

Unpolarized and polarized gluon pseudo-distributions at short distances: Forward case

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We present the results that are necessary in the ongoing lattice calculations of the unpolarized and polarized gluon parton distribution functions (PDFs) within the pseudo-PDF approach. We give a classification of possible two-gluon correlator functions and identify those that contain the invariant amplitude determining the gluon PDF in the light-cone $z^2 \rightarrow 0$ limit. One-loop calculations have been performed in the coordinate representation and in an explicitly gauge-invariant form. We made an effort to separate ultraviolet (UV) and infrared (IR) sources of the $\ln(-z^2)$ -dependence at short distances z^2 . The UV terms cancel in the reduced Ioffe-time distribution (ITD), and we obtain the matching relation between the reduced ITD and the light-cone ITD. Using a kernel form, we get a direct connection between lattice data for the reduced ITD and the normalized gluon PDF.

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1. Introduction

Lattice calculations of parton distribution functions (PDFs) are now a subject of considerable interest. Modern efforts aim at the extractions of PDFs $f(x)$ themselves rather than their x^N moments. On the lattice, this may be achieved by switching from local operators to equal-time correlators [1–4]. We use the pseudo-PDF approach [4], which is coordinate-space oriented, and parton distributions are extracted there by taking the short-distance $z_3 \rightarrow 0$ limit.

Since the $z_3 \rightarrow 0$ limit is singular, one needs matching relations to convert the Euclidean lattice data into the usual light-cone PDFs. Our goal is to outline the pseudo-PDF approach to the extraction of unpolarized gluon PDFs, and also to find one-loop matching conditions.

In the gluon case, the calculation is complicated by strict requirements of gauge invariance. In this situation, a very effective method is provided by the coordinate-representation approach of Ref. [5]. It is based on the background-field method and the heat-kernel expansion. It allows, starting with the original gauge-invariant bilocal operator, to find its modification by one-loop corrections. The results are obtained in an explicitly gauge-invariant form.

2. Matrix elements

2.1 Unpolarized case

The spin-averaged matrix element for operators comprised of two gluon fields with uncontracted indices is $M_{\mu\alpha;\nu\beta}(z, p) \equiv \langle p | G_{\mu\alpha}(z) [z, 0] G_{\nu\beta}(0) | p \rangle$, where $[z, 0]$ is the standard straight-line gauge link in the adjoint representation. Accounting for the antisymmetry of $G_{\rho\sigma}$ with respect to indices, and the available four-vectors p , and z , the decomposition of the matrix element into invariant amplitudes is:

$$\begin{aligned}
M_{\mu\alpha;\nu\beta}(z, p) = & (g_{\mu\nu}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\nu}) \mathcal{M}_{gg}(v, z^2) \\
& + (g_{\mu\nu}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\nu - g_{\alpha\nu}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\nu) \mathcal{M}_{pp}(v, z^2) \\
& + (g_{\mu\nu}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\nu - g_{\alpha\nu}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\nu) \mathcal{M}_{zz}(v, z^2) \\
& + (g_{\mu\nu}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\nu - g_{\alpha\nu}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\nu) \mathcal{M}_{zp}(v, z^2) \\
& + (g_{\mu\nu}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\nu - g_{\alpha\nu}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\nu) \mathcal{M}_{pz}(v, z^2) \\
& + (p_\mu z_\alpha p_\nu z_\beta - p_\alpha z_\mu p_\nu z_\beta - p_\mu z_\alpha p_\beta z_\nu + p_\alpha z_\mu p_\beta z_\nu) \mathcal{M}_{ppzz}(v, z^2), \quad (1)
\end{aligned}$$

where $v = -p \cdot z$ is the Ioffe time[6]. The light-cone distribution is obtained from $g^{\alpha\beta} M_{+\alpha,+\beta}(z, p)$, where z is taken to be in the ‘‘minus’’ direction $z = z_-$, so:

$$g^{\alpha\beta} M_{+\alpha,+\beta}(z_-, p) = -2p_+^2 \mathcal{M}_{pp}(v, z^2), \quad (2)$$

thus the PDF is determined by \mathcal{M}_{pp} :

$$-\mathcal{M}_{pp}(v, 0) = \frac{1}{2} \int_{-1}^1 dx e^{-ixv} x f_g(x). \quad (3)$$

The procedure is then to take projections of the matrix element that contain the \mathcal{M}_{pp} structure, and little of anything else. The projection that best meets this condition is $M_{0i;i0}$, whose decomposition

is $M_{0i;i0} = 2\mathcal{M}_{gg} + 2p_0^2\mathcal{M}_{pp}$. We can remove the contaminating \mathcal{M}_{gg} structure by adding the projection $M_{ij;ji}$, whose decomposition is $M_{ij;ji} = -2\mathcal{M}_{gg}$.

While all projections of the matrix element are individually multiplicatively renormalizable [7], they won't necessarily carry the same anomalous dimension; however, this addition works because both projections, $M_{0i;i0}$ and $M_{ij;ji}$, have the same anomalous dimension at one loop.

2.2 Polarized case

With polarized gluons we have the matrix element comprised of a gluon field strength tensor and a dual field strength tensor: $m_{\mu\alpha;\nu\beta}(z, p) \equiv \langle p, s | G_{\mu\alpha}(z) [z, 0] \tilde{G}_{\nu\beta}(0) | p, s \rangle$, where the dual tensor is defined by $\tilde{G}_{\mu\beta} \equiv \frac{1}{2}\epsilon_{\mu\beta\rho\lambda}G^{\rho\lambda}$. Taking the z -odd combination $M_{\mu\alpha;\nu\beta}(z, p) \equiv m_{\mu\alpha;\nu\beta}(z, p) - m_{\mu\alpha;\nu\beta}(-z, p)$, we perform a similar decomposition into invariant amplitudes, and find that the LC helicity distribution is defined by:

$$g^{\alpha\beta} \tilde{M}_{+\alpha;\beta+}(z_-, p) = -2p_+s_+ \left[\mathcal{M}_{ps}^{(+)}(\nu, 0) - \nu\mathcal{M}_{pp}(\nu, 0) \right] \quad (4)$$

where s_+ is related by $s_\mu = mS_\mu$ to the usual gluon polarization vector S_μ , and $\mathcal{M}_{ps}^{(+)} \equiv \mathcal{M}_{ps} + \mathcal{M}_{sp}$.

In the case of pseudodistributions, a promising combination of projections on this matrix element is $\tilde{M}_{0i;i0} + \tilde{M}_{ji;ij}$, whose decomposition is $2p_3p_0[\mathcal{M}_{ps}^{(+)}(\nu, z^2) - (1 + m^2/p_3^2)\nu\mathcal{M}_{pp}(\nu, z^2)]$. For $p_3 \rightarrow \infty$, this result approaches the decomposition structure in the LC case.

3. One-loop corrections

3.1 Link self energy and ultraviolet divergences

The link self energy correction, given by

$$\frac{g^2 N_c}{8\pi^2} \frac{\Gamma(1 - \epsilon_{UV}) (-z^2 \mu_{UV}^2 + i\epsilon)^{\epsilon_{UV}}}{(2\epsilon_{UV} - 1) \epsilon_{UV}} G_{\mu\alpha}(z) G_{\nu\beta}(0), \quad (5)$$

should, in principle, be zero on the light-cone. However, this result contains linear ($\epsilon_{UV} = 1/2$) and logarithmic ($\epsilon_{UV} = 0$) UV poles and a singularity on the light-cone $z^2 = 0$ after expansion in ϵ_{UV} . In order to account for this, we explicitly separate the z^2 dependence generated by the UV singular terms and those in the QCD (or DGLAP) evolution logarithms $\ln(-z^2 \mu_{IR}^2)$.

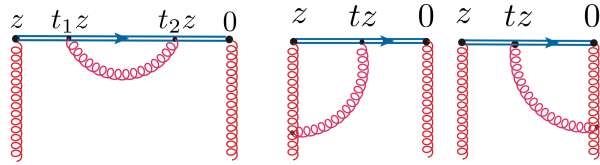


Figure 1: Self-energy-type correction for the gauge link and vertex diagrams with gluons coming out of the gauge link.

3.2 Vertex contributions

When one uses the background-field technique, with gluon propagator in the background-Feynman (bF) gauge [8], the three-gluon vertex differs from the usual Yang-Mills vertex. Therefore, the Feynman diagrams we use do not correspond one-to-one with the usual Feynman diagrams. A consequence of this is, where one would have two linear UV divergences that cancel after addition of two diagrams, in our case that same divergence cancels implicitly. This can be seen from the evolution part of the vertex correction:

$$\frac{g^2 N_c}{8\pi^2} \frac{\Gamma(d/2 - 2)}{(d-3)(-z^2)^{d/2-2}} \int_0^1 du [u^{3-d} - 1]_+ G_{\mu\alpha}(\bar{u}z) G_{\nu\beta}(0), \quad (6)$$

where the linear divergences present in the “ u^{3-d} ” part and the “ -1 ” part cancel.

The full uncontracted vertex calculation contains also a UV divergent and constant part, however this term is zero for the $M_{0i;i0}$ and the $M_{ij;ji}$ projection.

3.3 Box and self-energy contributions

The “box” correction is free of UV divergences, but gives a more complicated structure, generating a mixture of different operators corresponding to different projections of $G_{\mu\alpha}(\bar{u}z) G_{\nu\beta}(0)$. The full uncontracted result is too long for this paper, but it should be noted that the DGLAP part does not have the necessary plus-prescription form. To get it, one should add the contribution of the gluon self-energy diagrams. The relevant part of the result is the coefficient to the \mathcal{M}_{pp} structure, $2(-u^3 + u^2 - 2u + 1)$, which integrates to $1/6$.

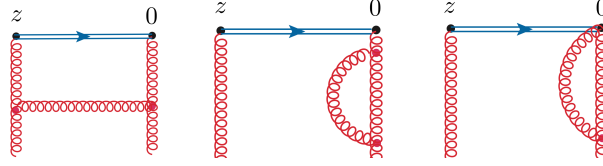


Figure 2: Box diagram and gluon self-energy-type insertions into the right leg.

The self-energy diagrams contain both UV and collinear divergences that generate logarithmic term $\ln(\mu_{IR}^2/\mu_{UV}^2)$. In order to obtain the necessary plus prescription with the “box” diagram, one can separate this term into the difference $\ln(z_3^2 \mu_{IR}^2) - \ln(z_3^2 \mu_{UV}^2)$.

The self-energy result is

$$\frac{g^2 N_c}{8\pi^2} \frac{1}{2-d/2} \left[2 - \frac{\beta_0}{2N_c} \right] G_{\mu\alpha}(z) G_{\nu\beta}(0), \quad (7)$$

where $\beta_0 = 11N_c/3$ in gluodynamics, and $\frac{1}{2-d/2}$ is to be replaced by $\ln(z_3^2 \mu_{IR}^2) - \ln(z_3^2 \mu_{UV}^2)$. Substituting in the value of β_0 , we get $1/6$, giving us the needed plus-prescription.

3.4 Polarized case

Because the one-loop calculation was performed at the operator level with uncontracted indices, the results for $G_{\mu\alpha}(z) \tilde{G}_{\rho\sigma}(0)$ can be easily obtained by contracting the previous results from this section with $\frac{1}{2}\epsilon_{\rho\sigma}^{\nu\beta}$. The DGLAP evolution structure and matching relation for the polarized

gluon pseudodistribution will be given in an upcoming paper. Here, we just list the results of the $\tilde{M}_{0i;i0} + \tilde{M}_{ji;j}$ combination:

$$\begin{aligned}
& \tilde{M}_{0i;i0}(z, p) + \tilde{M}_{ji;j}(z, p) \\
& \rightarrow \frac{g^2 N_c}{8\pi^2} \frac{16}{6} \left(\frac{1}{\epsilon_{UV}} + \log(z_3^2 e^\gamma) \right) (\tilde{M}_{0i;i0}(z, p) + \tilde{M}_{ji;j}(z, p)) \\
& + \frac{g^2 N_c}{8\pi^2} \int_0^1 du \left\{ 4\delta(\bar{u}) - 2\bar{u}u + 2 \left(\frac{1}{\bar{u}} - \bar{u} \right)_+ - \left[\frac{4u}{\bar{u}} + \frac{4\log(1-u)}{\bar{u}} \right]_+ \right. \\
& \left. + \left(\frac{1}{\epsilon_{IR}} - \log(z_3^2 e^\gamma) \right) \left[\{4u\bar{u} + 2[u^2/\bar{u}]_+\} - \frac{1}{2} \left(\frac{\beta_0}{N_c} + 6 \right) \delta(\bar{u}) \right] \right\} \\
& \times (\tilde{M}_{0i;i0}(uz, p) + \tilde{M}_{ji;j}(uz, p))
\end{aligned} \tag{8}$$

4. DGLAP evolution structure, unpolarized case

4.1 Reduced Ioffe-time distribution

In order to handle the link and self-energy UV divergences, we use the reduced ITD:

$$\mathfrak{M}(v, z_3^2) \equiv \frac{\mathcal{M}_{pp}(v, z_3^2)}{\mathcal{M}_{pp}(0, z_3^2)}. \tag{9}$$

This method works because our operator is multiplicatively renormalizable, and because the DGLAP evolution logarithm drops out of $\mathcal{M}_{pp}(0, z_3^2)$, so we're only and entirely removing the non-DGLAP related z_3 dependence.

4.2 Matching relations

Combining the one-loop gluon corrections, and the gluon-quark mixing term (that contains the gq evolution kernel $\mathcal{B}_{gq}(u) = 1 + (1-u)^2$), we obtain the matching relation (excluding higher

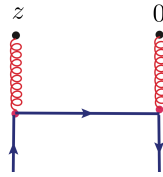


Figure 3: Gluon-quark mixing diagram.

twist terms, or rather keeping only \mathcal{M}_{pp}):

$$\begin{aligned}
\mathfrak{M}(v, z_3^2) \mathcal{I}_g(0, \mu^2) &= \mathcal{I}_g(v, \mu^2) - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \mathcal{I}_g(uv, \mu^2) \left\{ \ln(z_3^2 \mu^2 e^{2\gamma_E} / 4) \mathcal{B}_{gg}(u) \right. \\
& \left. + 4 \left[\frac{u + \log(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3} [1 - u^3]_+ \right\} \\
& - \frac{\alpha_s C_F}{2\pi} \ln(z_3^2 \mu^2 e^{2\gamma_E} / 4) \int_0^1 dw [\mathcal{I}_S(wv, \mu^2) - \mathcal{I}_S(0, \mu^2)] \mathcal{B}_{gq}(w), \tag{10}
\end{aligned}$$

where $\mathfrak{M}(\nu, z_3^2)$, the reduced ITD, is our ‘‘lattice function’’, and $\mathcal{I}_g(\nu, \mu^2)$ and $\mathcal{I}_S(u\nu, \mu^2)$ are the light-cone ITDs. The gluon Altarelli-Parisi kernel is given by

$$B_{gg}(u) = 2 \left[\frac{(1 - \bar{u}u)^2}{\bar{u}} \right]_+ . \quad (11)$$

The gluon light-cone ITD can be directly related to the light-cone PDF through

$$\mathcal{I}_g(\nu, \mu^2) = \frac{1}{2} \int_{-1}^1 dx e^{ix\nu} x f_g(x, \mu^2) . \quad (12)$$

Because $x f_g(x, \mu^2)$ is an even function of x , the real part of $\mathcal{I}_g(\nu, \mu^2)$ is given by the cosine transform of $x f_g(x, \mu^2)$, while the imaginary part vanishes.

The factor $\mathcal{I}_g(0, \mu^2) = \langle x \rangle_{\mu^2}$ is the fraction of the hadron momentum carried by the gluons. It should be found from an independent lattice calculation (see, e.g. [9]).

The matching relation can be cast into a new kernel form in terms of the light-cone PDFs:

$$\mathfrak{M}(\nu, z_3^2) = \int_0^1 dx \frac{x f_g(x, \mu^2)}{\langle x \rangle_{\mu^2}} R_{gg}(x\nu, z_3^2 \mu^2) + \int_0^1 dx \frac{x f_S(x, \mu^2)}{\langle x \rangle_{\mu^2}} R_{gq}(x\nu, z_3^2 \mu^2) , \quad (13)$$

where

$$R_{gg}(y, z_3^2 \mu^2) = \cos y - \frac{\alpha_s N_c}{2\pi} \left\{ \ln \left(z_3^2 \mu^2 e^{2\gamma_E} / 4 \right) R_B(y) + R_L(y) + R_C(y) \right\} , \quad (14)$$

and

$$R_{gq}(y, z_3^2 \mu^2) = -\frac{\alpha_s N_c}{2\pi} \ln \left(z_3^2 \mu^2 e^{2\gamma_E} / 4 \right) R_B(y) \quad (15)$$

The various R terms are given by cosine transformations of the gluon kernel, log term, constant term, and mixing kernel, and are all perturbatively calculable expressions.

Using lattice data and models for $f_g(x, \mu^2)$ and $f_S(x, \mu^2)$, one can fit their parameters and α_s .

5. Conclusion

We presented the results of the calculations necessary in the ongoing work to extract unpolarized gluon PDFs from the lattice using the method of pseudo-PDFs, and provide an update on the soon to be released results for the polarized case. Specifically, we demonstrated $M_{0i;i0} + M_{ij;ji}$ to be the most promising combination of matrix elements for obtaining the unpolarized gluon PDF, and $\tilde{M}_{0i;i0} + \tilde{M}_{ji;ij}$ to be the likely combination for obtaining the gluon helicity PDF. We gave the matching relations, in the unpolarized case, between the reduced pseudo-ITD and the light-cone ITD or light-cone PDF, demonstrating that lattice data and light-cone PDFs can be directly related.

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