the light meson spectrum, and there has been no direct lattice computation of the topological mass contribution itself.

In a recent paper [1], we have filled this gap. Based on a theoretical proposal in Refs. [30, 31], we directly computed the topological up-quark mass contribution by examining the dependence of the pion mass on the dynamical strange-quark mass. This calculation of the topological mass contribution has the advantage of avoiding any fitting procedures. Thus, it provides a complementary analysis of the $m_u = 0$ proposal and may finally lay it to rest. Note that both a positive and negative assessment of the $m_u = 0$ proposal provides important insights for model building beyond the SM: a positive assessment challenges other proposed solutions, including the axion [12–14] and Nelson-Barr [35, 36] mechanisms, while a negative assessment strengthens the case for these other solutions, which are searched for by several ongoing and planned experiments (see, e.g., Ref. [37]).

2. Method

The topological up-quark mass contribution $m_{\text{eff}} = m_d m_s / \Lambda_{\text{top}}$ (see Fig. 1) is known to be proportional to the down-quark mass m_d and the strange-quark mass m_s [38]. The proportionality constant is the inverse characteristic scale Λ_{top} of the topological effects, which is unknown and needs to be determined by lattice computations. In our work, we study the variation of the pion mass

$$M_\pi^2 = \beta_1(m_u + m_d) + \beta_2 m_s(m_u + m_d) + \text{higher orders},\tag{4}$$

with respect to the strange-quark mass [24]. This variation alters the second term in Eq. (4), which contains both the topological mass contribution (with $1/\Lambda_{\text{top}} < \beta_2$) and higher-order corrections in χPT that are proportional to m_s. The first term in Eq. (4) stays unaltered and can be used as a reference point in the following way. In order to solve the strong CP problem, the CP-conserving
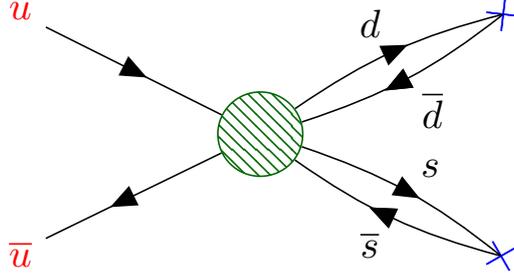
Figure 1: Topological up-quark mass contribution $m_{\text{eff}} = m_d m_s \Lambda_{\text{top}}$, where Λ_{top} is the characteristic scale of the non-perturbative topological vertex (green circle), such as generated by instantons.

topological mass contribution $m_{\text{eff}} = m_d m_s / \Lambda_{\text{top}} < \beta_2 m_d m_s$ must be large enough to mimic the m_u-contribution in the first term of Eq. (4). This gives the following constraint:

$$\frac{\beta_2}{\beta_1} \stackrel{!}{\approx} \frac{m_u}{m_s m_d} \approx 5 \,\text{GeV}^{-1} \tag{5}$$

at renormalization scale $\bar{\mu} = 2$ GeV in the $\overline{\text{MS}}$ scheme [21–24, 30, 31].

In our lattice computation [1], we assume equal and fixed masses of the lightest quarks, $m_u = m_d \equiv m_\ell$, and we vary the strange-quark mass $m_{s,i}$, which determines β_2/β_1 via [30, 31]

$$\frac{\beta_2}{\beta_1} = \frac{M_{\pi,1}^2 - M_{\pi,2}^2}{m_{s,1} M_{\pi,2}^2 - m_{s,2} M_{\pi,1}^2}\bigg|_{M_{\pi,i} \to 0}. \tag{6}$$

Here, $M_{\pi,i} = M_\pi(m_{s,i})$ is the charged pion mass defined in Eq. (4). Note that β_2/β_1 in Eq. (6) only becomes exact after the chiral extrapolation $M_{\pi,i} \to 0$, which ensures the cancellation of higher-order corrections in Eq. (4). As the ratio β_2/β_1 is independent of m_ℓ, we can reliably compute this ratio using pion masses $M_\pi(m_\ell, m_s)$ with m_ℓ larger than its physical value.

3. Lattice computation

In our computation of the topological up-quark mass contribution [1], we used dynamical up, down, strange, and charm quark flavors with degenerate masses of the lightest quarks. Our gauge configurations were generated by the Extended Twisted Mass (ETM) Collaboration, using the Iwasaki improved gauge action [39] and Wilson twisted mass fermions, $\psi(x) \to \exp(-i\omega\gamma_5\tau^3/s)\psi(x)$, $m_\psi \to \exp(i\omega\gamma_5\tau^3)m_\psi$, at maximal twist, $\omega = \pi/2$ [40, 41]. In Table 1, we list all the ensembles, pion masses, and quark masses that we used in our study (for more details, see the supplemental material of [1]). In particular, we used three pairs of ensembles (AX and AXs) with a lattice spacing value of $a = 0.0885(36)$ fm [42] and without a clover term in the action [43], as well as one ensemble (cA211.30.32) with $a = 0.0896(10)$ fm [1] and a clover term [44]. All seven ensembles stem from simulations of several thousand trajectories. This, together with their relatively coarse lattice spacing, ensures that topological sectors are well sampled.

Each of the three pairs of ensembles (AX and its corresponding AXs) has different values for m_s and m_c but otherwise identical parameters. The difference between the three pairs with $X = 60$,

Ensemble	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	aM_π	M_π [MeV]	m_s [MeV]
A60	0.006	0.15	0.190	0.17308(32)	386(16)	98(4)
A60s			0.197	0.17361(31)	387(16)	79(4)
A80	0.008		0.190	0.19922(30)	444(18)	98(4)
A80s			0.197	0.19895(42)	443(18)	79(4)
A100	0.010		0.190	0.22161(35)	494(20)	100(4)
A100s			0.197	0.22207(27)	495(20)	79(4)
cA211.30.32	0.003	0.1408	0.1521	0.12530(14)	276(3)	99(2)
cA211.30.32l		0.1402	0.1529	0.12509(16)	275(3)	94(2)
cA211.30.32h		0.1414	0.1513	0.12537(14)	276(3)	104(2)

Table 1: Parameters of the two different types of ensembles used in our computations. All dimensionful quantities are quoted in units of the lattice spacing a, unless denoted otherwise. Here, μ_ℓ is the bare mass of the light quarks. The parameters μ_σ and μ_δ determine the renormalized strange and charm quark masses via $m_s = (\mu_\sigma/Z_P) - (\mu_\delta/Z_S)$ and $m_c = (\mu_\sigma/Z_P) + (\mu_\delta/Z_S)$, where Z_P and Z_S are the pseudoscalar and scalar renormalization functions, respectively. The pion mass M_π is given both in units of a and in physical units. The strange-quark mass m_s is given at 2 GeV in the $\overline{\text{MS}}$ scheme. Table adapted from Ref. [1].

80, and 100 is the different values for m_ℓ corresponding to $M_\pi = 386$ MeV, $M_\pi = 444$ MeV and $M_\pi = 494$ MeV, respectively. We use these three different pairs of ensembles with different values for m_s and M_π to directly compute β_2/β_1 from Eq. (6).

For the ensemble cA211.30.32, the values for m_s and m_c are similar to the ones for the AX(s) ensembles, but the pion mass is much smaller, $M_\pi = 270$ MeV, thus closer to the physical value. The most crucial difference to the AX(s) ensembles is that the cA211.30.32 ensemble only has a single strange-quark mass value, which prevents a direct computation of β_2/β_1 from Eq. (6). Therefore, we compute the different $M_\pi(m_{s,i})$ and $m_{s,i}$ required in Eq. (6) through reweighting. Here, we call cA211.30.32h (cA211.30.32l) the reweighted ensemble with a 5% higher (lower) value for m_s than the original ensemble cA211.30.32. We also perform a cross-check of the reweighting procedure using the AX(s) ensembles, as shown in Fig. 2. Here, we split the reweighting factor in several steps, which allows us to compute M_π for three intermediate steps (green boxes) between the original values of m_s for the A60 (filled blue circle) and A60s (open blue circle) ensembles.

Thus, using these two different types of ensembles, AX(s) and cA211.30.32, allows us to both test the reweighting procedure and study the M_π-dependence of Eq. (6), which will later enable a reliable chiral extrapolation to determine β_2/β_1 for $M_\pi \to 0$.

4. Results

Our results for the ratio β_2/β_1 are shown in Table 2, which we obtained using Eq. (6) with the input values for M_π and m_s given in lattice units, see Table 1. We denote with cA211.30.32(h) the results obtained with input values from the original ensemble cA211.30.32 and the reweighted ensemble cA211.30.32h. Similarly, cA211.30.32(l) corresponds to the ensembles cA211.30.32 and cA211.30.32l, while cA211.30.32(h,l) corresponds to the ensembles
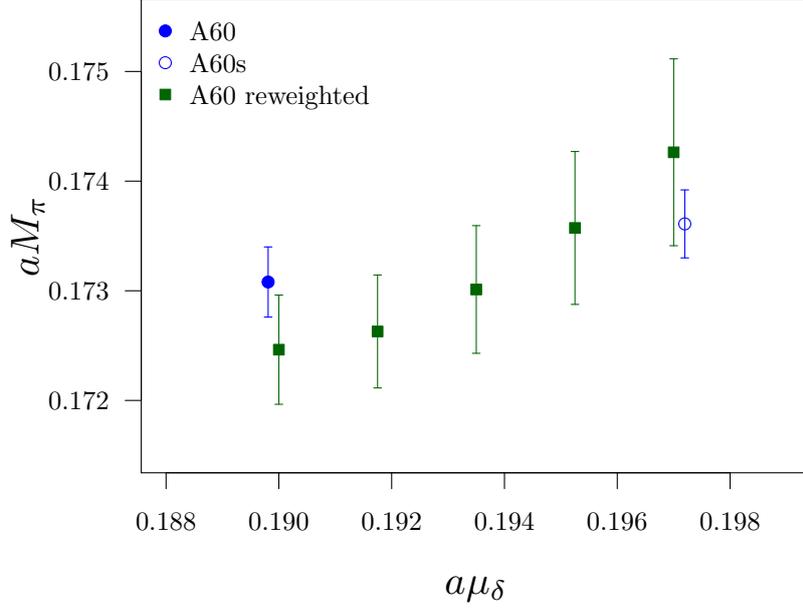
Figure 2: Pion mass M_π as a function of $a\mu_\delta$. Original results for the A60 (filled blue circle) and A60s (open blue circle) ensembles, compared to reweighting results (green boxes). Figure taken from Ref. [1].

cA211.30.32h and cA211.30.32l. Our results for β_1 are strictly positive, which is expected as this coefficient is proportional to the chiral condensate [45],

$$\frac{d\beta_1}{dm_s} \propto \frac{d}{dm_s}\langle\bar{\psi}\psi\rangle \propto \int_{\lim k\to 0} dx e^{ikx}\langle\bar{\psi}\psi(x), \bar{s}s(0)\rangle. \tag{7}$$

Note that our results for β_2 are compatible with zero, and β_2/β_1 is zero at the 1.5σ level.

In Fig. 3, we plot our results from Table 2 as a function of M_π^2. The blue (red) data points are the results for the AX(s) ensembles (the cA211.30.32 ensemble). The black line is the chiral extrapolation that eliminates higher-order corrections to β_2/β_1, as explained below Eq. (6). We choose a linear extrapolation due to the χPT prediction of [34]

$$\frac{\beta_2}{\beta_1} \approx \frac{\alpha_2}{\alpha_1 + (\alpha_3/\alpha_1)M_\pi^2} \approx \frac{\alpha_2}{\alpha_1} - \frac{\alpha_2\alpha_3}{\alpha_1^3}M_\pi^2, \tag{8}$$

modulo logarithmic corrections. Here, the coefficients $\alpha_{1,2,3}$ are combinations of low-energy constants with $\alpha_1 \gg (\alpha_3/\alpha_1)M_\pi^2$, $M_\pi^2 = \alpha_1 m_\ell + O(\alpha_{2,3})$, and $O(\alpha_{2,3})/(\alpha_1 m_\ell) \approx 0.1$. For the linear fit, we use all three data points from the AX(s) ensemble but only one data point from the cA211.30.32 ensemble (see the filled symbols in Fig. 3), because the three data points for the ensemble cA211.30.32 are strongly correlated. The fit has a p-value of 0.2 and yields a chirally extrapolated result of $\beta_2/\beta_1(M_\pi \to 0) = 0.63(25)$ GeV^{-1}. Taking into account the 1σ statistical uncertainty, this result excludes the value of $\beta_2/\beta_1 \approx 5$ GeV^{-1} that is required to solve the strong CP problem by more than 10σ, see Eq. (5).

To estimate the systematic uncertainties, the discretization errors for β_2/β_1 can be obtained by comparing our lattice results for m_s and M_π to the known continuum extrapolation values for the AX ensembles [42]. The resulting discretization errors are of the order of $5-10\%$, which implies
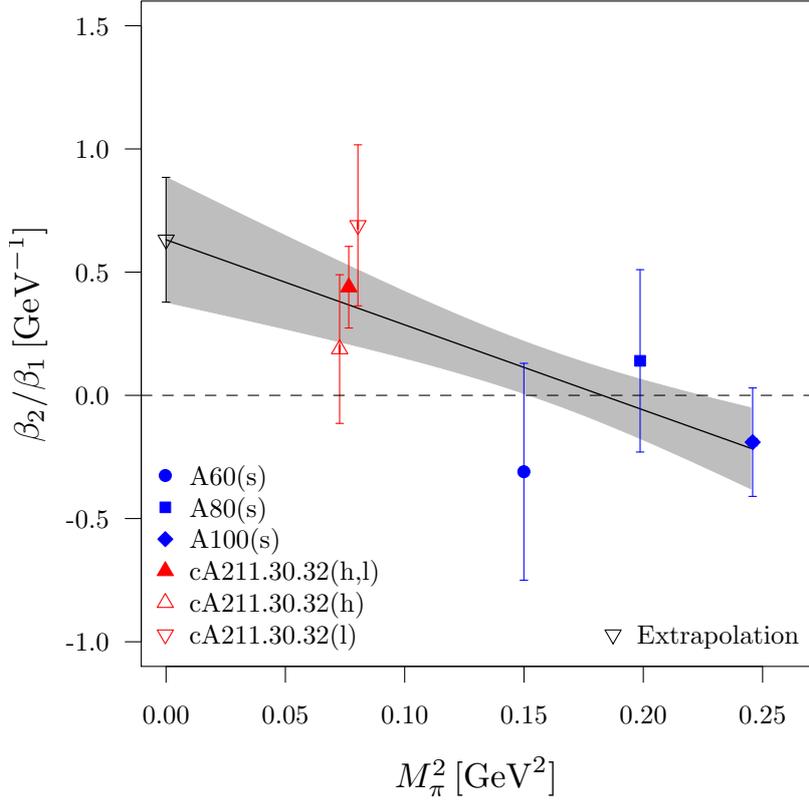
Figure 3: Results for the ratio β_2/β_1 as a function of M_π^2 in physical units. The ensembles are AX(s) without a clover term (blue) and cA211.30.32 with a clover term (red). For better legibility, the red data points are displaced horizontally. The solid line (black) with the 1σ error band (grey) denotes the linear chiral extrapolation in M_π^2, see Eqs. (6) and (8). Figure taken from Ref. [1].

errors of maximally 10% for β_2, 15% for β_1, and 20% for β_2/β_1, because most lattice artifacts cancel in the differences in Eq. (6). For the ensemble cA211.30.32, the lattice artifacts are even further reduced due to the inclusion of the clover term [1]. There are no finite-size effects for m_s, and the finite-size corrections for M_π are equal for $M_{\pi,1}^2$ and $M_{\pi,2}^2$, thus canceling for the ratio β_2/β_1.

In total, our chirally extrapolated value for β_2/β_1 has a 1σ statistical error and a conservatively estimated 20% systematic error from lattice artifacts. Thus, we arrive at the following result [1]:

$$\frac{\beta_2}{\beta_1} = 0.63(25)_{\text{stat}}(14)_{\text{sys}} \text{ GeV}^{-1} = 0.63(39) \text{ GeV}^{-1} \tag{9}$$

at $\bar{\mu} = 2$ GeV in the $\overline{\text{MS}}$ scheme. We note that β_2/β_1 receives logarithmic corrections [45, 46], which are of the same order as our result in Eq. (9); this renders the topological contribution to β_2/β_1 even smaller. We also note that a constant extrapolation in M_π^2 would have been equally well compatible with our data and would have given a substantially smaller result for β_2/β_1. Thus, our result can be considered as a conservative upper bound for β_2/β_1.

Ensemble	β_2 [GeV^2]	β_1 [GeV^3]	β_2/β_1 [GeV^{-1}]
A60(s)	$-0.0009(08)$	$0.0029(4)$	$-0.32(26)$
A80(s)	$0.0005(10)$	$0.0036(4)$	$0.15(30)$
A100(s)	$-0.0010(10)$	$0.0053(6)$	$-0.19(19)$
cA211.30.32(h)	$0.00007(11)$	$0.00039(5)$	$0.18(30)$
cA211.30.32(l)	$0.00026(11)$	$0.00037(5)$	$0.69(33)$
cA211.30.32(h,l)	$0.00033(12)$	$0.00076(5)$	$0.43(16)$

Table 2: Results for the coefficients β_2 and β_1 defined in Eq. (4) and their ratio β_2/β_1. All results are obtained from Eq. (6) and given in physical units at $\bar{\mu} = 2$ GeV in the $\overline{\text{MS}}$ scheme. Table taken from Ref. [1].

5. Conclusion

In our work [1], we have provided the first direct lattice computation of the topological up-quark mass contribution, by studying the dependence of the pion mass on the dynamical strange-quark mass. Using Wilson twisted mass fermions at maximal twist, the Iwasaki gauge action, and gauge configurations generated by the ETM Collaboration, we determined an upper bound for the strength of the topological mass contribution, $\beta_2/\beta_1 < 1.02$ GeV^{-1}, see Eq. (9). Our systematic error estimates are highly conservative, and our result is significantly lower than the value of $\beta_2/\beta_1 \approx 5$ GeV^{-1} required by the massless up-quark solution to the strong CP problem. Thus, our work excludes the massless up-quark solution, in agreement with previous results using direct χPT fits of the light meson spectrum. These findings strengthen the case for alternative solutions to the strong CP problem, including the axion solution, which are highly sought after experimentally.

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PoS(LATTICE2021)352