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Proton Decay Amplitudes with Physical Chirally-Symmetric Quarks

Jun-Sik Yoo,[a,b] **Sergey Syritsyn,**[a,c,*] **Yasumichi Aoki,**[d] **Peter Boyle,**[e] **Taku Izubuchi**[c,e] **and Amarjit Soni**[e]

[a] *Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*
[b] *Theory Center, Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan*
[c] *RIKEN-BNL Research Center, Brookhaven National Lab, Upton, NY, 11973, USA*
[d] *RIKEN Center for Computational Science, Kobe, 650-0047, Japan*
[e] *Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

E-mail: sergey.syritsyn@stonybrook.edu

Proton decay is a major prediction of Grand-Unified Theories (GUT) and its observation would indicate baryon number violation that is required for baryogenesis. Many decades of searching for proton decay have constrained its rate and ruled out some of the simplest GUT models. Apart from the baryon number-violating interactions, this rate also depends on transition amplitudes between the proton and mesons or leptons produced in the decay, which are matrix elements of three-quark operators. We report nonperturbative calculation of these matrix elements for the most studied two-body decay channels into a meson and antilepton done on a lattice with physical light and strange quark masses and lattice spacings $a \approx 0.14$ and 0.20 fm. We perform nonperturbative renormalization and excited state analysis to control associated systematic effects. Our results largely agree with previous lattice calculations done with heavier quark masses and thus remove ambiguity in ruling out some simple GUT theories due to quark mass dependence of hadron structure.

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*Speaker

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Introduction Proton decay is a $|\Delta B| = 1$ baryon number-violating process that has been predicted by Grand Unification Theories (GUT)[1–3] but has not been observed so far. Discovery of proton decay may potentially fulfil one of the three prerequisites to explain the Baryon asymmetry in the Universe [4][1], and also demand extension of the Standard Model to accomodate baryon number violation [6], potentially involving supersymmetry [7, 8].

Some of the most important proton decay channels are into an (anti)lepton and one or more mesons. At the lowest order, interactions leading to such decay are local operators [9, 10],

$$\mathcal{L}_{\text{eff}} = \sum_I C_I O_I \ + \text{h.c.}, \qquad O_I = \epsilon^{abc} (\bar{q}^{aC} P_{\chi_I} q^b)(\bar{\ell}^C P_{\chi'_I} q^c), \tag{1}$$

where chirality projectors $P_{\chi^{(\prime)}=R,L} = \frac{1\pm\gamma_5}{2}$ and the Wilson coefficients C_I depend on the specifics of an underlying unified theory. For small $m_{\bar{\ell}} \ll m_N$, the $p \to \Pi\bar{\ell}$ partial decay width is

$$\Gamma(p \to \Pi\bar{\ell}) = \frac{m_N}{32\pi}\Big[1 - \Big(\frac{m_\Pi}{m_N}\Big)^2\Big]^2 \Big|\sum_I C_I W^I_{\bar{\ell}}\Big|^2, \tag{2}$$

where $\Pi = \pi, K$ is a meson and $\bar{\ell} = e^+, \bar{\nu}, \mu^+$ is a lepton in the final state. The proton decay amplitudes $W_{\bar{\ell}} \approx W_0^I + O(m_\ell/m_N) \cdot W_1^I$ depend only on the quark component of the operators O_I (1). From dimensional analysis, $W^I_\ell \propto \Lambda^2_{\text{QCD}}$ and the proton decay rate is suppressed as $\Gamma \propto |c_I|^2 (\Lambda_{\text{QCD}}/\Lambda_{(\text{GUT})})^4$, where c_I are dimensionless GUT couplings and $\Lambda_{(\text{GUT})}$ is the relevant scale. The decay form factors $W^I_{0,1}(Q^2)$ are defined as

$$\bar{v}^C_{\ell\alpha}(\vec{q})\langle \Pi(\vec{p})|O^{\chi\chi'}_\alpha(q)|N(\vec{k})\rangle = (\bar{v}^C_\ell(\vec{q})\, P_{\chi'}\, [W^O_0(Q^2) - \frac{i\not{q}}{m_N} W^O_1(Q^2)] u_N(\vec{k})) \tag{3}$$

and must be determined at the decay kinematical point $Q^2 = -(E_N - E_\Pi)^2 + (\vec{k} - \vec{p})^2 = -m^2_\ell$. They depend on nonberturbative quark dynamics and have to be evaluated in ab initio QCD calculations.

Table 1: Parameters of lattice ensembles with I-DSDR gauge and (zMobius) Domain Wall fermion actions.

$L_x^3 \times L_t$	a^{-1} [GeV]	β	am_π	am_K	$m_\pi L$	N_{cfg}	N_{exact}	N_{approx}
$24^3 \times 64$	1.023(2)	1.633	0.1378(7)	0.5004(25)	3.31	140	1	32
$32^3 \times 64$	1.378(5)	1.75	0.1008(5)	0.3543(6)	3.25	112	1	32

We perform our QCD calculation on a lattice using physical values of quarks with chirally-symmetric action (see Tab. 1 and Ref. [11] for details). For full description of the contents in this report, see Ref. [12]. To make such expensive calculations affordable we (1) use "zMobius" fermion action, (2) use pre-calculated multigrid eigenvectors [13] to accelerate propagator calculation, and (3) employ *all-mode-averaging* sampling. In the latter, we evaluate 32 approximate samples per configuration with quark propagators computed employing truncated Conjugate-Gradient (s quark) combined with deflation (u, d quarks), as well as one exact sample to correct for bias in the approximate samples.

[1] There are viable alternatives such as leptogenesis [5].

Hadron spectrum First step of the analysis is to extract the meson (pion and kaon) and proton masses and energies as well as normalization of their operators from their two-point functions

$$C^{\Pi\Pi}(\vec{k},t) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \langle J_\Pi(x)\, J_\Pi^\dagger(0)\rangle, \tag{4}$$

$$C_+^{N\bar{N}} = \mathrm{Tr}\Big[\frac{1+\gamma_4}{2} C^{N\bar{N}}\Big], \qquad C_{\alpha\beta}^{N\bar{N}}(\vec{k},t) = \sum_{\vec{x}} e^{-i\vec{k}\vec{x}} \langle N_\alpha(x)\, \bar{N}_\beta(0)\rangle, \tag{5}$$

where $J_\Pi = \bar{d}\gamma_5 u$, $\bar{s}\gamma_5 u$, and $\bar{s}\gamma_5 d$ for π^+, K^+, and K^0, respectively, and $N = (u^T C\gamma_5 d)u$ for the proton. In Figure 1, we show their effective energies and 2-state fit results compared to the continuum dispersion relations extrapolated from their masses determined on the lattice.

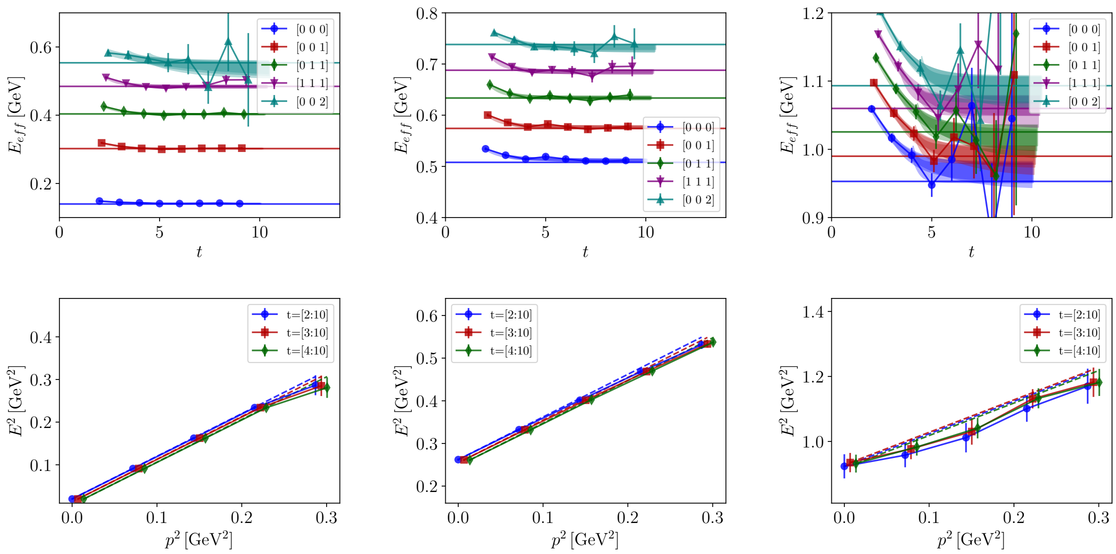

Figure 1: Two-state fits to pion (left), kaon (center), and nucleon (right) two-point correlation functions shown as effective energies (top) and ground-state dispersion relations (bottom) on the 24ID ensemble. The continuum dispersion relations $E^2(p) = E^2(0) + p^2$ for ground-state energies are shown with horizontal lines in the top panels and with dashed lines in the bottom panels.

Renormalization of proton decay operators is computed nonperturbatively using intermediate lattice momentum scheme similar to the traditional MOM or SMOM schemes. Momenta of the three quarks can be arranged in various combinations. In previous calculations (e.g., Ref. [14]), all three quarks carried the same momentum p, and thus the operator carried larger momentum $(3p)$. Such large difference in scales is challenging for our coarse lattices and may cause significant perturbative contributions, therefore we have adopted a scheme to which we refer as SYM3q, in which the quark momenta p, k, r add up to zero operator momentum $q = (p+k+r) = 0$. Conversion from SYM3q to the $\overline{\text{MS}}$ scheme to $O(\alpha_S^3)$ precision has been perturbatively computed in Ref. [15].

As commonly done, we compute amputated correlators $\Lambda_X(p,k,r)$ of each of the three-quark operator O_X with external *plane-wave* quark fields in the Landau gauge, and project onto the tree-level spin-color structure of operator O_Y. The renormalization matrix is then defined as

$$Z_q^{-3/2}\, Z_{ZX}^{3q}\, \Lambda_{XY}^{3q} = \delta_{ZY}\,. \tag{6}$$

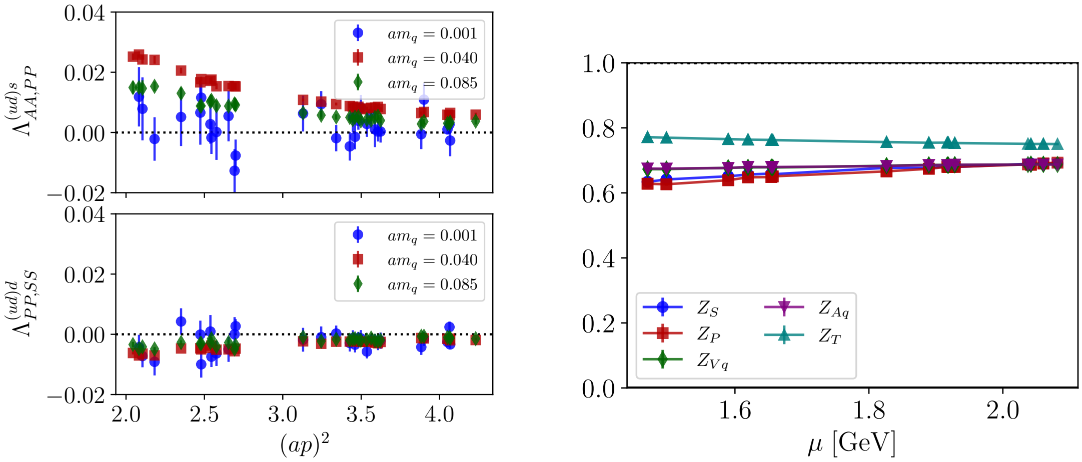

Figure 2: Quark mass and scale dependence of the mixing Green's functions $\Lambda^{(ud)s}_{AA,PP}$ and $\Lambda^{(ud)d}_{PP,SS}$ (left) and renormalization of quark-bilinear operators (right) on the 24ID ensemble.

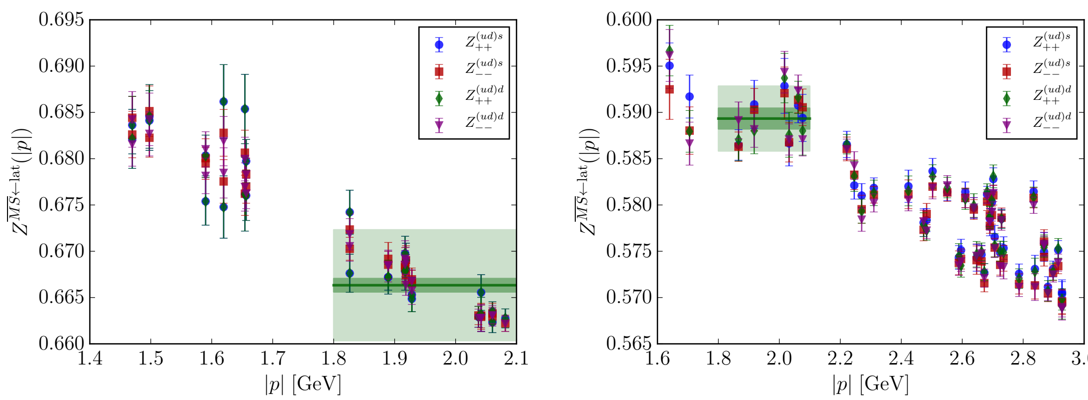

Figure 3: Diagonal conversion factors from SYM3q/SMOM$_{\gamma_\mu}$ to $\overline{\mathrm{MS}}$(2 GeV) scheme, in which perturbative running with intermediate scale $|p|$ has been removed. In absence of discretization, nonperturbative, and higher-order perturbative effects, it should be independent of $|p|$. The green bands indicate averages over the same momentum range on both ensembles, which is necessary for consistent continuum extrapolation.

We observe that our calculations respect chiral symmetry and mixing between chiral symmetry partners (scalar and pseudoscalar) is negligible. We study quark mass dependence and $SU(3)_f$-symmetric chiral limit of our renormalization factors, and observe that the mixing increases with the quark mass as expected. However, as shown in Fig. 2(left), the mixing becomes negligible already when the quark mass is set to the physical value of $m_{u,d}$. The magnitude of this mixing is comparable to the perturbative uncertainty that has only sub-percent effect and is therefore ignored.

The quark field renormalization factor Z_q is evaluated from renormalization of quark bilinears in the SMOM$_{\gamma_\mu}$ scheme in particular the axial charge[2]. In Figure 2(right), we show quark-bilinear

[2] Unlike in the traditional SMOM scheme, the Green's function of the axial-vector current is projected as $\Lambda_A = \frac{1}{48}\mathrm{Tr}\Lambda_{\gamma_\mu\gamma_5}\gamma_5\gamma_\mu$.

renormalization factors. The lattice renormalization factors are calculated as

$$Z^{\text{lat}}_{X,Y}(|p|) = [Z_A\Lambda_A(|p|)]^{3/2}\,[\Lambda^{3q}(|p|)]^{-1}_{X,Y}\,, \tag{7}$$

where Z_A is computed elsewhere using (partially) conserved axial current.

It is difficult to satisfy the SYM3q scheme condition for the lattice quark momenta, so we allow deviations from the strict continuum equality $p^2 = k^2 = r^2$ up to 5%. In addition, to minimize discretization effects, we select mostly diagonal-aligned momenta using criterion $\frac{\sum_\mu p_\mu^4}{(\sum_\mu p_\mu^2)^2} \le 0.4$. The final scale-independent conversion factors are shown in Fig. 3 as functions of the intermediate scale momentum $|p|$. We select final renormalization factors as averages over the same window of momenta $1.8 \le |p| \le 2.1$ GeV for both ensembles in order to perform consistent continuum extrapolations. The variance within this window indicates systematic uncertainty in our nonperturbative conversion factors, which is negligible compared to other uncertainties.

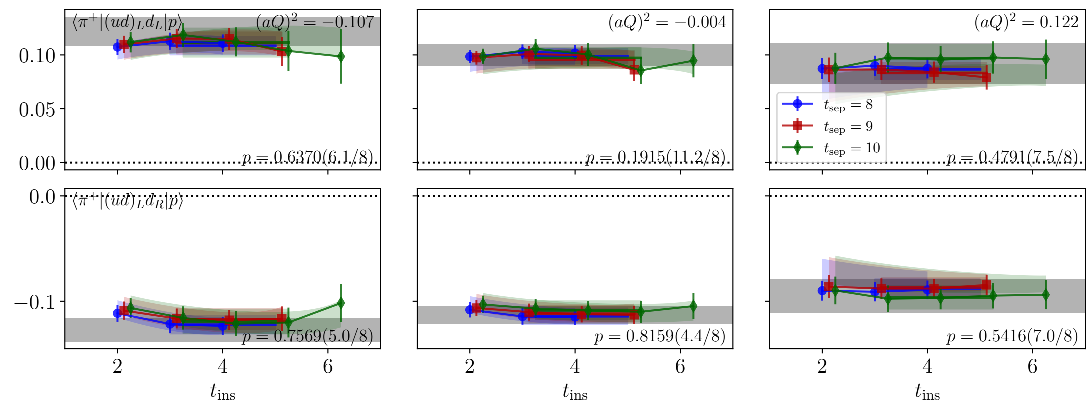

Figure 4: Two-state fits of renormalized $N \to \pi$ correlation functions for form factor W_0 at the three kinematic points on the 24ID lattices with $a \approx 0.20$ fm. The data points show the ratios (9), the color bands show 2-state fits, and the grey bands show the ground-state fit results. All data are normalized in the final $\overline{\text{MS}}$ scheme. The fit quality (p-value) is estimated using the Hoteling distribution.

Matrix elements are extracted from the three-point functions

$$C^{\Pi ON}_{\alpha\beta}(\vec p, \vec q; t_2, t_1) = \sum_{\vec y, \vec z} e^{-i\vec p\vec y - i\vec q\vec z + i\vec k\vec x}\langle J_\Pi(\vec y, x_4 + t_2)\, O^{\chi\chi'}_\alpha(\vec z, x_4 + t_1)\, \bar N_\beta(x)\rangle\,, \tag{8}$$

where the spin indices α, β are the lepton and proton polarization and the 4-momentum transfer $q = (k - p)$ is the lepton "recoil" that must satisfy the decay kinematics $q^2 = m_\ell^2$. We use the two spin projections $C^{\Pi ON}_{\mathcal P} = \text{Tr}[\mathcal P C^{\Pi ON}]$ with $\mathcal P = \frac12(1+\gamma_4)$ and $\frac12(1+\gamma_4)(\vec\gamma\cdot q)$, which yield linearly independent combinations of the proton decay form factors $W_{0,1}$ [12]. Ground-state matrix elements are obtained from the correlation functions (8) using 2-state fits as well, with state energies and overlaps determined in the spectrum analysis. We compare that to the alternative "ratio" method

$$R^O_{\mathcal P}(\vec p, \vec q; t_2, t_1) = \frac{\sqrt{Z_\Pi(\vec p) Z_N(\vec k)}\, C^{\Pi O\bar N}_{\mathcal P}(\vec p, \vec q; t_2, t_1)}{C^{\Pi\Pi}(\vec p, t_2 - t_1)\, C^{N\bar N}_+(\vec k, t_1)}\,, \tag{9}$$

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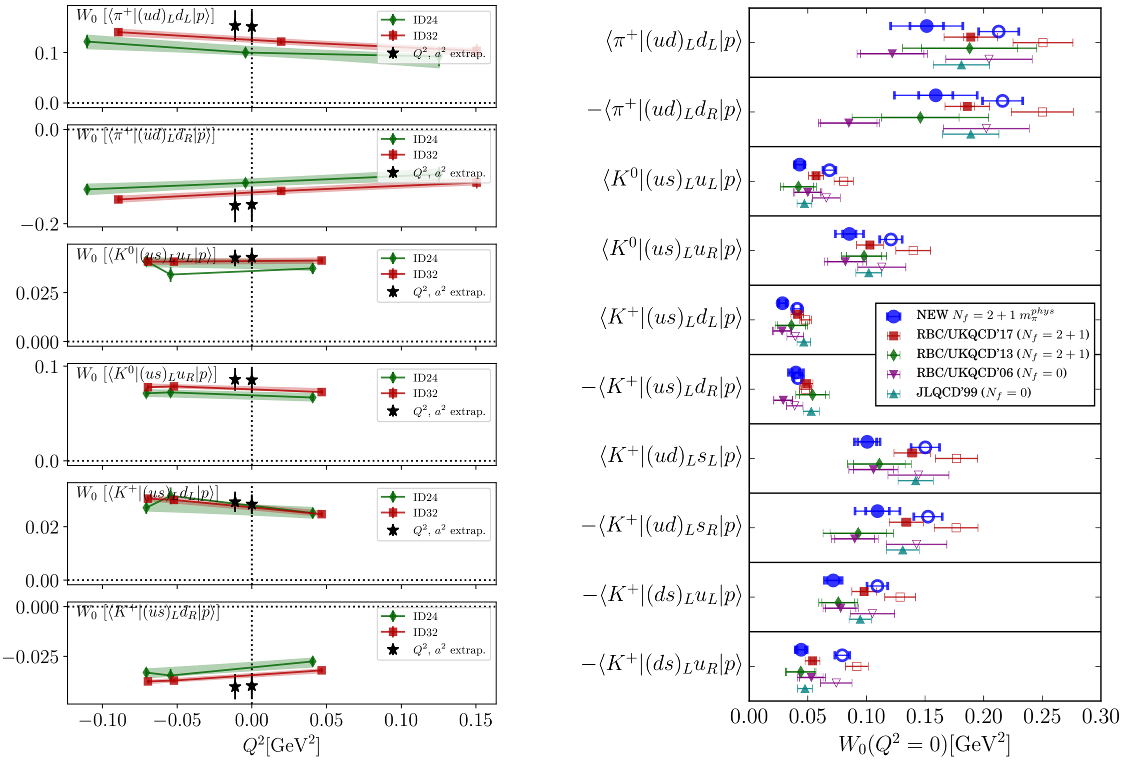

Figure 5: (Left) Momentum interpolation and continuum extrapolation of form factor W_0 for select six channels. (Right) Comparison of our results ("NEW") for the proton decay amplitudes $W_0(0)$ computed directly (filled symbols) and indirectly (open symbols) to previous determinations [16–18]. All results are renormalized to the $\overline{\text{MS}}(2\,\text{GeV})$ scheme.

where $R|_{t_1\approx t_2/2}$ is constructed to converge to the ground-state matrix element at large t_2. The agreement between the fits, the ratios, and the ground-state results in the pion channel shown in Fig. 4 indicates that excited-state effects are negligible. These effects are estimated as differences between the central values of the fits and the ratios with the largest $t_2 = 10a$. Due to discrete values of momenta in the finite volume, we calculate the matrix elements at three kinematical points and perform linear interpolation in Q^2 (see Fig. 5, left), and then perform a^2 extrapolation to the continuum limit. The systematic uncertainty of the continuum extrapolation is estimated as the difference between the extrapolated and the finer-ensemble (32ID) values. This estimate is likely conservative, but it is difficult to assess it more precisely with only two coarse lattice spacings.

Discussion Our final results are presented in Fig. 5 (right). We find reasonable agreement with earlier calculations that employed heavier quark masses[14], quenched [17] or chiral symmetry-breaking fermion action [18]. We also compare our results to estimates based on tree-level chiral perturbation theory using lattice nucleon-to-vacuum decay constants, and find that the latter yield larger values than the direct calculation. We do not observe any suppression as suggested in Ref. [19], confirming earlier limits on some the GUT models. Although conservatively estimated uncertainties in our results sufficiently small [12], they can be further improved by performing calculations at finer lattice spacings, larger volume, and with more statistics.

Our precision is mostly limited by statistical uncertainty and the conservative estimates of discretization errors. While the latter are most likely overestimated, imroving this uncertainty will require calculations with finer lattice spacings. On the other hand, our results with even overestimated uncertainties are sufficient for our main conclusion about the lack of suppression above. Another potential source of uncertainty is the finite volume. In our calculations, $m_\pi L \approx 3.3$, which is low compared to the current state-of-the-art calculations. While it is *very unlikely* that the main conclusion of our work is affected by finite volume effects, this may have to be addressed in subsequent calcuations.

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