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A new method for a lattice calculation of $\eta_c \to 2\gamma$

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We propose a direct method to calculate the decay width of a hadron decaying to two photons, which is conventionally obtained by an extrapolation for various off-shell form factors. The new method provides an simple way to examine the finite-volune effects. As an example, we apply the method to $\eta_c \to 2\gamma$. Using three $N_f = 2$ twisted-mass gauge ensembles with different lattice spacings, we obtain the final decay width $\Gamma_{\eta_c\gamma\gamma} = 6.57(15)_{\mathrm{stat}}(8)_{\mathrm{syst}}$ keV, where the systematic error accounts for the fine-tuning of the charm quark mass. This method can also be applied for other processes which involve the leptonic or radiative particles in the final states.

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PoS(LATTICE2021)618

1. Introduction

Charmonium decay is a crucial object in particle physics because of its multiscale features [1]. It presents an ideal laboratory for connecting the perturbative and nonperturbative QCD, and tests the validity limit of these various approximations and techniques which work on both sides. In the nonrelativistic limit, the two-photon decay width of charmonium is directly related to the wave function at the origin [2], which acts as a fundamental role in the evaluation of charmonium decay and spectrum. The lowest charmonium decay $\eta_c \to 2\gamma$ has attracted extensive attentions from both experiments [3, 4] and theories [5–10].

For the lattice calculation on the decay width of $\eta_c \to 2\gamma$, the traditional approach is to calculate various off-shell form factors [7] or amplitude squares [10] with specific photon virtualities. The physical result is finally obtained in a continuous extrapolation with photon virtualities approaching to zero. Inevitably, it leads to model-dependent systematic errors. Besides, it also needs more computational resources since many off-shell form factors with different photon momenta are required. In Ref. [11], we have proposed a method to compute the on-shell form factor straightforwardly without calculating these off-shell form factors. This method could significantly reduce the computational costs, especially for a lattice calculation aimming for percent precision. In this talk, we give a brief introduction to the method.

2. Methodology

In framework of lattice QCD, the on-shell amplitude of a hadron decaying to two-photon is related to an infinite-volume hadronic tensor $\mathcal{F}_{\mu\nu}(p)$, which is given by

$$\mathcal{F}_{\mu\nu}(p) = \int dt\, e^{m_H t/2} \int d^3\vec{x}\, e^{-i\vec{p}\cdot\vec{x}} \mathcal{H}_{\mu\nu}(t,\vec{x}), \tag{1}$$

where the hadronic function $\mathcal{H}_{\mu\nu}(t,\vec{x})$ is defined as

$$\mathcal{H}_{\mu\nu}(t,\vec{x}) = \langle 0|\mathrm{T}[J_\mu^{em}(x)J_\nu^{em}(0)]|H(k)\rangle, \tag{2}$$

here we denote the initial hadronic state as $|H(k)\rangle$ with the momentum $k = (im_H, \vec{0})$. The electromagnetic current is chosen as $J_\mu^{em} = \sum_q e_q \bar{q}\gamma_\mu q$ ($e_q = 2/3, -1/3, -1/3, 2/3$ for $q = u, d, s, c$). One of the photon momenta is indicated as $p = (im_H/2, \vec{p})$ with $|\vec{p}| = m_H/2$, which satisfies the on-shell condition. If the initial state is a pseudo-scalar particle, the hadronic tensor $\mathcal{F}_{\mu\nu}(p)$ can be parameterized as

$$\mathcal{F}_{\mu\nu}(p) = \epsilon_{\mu\nu\alpha\beta} p_\alpha k_\beta F_{H\gamma\gamma}. \tag{3}$$

By multiplying $\epsilon_{\mu\nu\alpha\beta} p_\alpha k_\beta$ to both sides, the on-shell form factor is extracted through

$$F_{H\gamma\gamma} = -\frac{1}{2m_H|\vec{p}|^2} \int d^4x\, e^{-ipx} \epsilon_{\mu\nu\alpha 0} \frac{\partial \mathcal{H}_{\mu\nu}(t,\vec{x})}{\partial x_\alpha}. \tag{4}$$

After averaging over the spatial direction for $\vec{p}$, $F_{H\gamma\gamma}$ would be obtained by

$$F_{H\gamma\gamma} = -\frac{1}{2m_H} \int d^4x\, e^{\frac{m_H}{2}t}\, \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|}\, \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(t,\vec{x}), \tag{5}$$

where $j_n(x)$ are the spherical Bessel functions. The decay width is then given as

$$\Gamma_{H\gamma\gamma} = \alpha^2 \frac{\pi}{4} m_H^3 F_{H\gamma\gamma}^2 \tag{6}$$

More recently, the infinite-volume reconstruction method has been proposed to reconstruct the infinite-volume hadronic function from the finite-volume ones [12–20]. In this work, we apply it for $\eta_c \to 2\gamma$ decay. For the time integral in Eq. (5), it is natural to introduce an integral truncation t_s, leading to the contribution of $F_{H\gamma\gamma}(t_s)$. For sufficiently large region $t > t_s$, the hadronic function $\mathcal{H}_{\mu\nu}(t,\vec{x})$ is dominated by the J/ψ state if only connected diagrams included. Such part of contribution is then calculated as

$$\delta F_{\eta_c\gamma\gamma}(t_s) = -\frac{1}{2m_{\eta_c}} \frac{e^{|\vec{p}|t_s}}{\sqrt{m_{J/\psi}^2 + |\vec{p}|^2} - |\vec{p}|} \times \int d^3\vec{x}\, \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(t_s,\vec{x}). \tag{7}$$

Finally, we combine these two parts together to obtain the total contribution

$$F_{\eta_c\gamma\gamma} = F_{\eta_c\gamma\gamma}(t_s) + \delta F_{\eta_c\gamma\gamma}(t_s), \tag{8}$$

In the simulation, the hadronic function in the small time region $t < t_s$ is calculated in a finite volume, as denoted by $\mathcal{H}_{\mu\nu}^L(t,\vec{x})$. The infinite-volume $\mathcal{H}_{\mu\nu}(t,\vec{x})$ is replaced by $\mathcal{H}_{\mu\nu}^L(t,\vec{x})$. Such replacement works well because i) the difference of $\mathcal{H}_{\mu\nu}^L(t,\vec{x})$ and $\mathcal{H}_{\mu\nu}(t,\vec{x})$ is caused by the exponentially supressed finite-volume effects. ii)such finite-volume effects are easy to be verified numerically by introducting a spatial integral truncation R and examining the R-dependence of $F_{H\gamma\gamma}$.

3. Lattice setup

The simulation is performed using three $N_f = 2$ flavour twisted mass gauge field ensembles generated by the Extended Twisted Mass Collaboration (ETMC) [21] with lattice spacing $a \simeq 0.0667, 0.085, 0.098$ fm. The ensemble parameters are reported in Table. 1. We tune the charm quark mass by setting the lattice result of 1) η_c mass to its physical value and 2) J/ψ mass to its physical value, respectively, and regard the deviation in both cases as systematic error in our analysis. For simplicity, we add the suffix "-I" and "-II" to the ensemble name to specify the cases of 1) and 2).

Table 1: Parameters of gauge ensembles. From left to right, we list the lattice spacing a, pion mass m_π, lattice volume=spatial^3×temporal, light twisted mass $a\mu_l$, the number of configurations and timeslice $N_{\rm conf} \times T_s$ and a series of time separation Δt used for extracting the ground-state contribution. All the Δt lies in the range of 0.7 – 1.6 fm.

Ensemble	a(fm)	m_π(MeV)	$L^3 \times T$	$a\mu_l$	$N_{\rm conf} \times T_s$	$\Delta t/a$
a98	0.098	365	$24^3 \times 48$	0.006	236×48	7:17
a85	0.085	315	$24^3 \times 48$	0.004	200×48	8:18
a67	0.0667	300	$32^3 \times 64$	0.003	197×64	10:24

The hadronic function $\mathcal{H}_{\mu\nu}$ is calculated by three-point correlation function $\Gamma^{(3)}_{\mu\nu}(x, y, t_i) = \langle J^{em}_\mu(x) J^{em}_\nu(y) O^\dagger_{\eta_c}(t_i)\rangle$ with $t_i = \min\{t_x, t_y\} - \Delta t$. Specifically, it is given as

$$\mathcal{H}_{\mu\nu}(t_x - t_y, \vec{x} - \vec{y}) = \Gamma^{(3)}_{\mu\nu}(x, y, t_i)/[(Z_0/2m_{\eta_c})e^{-m_{\eta_c}(t_y - t_i)}]. \tag{9}$$

where $Z_i = \frac{1}{\sqrt{V}}\langle i|O^\dagger_{\eta_c}|0\rangle (i = 0, 1)$ and $V = L^3$. Z_0 and m_{η_c} are obtained by fitting the two-point function $\Gamma^{(2)}_{\eta_c\eta_c}(t) = \langle O_{\eta_c}(t) O^\dagger_{\eta_c}(0)\rangle$ with a two-state form

$$\Gamma^{(2)}_{\eta_c\eta_c}(t) = V \sum_{i=0,1} \frac{Z_i^2}{2E_i}\left(e^{-E_i t} + e^{-E_i(T-t)}\right) \tag{10}$$

where $E_1, E_0 = m_{\eta_c}$ are the first- and ground-state energies of η_c meson. The η_c propagator is solved by using a stochastic Z_4 noise. We also apply the gauss smearing [22] for the η_c quark field and APE smearing [23] for the gauge field to reduce the excited-state effects. It is found that the excited-state contamination is significant in the region $\Delta t \lesssim 1.6$ fm when the precison reaches 1-3%. Such excited-state contribution can be taken into account by choosing a series of different Δt and the final form factor is extracted by a multi-state fit

$$F_{\eta_c\gamma\gamma}(\Delta t) = F_{\eta_c\gamma\gamma} + \xi\, e^{-(E_1 - E_0)\Delta t}, \tag{11}$$

where $F_{\eta_c\gamma\gamma}$ and ξ are two undeterminted parameters. We use local current $J^{em}_\mu(x) = Z_V e_c \bar{c}\gamma_\mu c(x)$ as the photon interpolating operator. The vector renormalization facotor Z_V is estimated by a modified ratio of the two-point function over three-point function

$$\frac{\Gamma^{(2)}_{\eta_c\eta_c}(t)}{\Gamma^{(3)}_{\eta_c\gamma_0\eta_c}(t)} = Z_V \cdot \left[1 + e^{-m_{\eta_c}(T-2t)}\right] \tag{12}$$

where $\Gamma^{(3)}_{\eta_c\gamma_0\eta_c} = \sum_{\vec{x}}\langle O_{\eta_c}(t) J^{(c)}_0(t/2, \vec{x}) O^\dagger_{\eta_c}(0)\rangle$ denotes the three-point function with a local operator $J^{(c)}_0(x) = \bar{c}\gamma^0 c(x)$ inserted between the initial and final η_c state with zero momentum. Compared with the conventional ratio [24], the modified one have accounted for the the around-of-world effect in $\Gamma^{(2)}_{\eta_c\eta_c}$, which leads to a distinct systematic effect at $t \simeq T/2$.

4. Lattice results

The lattice results of $F_{\eta_c\gamma\gamma}$ as a function of the truncation t_s with different separations Δt are shown in left panel of Figure. 1, here we take the ensemble a67-I as an example. The integral in Eq. (5) is performed with all $\vec{x}$ summed up and t truncated by t_s. We find that a temperal truncation at $t_s \simeq 1$ fm is a conservative choice. The results of $F_{\eta_c\gamma\gamma}$ at such truncation are shown in the right pannel. It shows that $F_{\eta_c\gamma\gamma}$ has an obvious Δt dependence, leading to a significant excited-state effect. Finally, we extract the ground-state contribution to $F_{\eta_c\gamma\gamma}$ by a two-state fit in Eq. (11).

The new method allows us to examine the finite-volume effects easily. For the integral in Eqs. (5) and (7), it is natural to introduce a spatial integral truncation R. The size of the integrand is exponentially suppressed as the $|\vec{x}|$ increases, since hadronic function $\mathcal{H}_{\mu\nu}(x)$ is dominated by the J/ψ state at large $|\vec{x}|$. The form factor $F_{\eta_c\gamma\gamma}$ as a function of R is shown in Figure. 2. It is

PoS(LATTICE2021)618

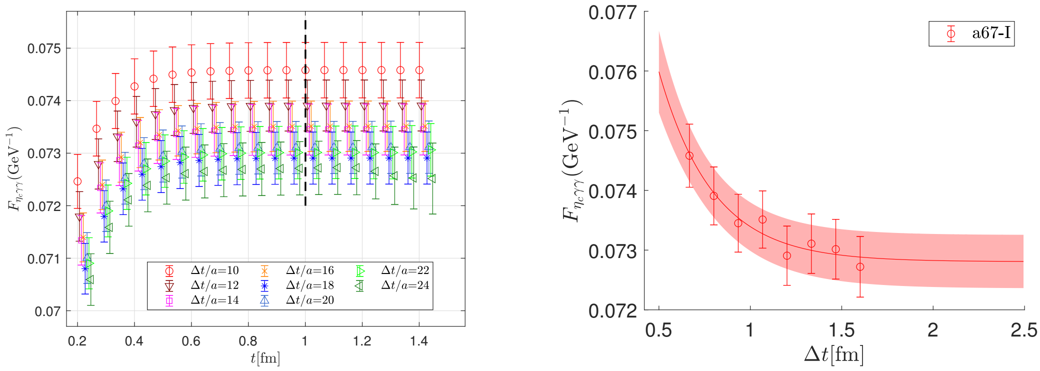

Figure 1: For ensemble a67-I: In the left panel, lattice results of $F_{\eta_c\gamma\gamma}$ as a function of the integral truncation t_s with different separation Δt. The dashed black line denotes the temporal cut $t_s \simeq 1$ fm; In the right panel, the ground-state extrapolation for the $F_{\eta_c\gamma\gamma}$.

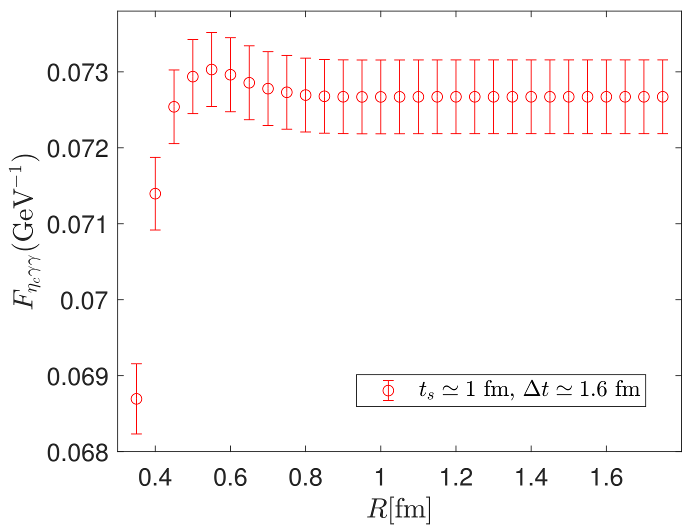

Figure 2: For ensemble a67-I, $F_{\eta_c\gamma\gamma}$ with $t_s \simeq 1$ fm and $\Delta t \simeq 1.6$ fm as a function of the spatial range truncation R.

clear that the plateau emerges at almost $R \simeq 0.8$ fm, indicating the hadronic function $\mathcal{H}_{\mu\nu}(x)$ at $|\vec{x}| \gtrsim 0.8$ fm has negligible contribution to $F_{\eta_c\gamma\gamma}$. All the lattice sizes of ensembles used in this work are greater than 2 fm, which is large enough to accommodate the hadronic system. Therefore, the finite-volume effects are well-controlled in our calculation.

The results of decay width $\Gamma_{\eta_c\gamma\gamma}$ are shown in Figure. 3. A linear extrapolated behavior is well consistent with the lattice values. In the continuum limit $a^2 \to 0$, we obtain

$$\Gamma_{\eta_c\gamma\gamma} = \begin{cases} 6.49(19)\ \text{keV} & \mu_c \text{ scaled by } \eta_c \\ 6.57(15)\ \text{keV} & \mu_c \text{ scaled by } J/\psi \end{cases} \tag{13}$$

where the systematic uncertainty due to fine-tuning of charm quark mass is smaller than the statistical

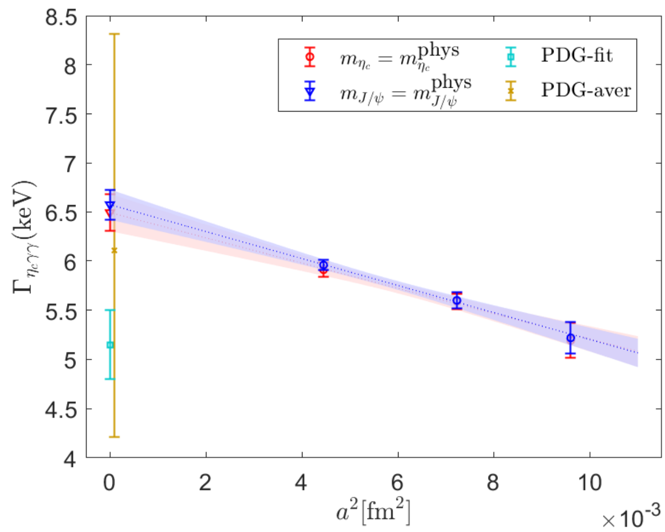

Figure 3: Decay width $\Gamma_{\eta_c\gamma\gamma}$ under three different lattice spacings. Experimental results of PDG-fit and PDG-aver are denoted by cyan square and orange cross, respectively. The latter is shifted horizontally for a clear comparison.

error. We take the value of $m_{J/\psi} \simeq m^{\text{phys}}_{J/\psi}$ as the final result and the deviation as the systematic error. Finally, the decay width is reported as

$$\Gamma_{\eta_c\gamma\gamma} = 6.57(15)_{\text{stat}}(8)_{\text{syst}}\,\text{keV}, \tag{14}$$

For the experimental results, there are two different values quoted by PDG:(i) world average of the experiments $B(\eta_c \to 2\gamma) = 1.9^{+0.7}_{-0.6} \times 10^{-4}$; (ii) combined fit value $B(\eta_c \to 2\gamma) = (1.61 \pm 0.12) \times 10^{-4}$. For simplicity, we denote the former as 'PDG-aver' and the latter as 'PDG-fit'. Our lattice result is larger than the PDG-fit by 28% with a 3.6-σ deviation, but compatible with PDG-aver as it carries a large uncertainty. The two PDG values are also presented in Figure. 3 for a comparison. A recent paper using Dyson-Schwinger equation obtains the decay width as $\Gamma = 6.3 \sim 6.4$ keV [6]. Another study from NRQCD including the next-to-next-to-leading-order perturbative correction gives the branching fraction as $B(\eta_c \to 2\gamma) = 3.1 \sim 3.3 \times 10^{-4}$ [5], which is larger than other theoretical and experimental results. For more detailed discussions on these discrepancies, we refer interested readers to Ref. [11].

5. Conclusion

In this work we propose a new method to calculate the matrix element of $\eta_c \to 2\gamma$, where the on-shell form factor is obtained straightforwardly by constructing an appropriate hadroinc scalar function. It is easy and efficient to examine the finite-volume effects with this method. Besides, it can also be applied for other processes which involve the leptonic or radiative particles in the final states, for example, $\pi_0 \to 2\gamma$ [25], $J/\psi \to 3\gamma$ [26] and radiative leptonic decays $K^- \to \ell^-\bar{\nu}\gamma$, $D_s^+ \to \ell^+\nu\gamma$, $B \to \ell^-\bar{\nu}\gamma$ [27, 28]. In the continuous extrapolation with three lattice spacings, we obtain the decay width $\Gamma(\eta_c \to 2\gamma) = 6.57(15)_{\text{stat}}(8)_{\text{syst}}$ keV, where the systematic effects of excited-state contamination, finite volume and fine-tuning of charm quark mass are under well control.

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References

[1] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011), 1010.5827.

[2] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995), hep-ph/9407339, [Erratum: Phys.Rev.D 55, 5853 (1997)].

[3] G. S. Adams *et al.*. (CLEO Collaboration), Phys. Rev. Lett. **101**, 101801 (2008), 0806.0671.

[4] M. Ablikim *et al.*. (BESIII Collaboration), Phys. Rev. D. **87**, 032003 (2013), 1208.1461.

[5] F. Feng, Y. Jia, and W.-L. Sang, Phys. Rev. Lett. **119**, 25201 (2017), 1707.05758.

[6] J. Chen, M. H. Ding, L. Chang and Y. X. Liu, Phys. Rev. D. **95**, 016010 (2017), 1611.05960.

[7] J. J. Dudek and R. G. Edwards, Phys. Rev. Lett. **97**, 172001 (2006), 0607140.

[8] T. Chen *et al.*. (CLQCD Collaboration), Eur. Phys. J. C. **76**, 358 (2016), 1602.00076.

[9] Y. Chen *et al.*. (CLQCD Collaboration), Chin.Phys.C. **44**, 083108 (2020), 2003.09817.

[10] C. Chuan, Y. Meng, and K.-L. Zhang, Phys. Rev. D. **102**, 034502 (2020), 2004.03907.

[11] Y. Meng, X. Feng, C. Liu, T. Wang and Z.-H. Zou , (2021), 2109.09381.

[12] X. Feng and L. C. Jin, Phys. Rev. D. **100**, 094509 (2019), 1812.09817.

[13] X. Y. Tuo, X. Feng, and L. C. Jin, Phys. Rev. D. **100**, 094511 (2019), 1909.13525.

[14] X. Feng, Y. Fu, and L. C. Jin, Phys. Rev. D. **101**, 051502 (2020), 1911.04064.

[15] X. Feng *et al.* Phys. Rev. Lett. **124**, 192002 (2020), 2003.09798.

[16] N. H. Christ, X. Feng, L. C. Jin, and C. T. Sachrajda, PoS. **LATTICE2019**, 259(2020).

[17] N. H. Christ *et al.* Phys. Rev. D. **103**, 014507 (2021), 2009.08287.

PoS(LATTICE2021)618

[18] P.-X. Ma *et al.* Phys. Rev. D. **103**, 114503 (2021), 2102.12048.

[19] X. Y. Tuo, X. Feng, L. C. Jin and T. Wang, (2021), 2103.11331.

[20] X. Feng, L.-C. Jin and M. J. Riberdy, (2021), 2108.05311.

[21] P. Boucaud *et al.*. (ETM Collaboration) Phys. Lett. B. **650**, 304 (2007),0701012; Comput.Phys.Commun. **179**, 695 (2008),0803.0224.

[22] S. Gusken, Nucl. Phys. Proc. Suppl **17**, 361 (1990).

[23] M. Albanese *et al.*. (APE Collaboration) Phys. Lett. B. **192**, 163 (1987).

[24] J. J. Dudek, R. G. Edwards and D. G. Richards, Phys. Rev. D. **73**, 074507 (2006), 061137.

[25] X. Feng *et al.* Phys. Rev. Lett. **109**, 182001 (2012), 1206.1375.

[26] Y. Meng, C. Liu and K. L. Zhang, Phys. Rev. D. **102**, 054506 (2020), 1910.11597.

[27] C. Kane,C. Lehner, S. Meinel and A. Soni, Pos . **LATTICE2019**, 134 (2019), 1907.00279.

[28] A. Desiderio *et al.* Phys. Rev. D. **103**, 014502 (2021), 2006.05358.