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H dibaryon away from the $SU(3)_f$ symmetric point

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We present the current status of our efforts in search of H dibaryon on N_f =2+1 CLS ensembles away from the $SU(3)$ flavor symmetric point. Utilizing the distillation framework (also known as LapH) in its exact and stochastic forms, we calculate two-point correlation matrices using large bases of bi-local two-baryon interpolators to reliably determine the low-energy spectra. We report the low lying spectrum on several moving frames for multiple ensembles with different lattice spacing and physical volumes. The status of finite-volume analysis to extract the scattering amplitudes is also discussed.

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1. Introduction

A variety of tetra- and pentaquark states (*e.g.* P_c, T_{cs}, T_{cc}) was discovered in recent years, raising the scientific interest in such systems. Even so, despite various experimental efforts, there are only two six quark systems (deuteron and $d^*(2380)$) that are established to date. The existence of a deeply bound $SU(3)$ flavor singlet dibaryon with scalar quantum numbers, referred to as H dibaryon, was conjectured in 1977 [1]. While there is no concrete experimental evidence in this regard, an upper bound of $\sim$ 7 MeV on the binding energy for such a state relative to the $\Lambda\Lambda$ threshold was reported based on the constraints from the Nagara event [2]. A recent study of the $\Lambda\Lambda$ interactions in p-p and p-Pb collisions also reports results compatible with the existence of a shallow bound state [3]. With higher statistics from future runs at the LHC, the scattering parameters are expected to get constrained further.

The first lattice QCD calculation addressing the existence of a bound H dibaryon was performed in 1985 [4]. Since then, there have been several lattice calculations to date. Apart from the calculations by the Mainz group, calculations with dynamical quarks were performed by only two groups: HALQCD [5] and NPLQCD [6, 7]. The calculation by the HALQCD collaboration was performed along the $SU(3)$ flavor symmetric line with varying pion masses. A calculation by the NPLQCD collaboration with an 800 MeV pion mass along the $SU(3)$ flavor symmetric line finds twice the binding energy as extracted by HALQCD at approximately the same pion mass. The NPLQCD collaboration reported a calculation with broken $SU(3)$ flavor symmetry in the other work. A general observation from these calculations is that the estimates for the binding energy decrease with decreasing pion masses. However, a clear consensus on the existence of such a state in the physical limit from lattice calculations has not been reached.

Lattice results from the Mainz group using N_f=2 ensembles indicate the existence of a bound H dibaryon at heavier than physical pion masses in an $SU(3)$ flavor symmetric and broken setup with a quenched strange quark [8]. Recent results from an extensive study using N_f=2+1 ensembles with five different lattice spacings also point to the existence of a shallow bound state, with significant cut-off dependence in the lattice estimates [9]. These calculations utilize the finite-volume quantization condition *à la* Lüscher to extract the infinite-volume binding energy. The results at the $SU(3)_f$ symmetric point were discussed in a separate talk [10]. In this talk, we present the status of Mainz efforts on H-dibaryon spectroscopy away from the $SU(3)_f$ symmetric point.

2. Methodology

Ensembles: We utilize the $N_f = 2+1$ ensembles generated as a part of the Coordinated Lattice Simulations (CLS) effort. These ensembles have been generated with a nonperturbatively $O(a)$ improved Wilson fermion action and a tree-level $O(a^2)$ improved Lüscher-Weisz gauge action. All ensembles discussed in this talk lie on the $\mathrm{Tr}(m) = 2m_{u/d} + m_s$ = const trajectory that goes through the physical point. The $SU(3)_f$ symmetric point on this trajectory is around m_π = 420 MeV. The valence quarks are realized using nonperturbatively improved Wilson-clover fermions. For those ensembles in which the gauge and fermion fields fulfill open boundary conditions in the time direction, we make the correlator measurements in the bulk of the lattice where the effects of finite temporal extent are sufficiently damped. We distribute the source time slices evenly along the

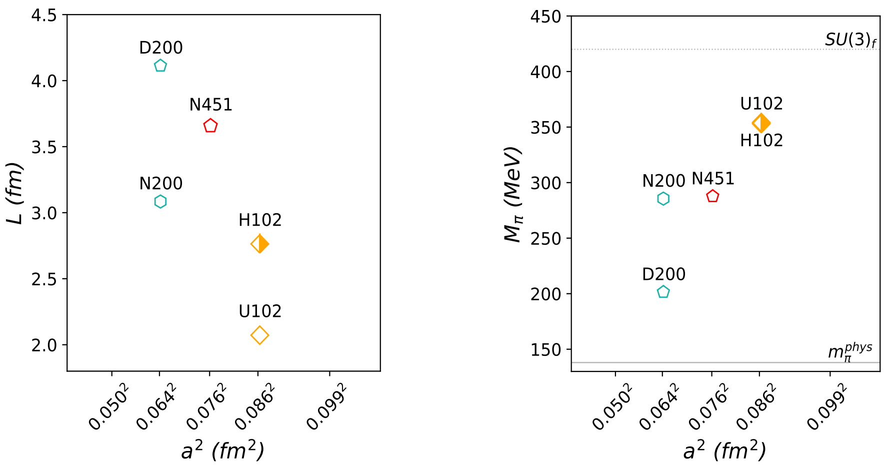

Figure 1: Left: Scatter plot of ensembles with y-axis referring to the physical lattice extension and the x-axis gives the info on the lattice spacings. Right: The same set of ensembles with the y-axis indicating the respective pion masses.

temporal dimension for the rest of the ensembles with periodic boundary conditions. In Figure 1, we show the list of ensembles for which we obtained the results presented here. More ensembles are in our production plan to constrain the infinite-volume physics with good control over systematics.

The left side of Figure 1 is a scatter plot of all the ensembles, with the y-axis referring to the physical lattice extension and the x-axis gives the info on the lattice spacings. The main reason for our choice of ensembles is to extract finite-volume spectra in multiple volumes to constrain the scattering amplitudes more precisely. The same ensembles are also shown with the y-axis indicating the respective pion masses on the right side of Figure 1. The dotted gray line represents the $SU(3)_f$ symmetric case, whereas the solid line at the bottom indicates the physical pion mass limit. As is evident from the figure, we utilize ensembles with different pion masses (equivalently different extents of $SU(3)_f$ symmetry breaking) to investigate the fate of H dibaryon at different physical situations.

ID	β	N_s	N_t	m_π[MeV]	N_{cfgs}	N_{LapH}	N_{tsrc}
U102	3.40	24	128	350	4861	20	5
H102	3.40	32	96	350	2005	48	4
N200	3.55	48	128	280	1712	68	8
N451	3.46	48	128	280	1011	108	8
D200	3.55	64	128	200	2001	448*	1

Table 1: The details of lattice QCD ensembles referred to in this talk. N_{LapH} is the number of Laplacian eigenvectors utilized for the distillation procedure, and N_{tsrc} is the number of source time slices used. *For the D200 ensemble, we utilize the stochastic LapH technique in which the Laplacian eigenvectors are interlaced with 16 dilution projectors and with full spin dilution.

Construction of correlation matrices: We employ the standard distillation technique to evaluate the correlation functions/matrices, except for the D200 ensemble. The large physical volume $V = (4.11\text{ fm})^3$ of the D200 ensemble demands a large number of Laplacian eigenmodes

N_{LapH} to be used in the distillation framework. To this end, the investigations on D200 are performed using the stochastic form of distillation technique to circumvent the huge computational demands due to the use of a large N_{LapH}. In Table 1, we present the relevant details of ensembles for which results are presented in this talk.

Interpolating operators: Throughout these calculations, we utilize only baryon-baryon interpolators in which each baryon is separately projected to definite momentum. The general form of the momentum projected single baryon operators looks like

$$\mathcal{B}_\mu(\mathbf{p},t)[q_1q_2q_3]=\sum_{\mathbf{x}}\epsilon_{abc}[q_1^{aT}(\mathbf{x},t)C\gamma_5P_+q_2^b(\mathbf{x},t)]\ [q_3^c(\mathbf{x},t)]_\mu\ \mathrm{e}^{i\mathbf{x}\cdot\mathbf{p}}. \quad (1)$$

Here C is the charge conjugation operator, and $P_+ = \frac{1}{2}(1+\gamma_0)$ projects the quark fields to positive parity. The two baryon operators are built from these single baryon interpolators using $\Gamma = C\gamma_5P_+$ and $\Gamma = C\gamma_iP_+$ to form the spin-zero and spin-one configurations, respectively, as follows

$$[\mathcal{B}^{(1)}\mathcal{B}^{(2)}](\mathbf{p}_1,\mathbf{p}_2,t) = \mathcal{B}^{(1)}(\mathbf{p}_1,t)\Gamma\mathcal{B}^{(2)}(\mathbf{p}_2,t). \quad (2)$$

At the $SU(3)_f$ symmetric point, the flavor of a system of two octet baryons can be characterized as belonging to the following irreducible representations (irreps), $\mathbf{8}\otimes\mathbf{8} = (\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27})_S\oplus(\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}})_A$ with H dibaryon a scalar in $\mathbf{1}_S$. Away from the $SU(3)_f$ symmetric point, the relevant quantum numbers are strangeness $S = -2$ and isospin $I = 0$, which has contributions from $\mathbf{1}_S$, $\mathbf{8}_S$, and $\mathbf{27}_S$. Using the (S, I) basis for individual baryons, the three relevant scattering channels are $\Lambda\Lambda$, $N\Xi$, and $\Sigma\Sigma$. We systematically include an interpolator for each low-lying noninteracting level from all three channels. Note that unlike $\Lambda\Lambda$ and $\Sigma\Sigma$, $N\Xi$ has nonidentical particles and thus appears in both symmetric and antisymmetric combinations. Owing to the reduced rotational symmetry on the lattice, we ensure that two-baryon operators transform according to the finite-volume symmetry group irreps. Combining flavor, single-baryon momenta, and spin yields a large set of interpolating operators, for which we compute correlation matrices $C_{ij}(t) = \langle O_i(t+t_{\mathrm{src}})O_j^\dagger(t_{\mathrm{src}})\rangle$. Correlation functions for the single baryon operators are also computed to determine the noninteracting finite-volume spectrum.

Spectrum extraction: The finite-volume spectrum is extracted from the correlation matrices by solving the Generalized EigenValue Problem (GEVP)

$$C_{ij}(t)v_j^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)C_{ij}(t_0)v_j^{(n)}(t,t_0). \quad (3)$$

Here the size of the correlation matrix (m) is as large as 28 in some of the finite-volume little group irreps we have considered. In the large time limit, the eigenvalue correlators $\lambda^{(n)}(t,t_0)$ are saturated by the lightest m states and can be shown to have an asymptotic form of $\lambda^{(n)}(t,t_0) \propto e^{-E_nt}$. An early t_0 is chosen such that the noise in $C(t_0)$ does not enter the eigensolutions while also ensuring that the extracted finite-volume spectrum is robust with its variation. The eigenvalues at sufficiently large times are then fit with a single exponential to extract the energy spectrum.

The best fits are chosen based on a comparative study between fits to the eigenvalue correlators $\lambda^{(n)}$ and their ratios $[r^{(n)} = \lambda^{(n)}/(C_{\mathcal{B}^{(1)}}C_{\mathcal{B}^{(2)}})]$ with a nearby noninteracting level $[\mathcal{B}^{(1)}\mathcal{B}^{(2)}]$. In Figure 2, we present the effective energy difference ($\Delta\mathrm{E}_{\mathrm{eff}}$) given by $\ln(\frac{r^{(n)}(t)}{r^{(n)}(t+1)})$ along with the energy splitting estimates from the single exponential fits to $\lambda^{(n)}(t)$ [exp] and $r^{(n)}(t)$ [r-exp], for

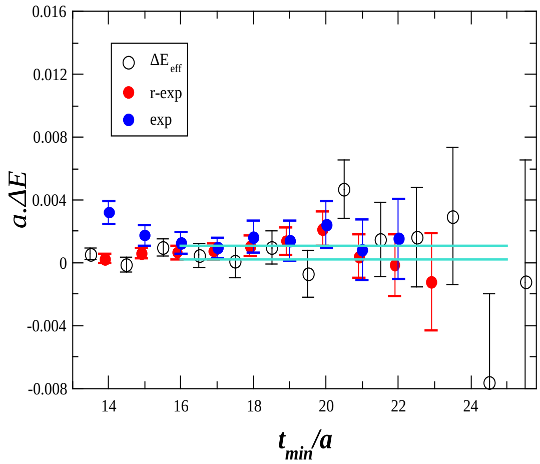

Figure 2: Comparative study of single exponential fits to $\lambda^{(n)}(t)$ [exp] and $r^{(n)}(t)$ [r-exp] for the first excited state in the A_1 irrep of $P^2 = 2$ moving frame in the N200 ensemble. $\Delta E_{\rm eff}$ is the effective energy difference and $t_{\rm min}$ refers to the boundary of the chosen fit range close to the source time slice. The cyan horizontal line indicates the chosen fit.

the first excited state in the $P^2 = 2$ moving frame on the N200 ensemble. The energy splittings from the fits to $\lambda^{(n)}$ are built using the energies for single hadrons determined from separate fits to the single hadron correlators ($C_{\mathcal{B}^{(1)}}$ & $C_{\mathcal{B}^{(2)}}$). Our final choices are generally made with the ratio fits, and such a comparative study ensures that the chosen fit ranges are robust in terms of the ground state signal saturation.

3. Results

In Figures 3, 4, and 5, we present the finite-volume energy spectrum on the five ensembles listed in the previous section. The energy spectrum in the center-of-momentum frame is shown along the y-axis in units of the elastic threshold ($2m_\Lambda$). In these units, the elastic threshold always appears at the value 1. The x-axis refers to the physical lattice size in femtometers, and different panes stand for different finite-volume little group irreps. Upon breaking of the $SU(3)_f$ symmetry, there are three relevant 2-particle scattering channels ($\Lambda\Lambda$, $N\Xi$ and $\Sigma\Sigma$). The black and gray curves show the related noninteracting finite-volume levels. The solid curves refer to $\Lambda\Lambda$, the dashed curves stand for $N\Xi$, and the dot-dashed are $\Sigma\Sigma$. The operators related to the black curves are included in the analysis, and those related to the gray curves are not. The lowest three-particle scattering threshold $N\Xi\pi$ is also shown in the figures.

In Figure 3, we present the finite-volume energy spectrum for the ensembles with m_π = 350 MeV. Due to the proximity of the $SU(3)_f$ symmetric point, the thresholds of the three scattering channels are close to each other. Currently, we have results from two ensembles at the same lattice spacing. The energy spectrum for the m_π = 280 MeV ensembles is shown in Figure 4. In this case, we have data at two different lattice spacings. For the ensemble with a larger physical volume, we have utilized a larger basis of baryon-baryon interpolators to extract an equally large tower of excited states across all the finite-volume irreps. Note that with decreasing pion mass, the extent of $SU(3)_f$ symmetry breaking increases. Consequently the energy splitting between the thresholds

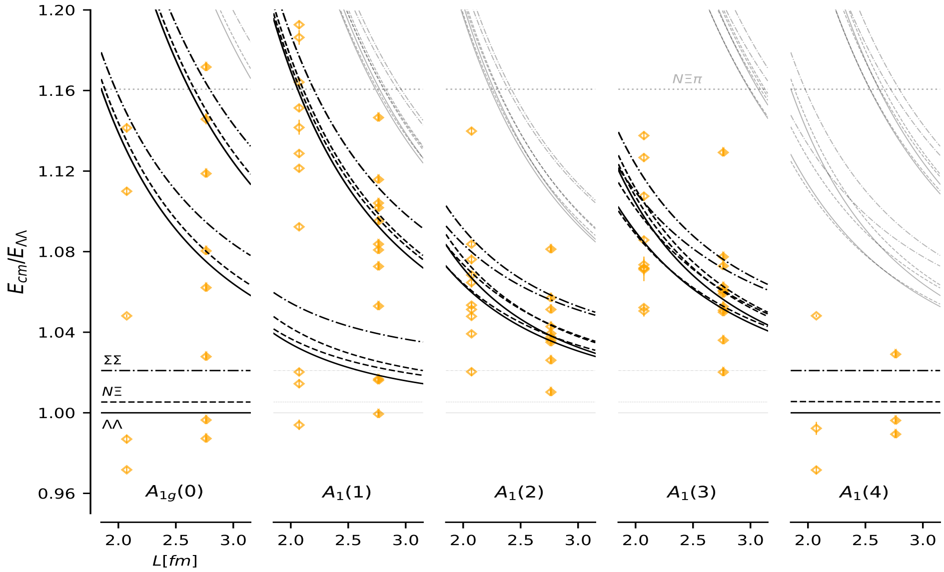

Figure 3: Energy spectrum of $I = 0, S = -2$ dibaryons in the trivial finite-volume irreps (A_1) in the ensembles with $m_\pi \sim 350$ MeV. Half-filled (unfilled) markers refer to the H102 (U102) ensemble.

of two-baryon scattering channels also increases. Larger energy splittings between the scattering channels are evident in the finite-volume spectrum for the ensemble with $m_\pi = 200$ MeV, which is shown in Figure 5.

Following the reliable extraction of the finite-volume energy spectra, the next thing to do is to extract the infinite-volume physics. We follow a procedure to extract the two-particle scattering amplitudes from the finite-volume spectrum through the quantization condition [11]

$$\det(K^{-1} - B) = 0, \tag{4}$$

first derived by Lüscher for elastic scattering of two spinless particles in the rest frame [12]. With three low lying 2-baryon scattering channels ($\Lambda\Lambda$, $N\Xi$ and $\Sigma\Sigma$) in the broken $SU(3)_f$ symmetry scenario, one has to deal with a scattering matrix of dimension >3. Assuming that higher partial wave contributions do not influence the s-wave scattering in the moving frames, one could work with a 3x3 scattering matrix. One could further simplify the problem by assuming that effects from the $\Sigma\Sigma$ channel are negligible. However, the applicability of this assumption is limited to lighter m_π scenarios, owing to the greater extent of $SU(3)_f$ symmetry breaking. $N\Xi$, being a channel with nonidentical particles, allows mixing of spin sectors ($S = 0$ and $S = 1$), which in turn allows for physical mixing of higher partial waves unlike in the $SU(3)_f$ symmetric case. Note that in moving frames, the first higher partial wave that can contribute to the finite-volume spectra is the p-wave. Relaxing the assumptions on neglecting higher partial wave effects complicates the problem of quantization further due to an enlarged scattering matrix.

The extracted finite-volume energy spectra are very dense, and several energy levels are nearly degenerate. Standard procedures such as minimizing the Determinant Residual [13] or a χ^2 defined from the extracted finite-volume energy spectrum and the reconstructed energy spectrum from the zeros of the quantization determinant [14] are reaching their limits with such a dense

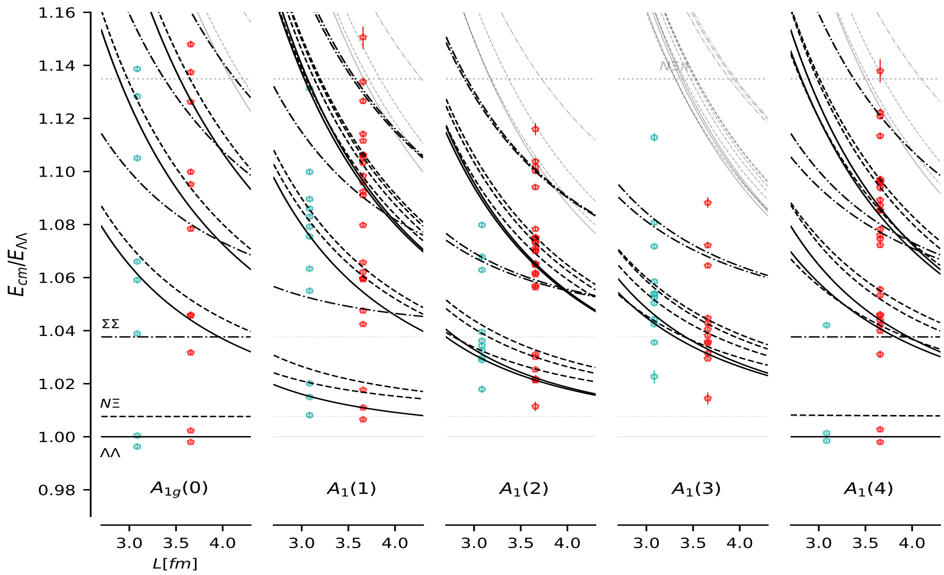

Figure 4: Same as in Figure 3, but for ensembles with $m_\pi \sim 280$ MeV. Red (cyan) markers refer to the N451 (N200) ensemble.

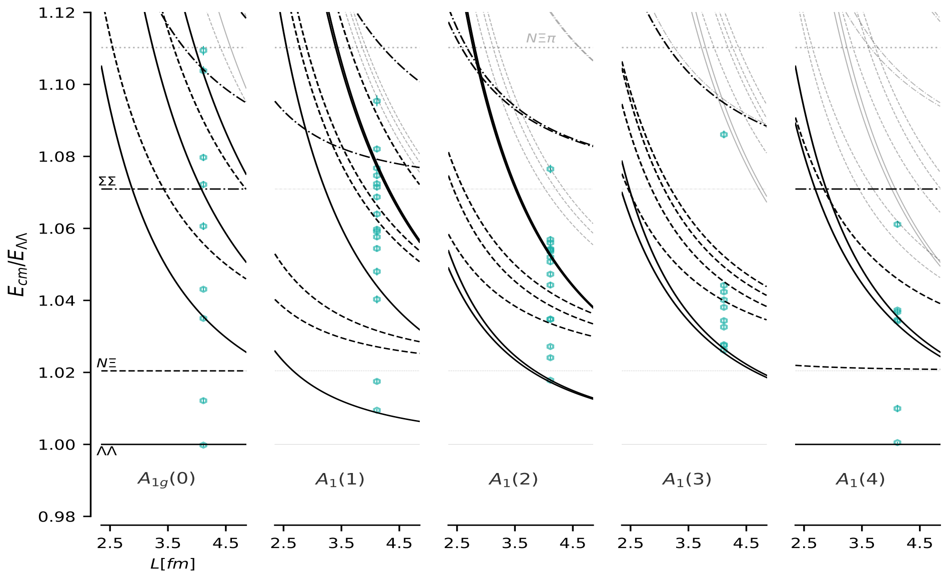

Figure 5: Same as in Figure 3, but for the D200 ensemble, which has $m_\pi \sim 200$ MeV.

spectrum. Currently, we are working on realizing a newer analysis procedure utilizing the eigenvalue decomposition of the quantization matrix [15], which we believe is the way to go forward with a complicated system such as this[5]. In addition to the fact that this is a system involving multi-channel scattering, we also need to be cautious about various systematic uncertainties that could be crucial. Our experience from the studies made at the $SU(3)_f$ symmetric point suggests that there could be large discretization effects [9]. Furthermore, the experimental bounds and the lessons from our studies at the $SU(3)_f$ symmetric point suggest that the continuum binding energy of H dibaryon, if it exists, could be very small. There is no reason to expect a different scenario in the $SU(3)_f$ broken situation, at least for the chosen discretization. These observations call for lattice calculations with good control over the systematic uncertainties. To this end, we plan to extend our investigations to several ensembles over a wide range of lattice spacings and volumes.

4. Summary

We have reported preliminary results for H dibaryon spectroscopy away from the $SU(3)_f$ symmetric point, obtained by applying the distillation framework on a set of ensembles with $N_f = 2 + 1$ flavors of $O(a)$-improved Wilson quarks, generated by CLS. We are able to resolve a dense spectrum of finite-volume energy levels at several values of the pion mass. Current efforts focus on the extraction of infinite-volume scattering amplitudes by applying the finite-volume quantization condition. We will also extend our analysis to dibaryon systems other than the H dibaryon, for which the correlator data have already been computed.

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[5] We utilize the *TwoHadronsInBox* package to realize the quantization condition [13].

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