

## S parameter from a prototype composite-Higgs model

---

**Yigal Shamir,<sup>a,\*</sup> Maarten Golterman,<sup>b</sup> William I. Jay,<sup>c</sup> Ethan T. Neil<sup>d</sup> and Benjamin Svetitsky<sup>a</sup>**

<sup>a</sup>*Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University,  
69978 Tel Aviv, Israel*

<sup>b</sup>*Department of Physics and Astronomy, San Francisco State University,  
San Francisco, CA 94132, USA*

<sup>c</sup>*Theoretical Physics Department, Fermi National Accelerator Laboratory,  
Batavia, Illinois, 60510, USA*

<sup>d</sup>*Department of Physics, University of Colorado,  
Boulder, CO 80309, USA*

*E-mail:* [shamir@tauex.tau.ac.il](mailto:shamir@tauex.tau.ac.il)

We have calculated the low-energy constant  $L_{10}$  in a prototype composite Higgs model with dynamical fermions in two different representations of the gauge group. The resulting contribution of the new strong sector to the  $S$  parameter is consistent with current bounds on the vacuum misalignment parameter. We end with a brief discussion of future directions.

*The 38th International Symposium on Lattice Field Theory, LATTICE2021 26th-30th July, 2021  
Zoom/Gather@Massachusetts Institute of Technology*

---

\*Speaker

## 1. Introduction

According to the composite Higgs paradigm, the Higgs boson is a pseudo Nambu-Goldstone boson of a new strong interaction [1, 2]. This protects its mass from receiving large radiative corrections. Hypercolor—the new strong interaction—may be operative at the few TeV scale (see Refs. [3–5] for reviews). One can also use the hypercolor interaction to generate a partially-composite top quark via linear coupling to a hypercolor baryon [6].

We have been studying a prototype composite-Higgs model using lattice techniques [7–12]. The model is an SU(4) gauge theory with 2 Dirac fermions in the fundamental representation, together with 2 Dirac fermions in the sextet, or two-index antisymmetric, representation. Since the sextet is real, this is equivalent to 4 Majorana fermions. Our work was the first lattice calculation with dynamical fermions in two different representations. For similar lattice calculations, see Refs. [13, 14]. For lattice work related to composite-Higgs models based on an Sp(4) gauge theory, see Refs. [15–18].

In itself, the fermion content of the prototype model is not enough to meet phenomenological requirements. But it is quite close to one of the composite-Higgs models proposed by Ferretti and Karateev [19], labeled M6 in Ref. [20], which contains 3 fundamental Dirac fermions and 5 sextet Majorana fermions (see also Refs. [21, 22]). Note that the Ferretti-Karateev models are partially, but not fully, ultraviolet complete: the origin of the 4-fermion couplings needed to generate a partially composite top quark remains unspecified. For a proposal for a full ultraviolet completion, see Ref. [23].

Here we report on our calculation of the low-energy constant  $L_{10}$  [12], which largely follows the QCD calculations of Refs. [24, 25]. We have calculated  $L_{10}$  in the sextet sector of our two-representation theory, since in the M6 model the electro-weak symmetries are embedded in the unbroken global symmetry of that sector. Our result for  $L_{10}$  allows us to constrain the contribution of the hypercolor theory to the  $S$  parameter in terms of the vacuum misalignment parameter [3–5].

## 2. Chiral perturbation theory

In the continuum, the correlator of a left-handed and a right-handed current is decomposed as

$$\langle J_{L\mu} J_{R\nu} \rangle = (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2), \quad (1)$$

with transverse function  $\Pi^{(1)}$  and longitudinal function  $\Pi^{(0)}$ . The transverse function is given by

$$\Pi^{(1)} = \frac{F^2}{q^2} + \hat{\Pi}(q^2), \quad (2)$$

where the leading-order pole reflects a kinematical singularity. To next-to-leading order (NLO) in chiral perturbation theory and for  $N$  Majorana fermions [26, 27]

$$\hat{\Pi}(q^2) = \frac{N+2}{96\pi^2} \left[ \frac{1}{3} + \log \left( \frac{M^2}{\mu^2} \right) - H(s) \right] + 8L_{10}, \quad (3)$$

which includes the dependence on  $L_{10}$ . Here  $M$  is the pion mass and  $\mu$  is the renormalization scale. The momentum dependence enters via

$$H(s) = 2s^2 + s^3 \log \left( \frac{s-1}{s+1} \right), \quad (4)$$

with  $s = \sqrt{1 + 4M^2/q^2}$ . The function  $H(s)$  has no free parameters. We will also need the difference

$$\Pi^{(1-0)} = \Pi^{(1)} - \Pi^{(0)} = \frac{F^2}{q^2 + M^2} + \hat{\Pi}(q^2). \quad (5)$$

Turning to the lattice calculation, our ensembles were generated with dynamical Wilson-clover fermions for both representations [7]. nHYP smearing was used in the fermion action [28, 29], and an NDS term was added to the gauge action to further improve the performance of the smeared links [30]. In view of the importance of chiral symmetry for the calculation of  $L_{10}$  we used staggered valence fermions, *i.e.*, we did a mixed-action calculation.  $J_{L\mu}$  and  $J_{R\nu}$  were constructed from the standard vector and axial staggered currents, except again with nHYP links. We made the usual replacement  $q_\mu \rightarrow \hat{q}_\mu = (2/a) \sin(aq_\mu/2)$ , with  $a$  the lattice spacing. The renormalization scale was  $1/\mu^2 = t_0$ , where  $t_0$  is the gradient-flow scale.

The pole terms, *i.e.*, the first term on the right-hand sides of Eqs. (2) and (5), involve the pure valence pion, so that  $M = M_{v\nu}$  and  $F = F_{v\nu}$ . At NLO, the pion in the loop is a mixed sea-valence pion, hence, in Eq. (3), as well as inside the argument  $s$  of  $H(s)$ ,  $M = M_{v,s}$ , with

$$M_{v,s}^2 = \frac{M_{s,s}^2 + M_{v\nu}^2}{2} + \frac{a^2}{t_0^2} \Delta_{\text{mix}}. \quad (6)$$

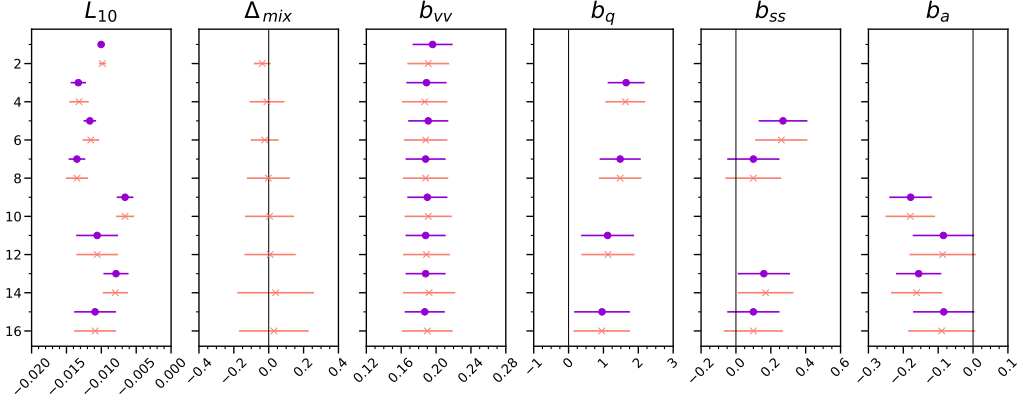
Here  $\Delta_{\text{mix}} \geq 0$  is a new parameter peculiar to the mixed-action setting [31–34]. In order to explore the possible effects of the next order in chiral perturbation theory, we have augmented Eq. (3) by the (NNLO) analytic terms,

$$t_0 \left( b_q q^2 + b_{s,s} M_{s,s}^2 + b_{v\nu} M_{v\nu}^2 \right) + b_a \frac{a^2}{t_0}. \quad (7)$$

### 3. Results

We used twelve  $16^3 \times 32$  ensembles and three  $24^3 \times 48$  ensembles taken from Ref. [7], with gradient-flow scale  $t_0/a^2$  in the range 0.9–2.7, and sea-pion mass  $\sqrt{t_0}M_{s,s}$  in the range 0.2–0.58. After some exploratory studies, we settled on a calculation with 7 valence masses  $am_v$  in the range 0.01–0.05. We successfully fitted  $\Pi^{(1-0)}$ , whose correlations turned out to be smaller than those of  $\Pi^{(1)}$ , always including  $L_{10}$  and  $b_{v\nu}$  in the fit. In order to limit other sources of higher-order corrections apart from the valence mass, we evaluated the correlators only at the smallest time-like momentum. We tried all 16 combinations of the 4 remaining parameters ( $\Delta_{\text{mix}}$ ,  $b_q$ ,  $b_a$  and  $b_{s,s}$ ), always obtaining a good  $p$ -value.

The results are shown in Fig. 1. The NLO mixed-action parameter  $\Delta_{\text{mix}}$  was always consistent with zero, so we focus on the fits without  $\Delta_{\text{mix}}$ . The value of  $b_{v\nu}$  was stable and more than  $5\sigma$  away from zero, which explains why this NNLO parameter had to be included in all the fits. Fit No. 1 includes only  $L_{10}$  and  $b_{v\nu}$ , and has a very small statistical error (about the size of the data point). The remaining 7 fits with  $\Delta_{\text{mix}} = 0$  include different subsets of the other NNLO parameters  $b_q$ ,  $b_a$  and  $b_{s,s}$ . They give rise to considerable variation in the value of  $L_{10}$ , as well as to increasing statistical error. The results of fits 3, 5, and 9, which include only a single additional NNLO parameter suggest that our main source of uncertainty is systematic. We used the central values of



**Figure 1:** Sixteen fits of  $\Pi^{(1-0)}$  to data from all 7 valence masses. All fits include  $L_{10}$  and  $b_{vv}$  as parameters but have different combinations of  $\Delta_{\text{mix}}$  and the other NNLO parameters  $b_q$ ,  $b_{ss}$  and  $b_a$ . Fits without  $\Delta_{\text{mix}}$  are shown in purple, and with  $\Delta_{\text{mix}}$  in orange.

these fits to bracket the systematic error of our final result, and their statistical errors to estimate the statistical error of this result, obtaining

$$L_{10} = -0.0100(12)_{\text{stat}}(35)_{\text{syst}} . \quad (8)$$

The central value of our final result coincides with that of fit 1, which contains only  $L_{10}$  and  $b_{vv}$ . Changing the renormalization scale  $\mu$  in Eq. (3) from  $1/\sqrt{t_0}$  to the sextet vector meson mass shifts the central value of  $L_{10}$  by about  $-0.00035$ , a 3.5% shift.

The contribution of the hypercolor sector to the  $S$  parameter is

$$S_{\text{HC}} = \xi S_{\text{NLO}} . \quad (9)$$

Here  $\xi = 2v^2/F_6^2$  is the vacuum misalignment parameter, where  $v = 246$  GeV is the vacuum expectation value of the Higgs field in the Standard Model, and  $F_6$  is the decay constant of Nambu-Goldstone bosons made of the sextet fermions in the chiral limit.  $S_{\text{NLO}}$  is given by [26, 35]

$$S_{\text{NLO}} = -2\pi \lim_{q^2 \rightarrow 0} \hat{\Pi}(q^2) = -\frac{N+2}{24\pi} \left( 1 + \log \left( \frac{M^2}{\mu^2} \right) \right) - 16\pi L_{10} . \quad (10)$$

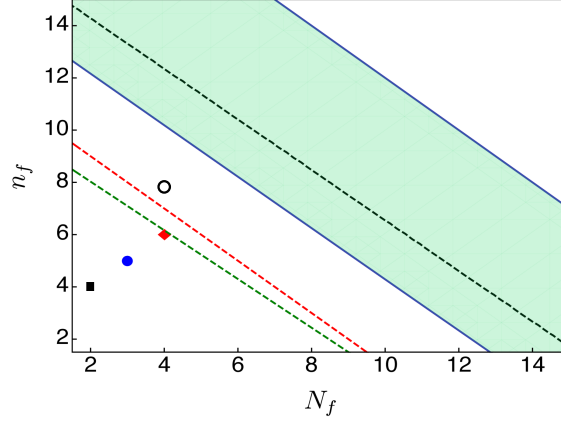
Assuming our result for  $L_{10}$  can be used in the M6 model (for which  $N = 5$  in the above equation), we find for that model

$$S_{\text{NLO}} = 0.8(2) , \quad (11)$$

where the error is dominated by the systematic error of  $L_{10}$ . In order to arrive at this result we have assumed for simplicity that the 14 pseudo Nambu-Goldstone bosons of the SU(5)/SO(5) coset have a common mass  $M$  about the size of the Higgs mass or somewhat larger. The uncertainty due to the actual spread of (non-degenerate) masses is expected to be much smaller than the already quoted error. Also, we assumed  $F_6 = 1.1$  TeV, the lowest value consistent with the commonly quoted upper bound  $\xi \leq 0.1$  [3–5]. The current experimental estimate is  $S = -0.01(10)$ , which implies a  $1\sigma$  upper bound of 0.09. This yields an independent  $1\sigma$  bound

$$\xi \leq \frac{0.09}{0.8(2)} = 0.11(3) , \quad (12)$$

which is compatible with the upper bound mentioned above.



**Figure 2:** Estimates for the location of the conformal window of the SU(4) gauge theory with  $N_f$  Dirac fermions in the fundamental representation and  $n_f$  Majorana fermions in the sextet representation. The uppermost line is the limit of asymptotic freedom. The green band represents the analytical estimate of Ref. [37] for the conformal window, while the dashed lines are other analytical estimates of the bottom of the window. Black square: 2+2 model; Blue circle: M6 model; red diamond: M11 model; open circle: 4+4 model. Adapted from Ref. [37].

#### 4. Future prospects

We previously reported an acute problem with our prototype composite-Higgs model [10, 36]. The problem has to do with the partially-composite top, which should receive its mass via direct coupling to a hypercolor baryon with the same Standard-Model quantum numbers as the top quark. This coupling is facilitated by a 4-fermion operator, schematically,  $tQqq$ , where  $t$  is the top quark,  $Q$  is a sextet hypercolor fermion, while  $qq$  is a pair of fundamental hypercolor fermions. The hypercolor singlet operator  $Qqq$  generates a “chimera” baryon.

The 4-fermion coupling  $G/\Lambda_{UV}^2$  (with dimensionless  $G$ ) is assumed to arise from physics at a new ultraviolet scale  $\Lambda_{UV}$ . In order to meet flavor constraints,  $\Lambda_{UV}$  should be much higher than the hypercolor scale  $\Lambda_{HC}$  [3–5]. We calculated the matrix element of  $Qqq$  between the vacuum and a chimera state, finding a very small value. Demanding the hierarchy  $\Lambda_{UV} \gg \Lambda_{HC}$ , our result implies that the top quark can only receive a mass which is smaller by several orders of magnitude than its actual mass. Conversely, if we insist that the top quark receive the correct mass, then  $\Lambda_{UV}$  would have to be *smaller* than  $\Lambda_{HC}$ . Either way, the prototype model is ruled out as a realistic composite-Higgs model.

The remedy might come from enlarging the fermion content of the hypercolor theory, driving it close to the conformal window, or even into it. Infrared conformal theories, as well as walking theories, have anomalous dimensions that (a) can be large, and (b) can stay roughly constant over many energy decades. In particular, a large anomalous dimension for the chimera operator  $Qqq$  could enhance the 4-fermion coupling  $G$  by several orders of magnitude, eventually leading to a phenomenologically acceptable partially-composite top.

Figure 2, adapted from Ref. [37], shows various analytical estimates for the conformal window for the SU(4) gauge theory in the fundamental–sextet plane. Our prototype “2 + 2 model” (with 2 Dirac fermions in each of the fundamental and sextet representations) is indicated by the black

square near the lower-left corner. Doubling the number of fermions in each representation brings us to the open circle, or “4 + 4 model.” One can see that the 2 + 2 model lies well below the sill of the conformal window according to all analytical estimates. By contrast, the 4 + 4 model would be inside the conformal window according to some estimates, or slightly below it according to the others. In addition, two models from the Ferretti-Karateev list: the M6 model mentioned before, as well as the M11 model, can be reached from the 4 + 4 model by giving some of the fermions a large mass.

In summary, the 4+4 model can be infrared conformal or near conformal according to analytical estimates, making it a promising candidate for a composite Higgs model. It would be interesting to study the 4 + 4 model using similar methods to those we have employed for the 2 + 2 model.

### Acknowledgements

Our calculations of staggered fermion propagators and currents were carried out with code derived from version 7.8 of the publicly available code of the MILC collaboration [38]. Computations for this work were carried out with resources provided by the USQCD Collaboration, which is funded by the Office of Science of the U.S. Department of Energy. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Awards No. DE-SC0010005 (Colorado) and DE-SC0013682 (SFSU), and by the Israel Science Foundation under grant No. 491/17 (Tel Aviv). Fermilab is operated by the Fermi Research Alliance, LLC under contract No. DE-AC02-07CH11359 with the U.S. Department of Energy.

### References

- [1] H. Georgi and D. B. Kaplan, *Composite Higgs and Custodial SU(2)*, Phys. Lett. B **145**, 216 (1984).
- [2] M. J. Dugan, H. Georgi and D. B. Kaplan, *Anatomy of a Composite Higgs Model*, Nucl. Phys. B **254**, 299 (1985).
- [3] R. Contino, *The Higgs as a Composite Nambu-Goldstone Boson*, in *Physics of the Large and the Small, TASI 09, Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 2009* (World Scientific, Singapore, 2011), pp. 235–306 [arXiv:1005.4269 [hep-ph]].
- [4] B. Bellazzini, C. Csáki and J. Serra, *Composite Higgses*, Eur. Phys. J. C **74**, no. 5, 2766 (2014) [arXiv:1401.2457 [hep-ph]].
- [5] G. Panico and A. Wulzer, *The Composite Nambu-Goldstone Higgs*, Lect. Notes Phys. **913**, pp.1 (2016) [arXiv:1506.01961 [hep-ph]].
- [6] D. B. Kaplan, *Flavor at SSC energies: A New mechanism for dynamically generated fermion masses*, Nucl. Phys. B **365**, 259 (1991).

- [7] V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir and B. Svetitsky, *Spectroscopy of  $SU(4)$  composite Higgs theory with two distinct fermion representations*, Phys. Rev. D **97**, no. 7, 074505 (2018) [arXiv:1710.00806 [hep-lat]].
- [8] V. Ayyar, T. DeGrand, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir and B. Svetitsky, *Baryon spectrum of  $SU(4)$  composite Higgs theory with two distinct fermion representations*, Phys. Rev. D **97**, no. 11, 114505 (2018) [arXiv:1801.05809 [hep-ph]].
- [9] V. Ayyar, T. DeGrand, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir and B. Svetitsky, *Finite-temperature phase structure of  $SU(4)$  gauge theory with multiple fermion representations*, Phys. Rev. D **97**, no. 11, 114502 (2018) [arXiv:1802.09644 [hep-lat]].
- [10] V. Ayyar, T. DeGrand, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir and B. Svetitsky, *Partial compositeness and baryon matrix elements on the lattice*, Phys. Rev. D **99**, no. 9, 094502 (2019) [arXiv:1812.02727 [hep-ph]].
- [11] V. Ayyar, M. Golterman, D. C. Hackett, W. Jay, E. T. Neil, Y. Shamir and B. Svetitsky, *Radiative Contribution to the Composite-Higgs Potential in a Two-Representation Lattice Model*, Phys. Rev. D **99**, no.9, 094504 (2019) [arXiv:1903.02535 [hep-lat]].
- [12] M. Golterman, W. I. Jay, E. T. Neil, Y. Shamir and B. Svetitsky, *Low-energy constant  $L_{10}$  in a two-representation lattice theory*, Phys. Rev. D **103**, no.7, 074509 (2021) [arXiv:2010.01920 [hep-lat]].
- [13] G. Cossu, L. Del Debbio, M. Panero and D. Preti, *Strong dynamics with matter in multiple representations:  $SU(4)$  gauge theory with fundamental and sextet fermions*, Eur. Phys. J. C **79**, no.8, 638 (2019) [arXiv:1904.08885 [hep-lat]].
- [14] G. Bergner and S. Piemonte, *Lattice simulations of a gauge theory with mixed adjoint-fundamental matter*, Phys. Rev. D **103**, no.1, 014503 (2021) [arXiv:2008.02855 [hep-lat]].
- [15] E. Bennett, D. K. Hong, J. W. Lee, C. J. D. Lin, B. Lucini, M. Piai and D. Vadicchino,  *$Sp(4)$  gauge theory on the lattice: towards  $SU(4)/Sp(4)$  composite Higgs (and beyond)*, JHEP **03**, 185 (2018) [arXiv:1712.04220 [hep-lat]].
- [16] E. Bennett, D. K. Hong, J. W. Lee, C. J. D. Lin, B. Lucini, M. Piai and D. Vadicchino,  *$Sp(4)$  gauge theories on the lattice:  $N_f = 2$  dynamical fundamental fermions*, JHEP **12**, 053 (2019) [arXiv:1909.12662 [hep-lat]].
- [17] E. Bennett, D. K. Hong, J. W. Lee, C. J. D. Lin, B. Lucini, M. Mesiti, M. Piai, J. Rantaharju and D. Vadicchino,  *$Sp(4)$  gauge theories on the lattice: quenched fundamental and antisymmetric fermions*, Phys. Rev. D **101**, no.7, 074516 (2020) [arXiv:1912.06505 [hep-lat]].
- [18] Bennett, J. Holligan, D. K. Hong, J. W. Lee, C. J. D. Lin, B. Lucini, M. Piai and D. Vadicchino, *Glueballs and strings in  $Sp(2N)$  Yang-Mills theories*, Phys. Rev. D **103**, no.5, 054509 (2021) [arXiv:2010.15781 [hep-lat]].

- [19] G. Ferretti and D. Karateev, *Fermionic UV completions of composite Higgs models*, JHEP **1403**, 077 (2014) [arXiv:1312.5330 [hep-ph]].
- [20] A. Belyaev, G. Cacciapaglia, H. Cai, G. Ferretti, T. Flacke, A. Parolini and H. Serodio, *Di-boson signatures as Standard Candles for Partial Compositeness*, JHEP **1701**, 094 (2017) [arXiv:1610.06591 [hep-ph]].
- [21] G. Ferretti, *UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group*, JHEP **1406**, 142 (2014) [arXiv:1404.7137 [hep-ph]].
- [22] G. Ferretti, *Gauge theories of Partial Compositeness: Scenarios for Run-II of the LHC*, JHEP **1606**, 107 (2016) [arXiv:1604.06467 [hep-ph]].
- [23] G. Cacciapaglia, S. Vatani and C. Zhang, *The Techni-Pati-Salam Composite Higgs*, Phys. Rev. D **103**, 055001 (2021) [arXiv:2005.12302 [hep-ph]].
- [24] E. Shintani *et al.* [JLQCD Collaboration], *S-parameter and pseudo-Nambu-Goldstone boson mass from lattice QCD*, Phys. Rev. Lett. **101**, 242001 (2008) [arXiv:0806.4222 [hep-lat]].
- [25] P. A. Boyle *et al.* [RBC and UKQCD Collaborations], *The S Parameter in QCD from Domain Wall Fermions*, Phys. Rev. D **81**, 014504 (2010) [arXiv:0909.4931 [hep-lat]].
- [26] J. Bijnens and J. Lu, *Two-Point Functions and S-Parameter in QCD-like Theories*, JHEP **1201**, 081 (2012) [arXiv:1111.1886 [hep-ph]].
- [27] T. DeGrand, M. Golterman, E. T. Neil and Y. Shamir, *One-loop Chiral Perturbation Theory with two fermion representations*, Phys. Rev. D **94**, no. 2, 025020 (2016) [arXiv:1605.07738 [hep-ph]].
- [28] A. Hasenfratz and F. Knechtli, *Flavor symmetry and the static potential with hypercubic blocking*, Phys. Rev. D **64**, 034504 (2001) [arXiv:hep-lat/0103029 [hep-lat]].
- [29] A. Hasenfratz, R. Hoffmann and S. Schaefer, *Hypercubic smeared links for dynamical fermions*, JHEP **05**, 029 (2007) [arXiv:hep-lat/0702028 [hep-lat]].
- [30] T. DeGrand, Y. Shamir and B. Svetitsky, *Suppressing dislocations in normalized hypercubic smearing*, Phys. Rev. D **90**, no.5, 054501 (2014) [arXiv:1407.4201 [hep-lat]].
- [31] O. Bär, G. Rupak and N. Shoresh, *Chiral perturbation theory at  $O(a^2)$  for lattice QCD*, Phys. Rev. D **70**, 034508 (2004) [arXiv:hep-lat/0306021 [hep-lat]].
- [32] O. Bär, C. Bernard, G. Rupak and N. Shoresh, *Chiral perturbation theory for staggered sea quarks and Ginsparg-Wilson valence quarks*, Phys. Rev. D **72**, 054502 (2005) [arXiv:hep-lat/0503009 [hep-lat]].
- [33] J. W. Chen, M. Golterman, D. O'Connell and A. Walker-Loud, *Mixed Action Effective Field Theory: An Addendum*, Phys. Rev. D **79**, 117502 (2009) [arXiv:0905.2566 [hep-lat]].



- [34] O. Bär, M. Golterman and Y. Shamir, *Flavor symmetry breaking in lattice QCD with a mixed action*, Phys. Rev. D **83**, 054501 (2011) [arXiv:1012.0987 [hep-lat]].
- [35] M. E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, Phys. Rev. D **46**, 381-409 (1992)
- [36] B. Svetitsky, V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil and Y. Shamir, *Towards a Composite Higgs and a Partially Composite Top Quark*, PoS **LATTICE2019**, 241 (2019) [arXiv:1911.10867 [hep-lat]].
- [37] B. S. Kim, D. K. Hong and J. W. Lee, *Into the conformal window: Multirepresentation gauge theories*, Phys. Rev. D **101**, 056008 (2020) [arXiv:2001.02690 [hep-ph]].
- [38] MILC Collaboration, <http://www.physics.utah.edu/~detar/milc/>