Chromo-electric screening length in 2+1 flavor QCD

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We study the Polyakov loop as well as the correlators of real and imaginary parts of the Polyakov loop in 2+1 flavor QCD at finite temperature. We use hypercubic (HYP) smearing to improve the signal in the lattice calculations and to obtain reliable results for the correlators at large distances. From the large distance behavior of the correlators we estimate the chromo-electric screening length to be $(0.38 - 0.44)/T$. Furthermore, we show that the short distance distortions due to HYP smearing do not affect the physics of interest.

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1. Introduction

Polyakov loop and Polyakov loop correlators are central for understanding deconfinement and color screening in gauge theories [1, 2] (see also Refs. [3, 4] for a historic reviews). They are related to the free energy of a static quark, $F_Q(T)$ and the free energy of a static quark anti-quark pair, $F_{Q\bar{Q}}(r, T)$ separated by some distance r. In SU(N) gauge theories the Polyakov loop and Polyakov loop correlators are order parameters for deconfinement. In particular, the free energy of a static quark is infinite in the confined phase. In gauge theories with fundamental fields, e.g. QCD this is no longer the case, $F_Q(T)$ is finite even at low temperatures because the static quark is screened by the dynamical quarks in the medium. At sufficiently low temperatures $F_Q(T)$ is related to the binding energy of static-light hadrons [5]. The free energy of a static quark anti-quark pair decays exponentially at large distances. Above the crossover temperature, the screening mass, that governs the exponential decay of the correlators is proportional to the temperature, up to possible sub-leading logarithmic corrections. This means that the chromo-electric screening length that is the inverse of the screening mass becomes shorter and shorter with increasing temperature.

Extracting the screening mass or equivalently the chromo-electric screening length from the Polyakov loop correlator on the lattice is challenging because of the poor signal to noise ratio. The problem is particularly severe in QCD with light dynamical quarks, and most of the lattice calculations of the screening masses have been performed in pure gauge theory [6–8]. It has been suggested to improve the signal to noise ratio for the Polyakov loop correlator by using smeared gauge links when calculating the Polyakov loop [9]. In this contribution we report on lattice QCD calculations of Polyalov loop and Polyakov loop correlators using HYP smearing [10].

We performed calculations of the Polyakov loop and Polyakov loop correlators in 2+1 flavor QCD using highly improved staggered quark (HISQ) action [11] with physical strange quark mass, m_s and light quark masses $m_l = m_s/20$, which correspond to the pion mass of 161 MeV in the continuum limit. The gauge configurations used in this study have been generated by the HotQCD and TUMQCD collaborations [12–16] on lattices with temporal extent $N_\tau = 4$, 6, 8, 10, 12 and 16 and the ratio of spatial to temporal extent (aspect ratio) $N_\sigma/N_\tau = 4$. For $N_\tau = 4$ and 6 we also consider the aspect ratio $N_\sigma/N_\tau = 6$. Our calculations span a temperature range from 110 MeV to 6 GeV. We use 1, 2, 3 and 5 steps of HYP smearing. On the finest lattices we also use 8 steps of HYP smearing.

2. Polyakov loop with HYP smearing

HYP smearing distorts the short distance physics. So it is important to understand the effects of these distortions of F_Q and the Polyakov loop correlators. First we would like to understand how the HYP smearing affects the temperature dependence of F_Q. The free energy of a static quark calculated with n steps of HYP smearing is denoted by F_Q^n. In Fig. 1 we show the difference $\Delta F_Q^{nm} = F_Q^n - F_Q^m$ for different number of HYP smearing steps as function of N_τ. Smearing changes the coefficient of the $1/a$ divergence in the free energy of a static quark and therefore, F_Q^n is different for different n. However, unless excessive amount of smearing is applied the temperature dependence of F_Q^n should not depend on the number of smearing steps, n. Therefore, ΔF_Q^{nm} should not depend on N_τ if cutoff effects are negligible (recall $T = 1/(aN_\tau)$). From Fig. 1 we see that
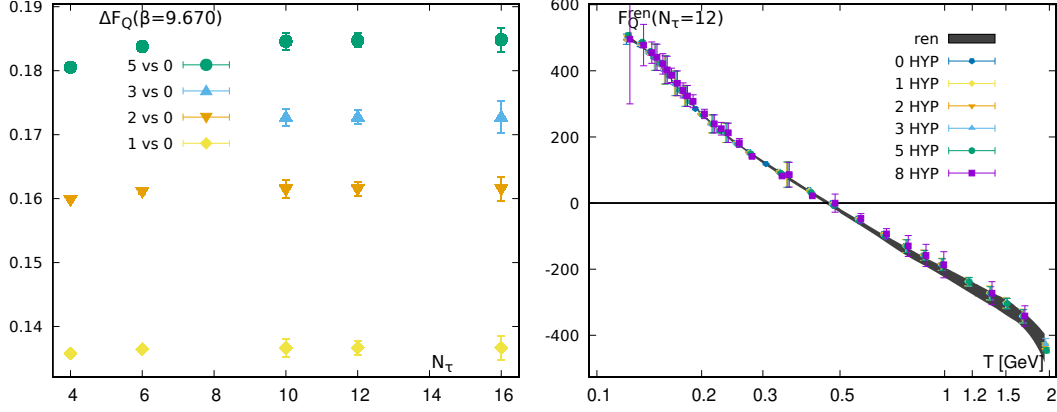
Figure 1: The N_τ dependence of ΔF_Q^{nm} for different smearing levels n and m (left) and the temperature dependence of F_Q for $N_\tau = 12$ for different smearing levels (right). The band shows the continuum extrapolated unsmeared result.

ΔF_Q^{nm} is independent of N_τ within errors for $N_\tau > 4$. In Fig. 1 we also show the temperature dependence of F_Q for different levels of smearing and $N_\tau = 12$. We compare our numerical results with the continuum extrapolated results on F_Q without smearing in the standard renormalization scheme [13]. The results obtained with different number of HYP smearings steps have been shifted by a constant to match with the unsmeared results on F_Q. We clearly see that the temperature dependence is not affected by HYP smearing. Thus there is no problem with over-smearing in our study, i.e. the amount of smearing used is not excessive.

3. Polyakov loop correlators and screening masses

We study the Polyakov loop correlators

$$C_{PL}(r,T) = \langle L(r) \cdot L(0) \rangle, \tag{1}$$

as well as the correlators of the real and imaginary parts of the Polyakov loop

$$C_{PL}^R(r,T) = \langle \mathrm{Re}L(r) \cdot \mathrm{Re}L(0) \rangle, \tag{2}$$

$$C_{PL}^I(r,T) = \langle \mathrm{Im}L(r) \cdot \mathrm{Im}L(0) \rangle. \tag{3}$$

We have $C_{PL}(r,T) = C_{PL}^R(r,T) + C_{PL}^I(r,T)$. The main reason to study $C_{PL}^R(r,T)$ and $C_{PL}^I(r,T)$ separately is that these correlators have quite different large distance behaviors governed by different screening masses, m_R and m_I [17]. This also can be seen in the purely pertubative picture, where the dominant contribution to C_{PL}^R comes from the two gluon exchange, while the leading contribution to C_{PL}^I comes from the three gluon exchange. The perturbative picture implies a different behavior of C_{PL}^R and C_{PL}^I also at short distances. In the weak coupling picture $m_I/m_R = 3/2$. Furthermore, at leading order (LO) $m_R = 2m_D$ and $m_I = 3m_D$, with m_D being the LO chromo-electric Debye mass. The correlator of the real part of the Polyakov loop can couple to magnetic gluons [17, 18] and therefore, the long distance behavior of this correlator can be governed by the magnetic

screening mass instead of electric screening mass. However, for the temperature range of interest the contribution from electric gluon dominates [19, 20].

It is convenient to consider the subtracted free energy

$$F_{Q\bar{Q}}^{sub}(r,T) = -T \ln \frac{C_{PL}(r,T)}{\langle L \rangle^2} = F_{Q\bar{Q}}(r,T) - 2F_Q(T), \tag{4}$$

which is independent of the number of smearing levels used in the calculations except for small distances. At large distances $F_{Q\bar{Q}}^{sub}(r,T)$ decays exponentially with the decay rate given by m_R. We also define the normalized correlators of real and imaginary part of the Polyakov loop

$$\tilde{C}_{PL}^R(r,T) = \frac{C_{PL}^R(r,T) - \langle L \rangle^2}{\langle L \rangle^2}, \tag{5}$$

$$\tilde{C}_{PL}^I(r,T) = \frac{C_{PL}^I(r,T)}{\langle L \rangle^2}. \tag{6}$$

As the subtracted free energy the normalized correlators turn out to be not very sensitive to the number of smearing steps used in the calculations except at short distances. The subtracted free energy can be written in terms of the normalized correlators as

$$F_{Q\bar{Q}}^{sub}(r,T)/T = \ln(1 + \tilde{C}_{PL}^R(r,T) + \tilde{C}_{PL}^I(r,T)). \tag{7}$$

Since we find that $\tilde{C}_{PL}^I(r,T) \ll \tilde{C}_{PL}^R(r,T)$ the subtracted free energy is dominated by the contribution from $\tilde{C}_{PL}^R(r,T)$ and can be used as proxy for $\tilde{C}_{PL}^R(r,T)$. Our results for $F_{Q\bar{Q}}^{sub}(r,T)$ calculated with different numbers of HYP smearings for $N_\tau = 12$ lattices and $T = 1938$ MeV are shown in Fig. 2. The lattice results have been multiplied by r^2T because of the weak coupling expectations. The smeared results are compared with the previously calculated unsmeared $F_{Q\bar{Q}}^{sub}(r,T)$ [15]. We see from the figure that even one or two levels of HYP smearing significantly improves the signal compared to the unsmeared case, however, as we go to larger and larger distances more smearing steps are needed to reduce the errors. The smearing clearly alters the behavior of $F_{Q\bar{Q}}^{sub}(r,T)$ at short distances but for $rT > 0.4$ $F_{Q\bar{Q}}^{sub}(r,T)$ is independent of smearing level up to three HYP smearing steps. For five HYP smearing steps the effect of smearing becomes negligible only for $rT > 0.6$. We obtain similar results for other N_τ values and temperatures. The r/a value for which smearing effects disappear is independent of N_τ.

We also compare our lattice results with weak coupling expectations. In a previous study we showed that the weak coupling calculations can describe the lattice results of $F_{Q\bar{Q}}^{sub}(r,T)$ in the short distance region $rT < 0.4$ [15]. A detailed comparison of the lattice results $F_{Q\bar{Q}}^{sub}(r,T)$ with the weak coupling calculations at larger distances turned out to be challenging because of the large statistical errors of the lattice results for $N_\tau > 4$. As discussed in Ref. [15] at distances $rT > 0.4$ the appropriate scale hierarchy for the weak coupling calculation is $r \sim 1/m_D$. The NLO result for this scale hierarchy was obtained in Ref. [21] and also confirmed later in Ref. [22]. In Fig. 2 we show the comparison of the lattice results with the LO and NLO weak coupling results. The shape of LO and NLO results is similar to the shape of the lattice results, but there is no quantitative agreement. While adding the NLO correction moves the weak coupling results closer to the lattice one, it is still significantly below the lattice result.
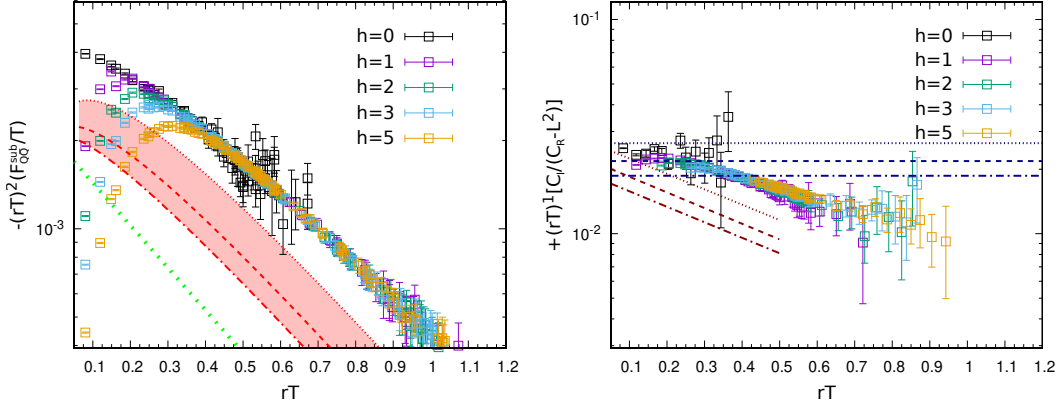
Figure 2: The subtracted free energy $F_{Q\bar{Q}}^{sub}(r,T)$ (left) and $R_{PL}^I(r,T)$ (right) as a function of the separation, rT for $N_\tau = 12$ and $T = 1938$ MeV obtained for different number of HYP smearings (h). In the left panel the LO result (green line) uses renormalization scale $\mu = 4\pi T$, while for the NLO result (red band) the renormalization scale has been varied from πT to $4\pi T$. In the right panel we show the LO result for R_{PL}^I for the renormalization scales $\mu = \pi T$, $2\pi T$ and $4\pi T$ (from top to bottom). The horizontal lines correspond to the LO result without screening.

In order to study the correlator of the imaginary part of the Polyakov loop we consider the ratio

$$R_{PL}^I = \frac{C_{PL}^I}{C_{PL}^R - \langle L \rangle^2}. \tag{8}$$

Again, this ratio is insensitive to smearing. In Fig. 2 we show it for unsmeared case as well as for calculations obtained for 1, 2, 3 and 5 steps of HYP smearings for $N_\tau = 12$ and $T = 1938$ MeV. We see that we were able to obtain reasonable results for R_{PL}^I up to distances $rT \simeq 1$ when 5 steps of HYP smearing are used. Furthermore, the smeared results on R_{PL}^I smoothly connect to the unsmeared results at short distances. In particular, the results with 1 HYP smearing agree with unsmeared results up to quite small distances. We also compare our lattice results of R_{PL}^I with the LO results with or without screening for renormalization scales $\mu = \pi T$, $2\pi T$ and $4\pi T$, shown as lines. We see that at distances $rT < 0.4$ lattice results agree well with the LO result that does not include screening (horizontal lines). At larger distances the lattice data follow the same trend as the LO result with screening but there are clear quantitative differences.

By fitting the large distance behavior of $\tilde{C}_{PL}^R(r,T)$ and $\tilde{C}_{PL}^I(r,T)$ to a $A/r \exp(-m_{R,I}r)$ form, we obtain the corresponding screening masses, m_R and m_I. Our results on the screening masses as functions of the temperature are shown in Fig. 3. We see a rapid decrease of the screening masses in temperature units around the crossover temperature, $T_c \simeq 156$ MeV [23], followed by a roughly constant behavior up to $T \simeq 1.5$ GeV. For $T < 400$ MeV our results for the screening masses are quite similar to the results of Ref. [9]. The decrease of m_R/T and m_I/T predicted by the weak coupling calculations is only seen for $T > 1.5$ GeV. For the ratio of the screening masses we find $m_I/m_R = 1.75$ which is only 17% larger than the weak coupling expectation. Therefore, we can define the chromo-electric screening length as $2/m_R$ or $3/m_I$, which for 170 MeV $< T < 1.5$ GeV corresponds to $(0.38 - 0.44)/T$.
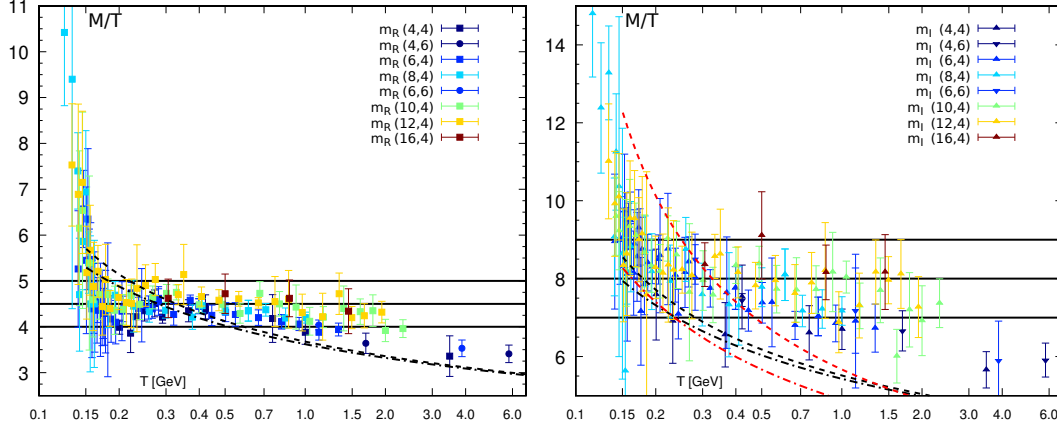
Figure 3: The screening masses, m_R/T (left) and m_I/T (right) as a function of the temperature calculated for different N_τ and N_σ/N_τ indicated in the labels by the numbers in the parenthesis. The solid horizontal lines indicate the approximate value of the screening masses together with the estimated uncertainty. The dashed lines correspond to the perturbative result.

4. Conclusions

In this contribution we studied the Polyakov loop and Polyakov loop correlators using HYP smearing. We found that HYP smearing significantly improves the signal and the distortions due to smearing can be controlled. The correlator of the imaginary part of the Polyakov loop is very well described by the leading order perturbative result for $rT < 0.4$. We determined the screening masses of the correlators of the real and imaginary parts of the Polyakov loop and found $m_R \simeq 4.5T$ and $m_I \simeq 8T$, respectively for 170 MeV $< T < 1.5$ GeV, implying a chromo-electric screening length of $(0.38 - 0.44)/T$ in this region.

Acknowledgments

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