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Lattice Study of the Dibaryon System

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We presented a lattice-QCD calculation of dibaryon matrix elements with weak current insertions by using point-splitting interpolating field operators. Our method of calculating two-point, three-point and four-point correlation functions is described in detail and we make a comparison between the correlation functions obtained from our point-splitting operators and point operators and report the preliminary results. We found that the excited-state contamination is reduced by using point-splitting operators and our lattice result of proton axial charge is consistent with the results from RBC/UKQCD collaborations[1] and the matrix elements of proton-proton fusion and double beta decay agree with the results from NPLQCD collaboration[2, 3].

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1. Introduction

Dibaryon system plays an essential role in our understanding of fundamental physics and many-body system. In recent years, *ab initio* nuclear many-body methods have made significant progress and are entering an era of precision nuclear physics[4]. How to constrain the uncertainties from the nuclear Hamiltonian, which now dominates over the uncertainties from many-body methods, has become a crucial question[4]. One of the possible ways to constrain the nuclear forces is to match effective field theory (EFT) interactions to corresponding lattice QCD results, which is especially important for the observables that are difficult to access experimentally[4, 5]. One of the examples is the neutrinoless double beta decay. Once detected, it would prove that neutrinos are Majorana fermions. For the double beta decays of the pion, the lattice results of the decay amplitudes can be matched with EFT formula to obtain the relevant low energy constants [6–9]. For the more complicated dibaryon system, the pioneering calculation of two-neutrino double beta decay has been carried out by NPLQCD Collaboration [2, 3]. For the neutrinoless double beta decay of the dibaryon system, it is computationally more demanding. In Ref. [10] we propose to reduce the finite-volume effects by using a massive neutrino. In this work, we report the progress made in the realistic lattice calculation of the dibaryon system.

The main difficulty of performing lattice calculation on dibaryon system comes from two aspects. First, the contraction of dibaryon correlation functions is much more complicated than mesons or single baryon systems so an effective algorithm is essential in order to complete the calculation with a reasonable time. The second difficulty is due to the well-known signal-to-noise problem that the dibaryon correlation functions become very noisy at large time slices while the data at small time slices suffer from strong excited-state contamination. Several multi-baryon contraction algorithms have been proposed[11, 12] and the pioneering calculation of proton-proton fusion and double beta decay matrix elements has been performed by NPLQCD collaboration[2, 3]. In addition, the HAL QCD collaboration proposed alternative approach[13–15] to overcome these difficulties.

In this work, we adopt the point-splitting interpolating field operators to reduce the excited-state contamination in our lattice calculation. We will present our contraction algorithm of calculating dibaryon two-point correlation functions and its extension to three-point and four-point correlation functions with weak current insertions.

2. Methodology

In this work, we introduced point-splitting operators (PS) in addition to traditional point operators (P)[1]. For point operators, all six quarks are located at one point while for point-splitting operators, we put one baryon at one point and the other baryon at another point. For example, the

[1]The point operators here are called hexquark operators and the point-splitting operators are called bilocal operators in the recent work by NPLQCD collaboration[16].

point and point-splitting interpolating field operators of deuteron with spin $s_z = 1$ are defined as,

$$d(s_z = 1, \mathrm{P}) = \sum_x p(\uparrow, x) n(\uparrow, x), \tag{1}$$

$$d(s_z = 1, \mathrm{PS}) = \sum_{x_1, x_2} \frac{1}{2} (p(\uparrow, x_1) n(\uparrow, x_2) + p(\uparrow, x_1) n(\uparrow, x_2)) \tag{2}$$

The main purpose of introducing point-splitting operators is to reduce the excited-state contamination. This can be seen by expressing these two kinds of operators in momentum space:

$$\tilde{d}(s_z = 1, \mathrm{P}) = \sum_k \tilde{p}(\uparrow, k) \tilde{n}(\uparrow, -k), \tag{3}$$

$$\tilde{d}(s_z = 1, \mathrm{PS}) = \frac{1}{2} (\tilde{p}(\uparrow, 0) \tilde{n}(\uparrow, 0) + \tilde{p}(\uparrow, 0) \tilde{n}(\uparrow, 0)), \tag{4}$$

where $\tilde{d}, \tilde{p}, \tilde{n}$ are operators of deuteron, proton and neutron in momentum space. We will demonstrate numerically in section 3.2 that the point-splitting operators cause much smaller excited-state effects. Since the contraction for point-splitting operators are more complicated, we developed an improved algorithm based on the idea from [11, 12].

To calculate three-point and four-point correlation functions, we employ isospin rotation method together with the sequential-source propagators. For the three point correlation function of $pp \to d$ process, the initial state is $|I, I_z\rangle = |1, 1\rangle$ and the final state is $|I, I_z\rangle = |0, 0\rangle$. Wigner-Eckart theorem told us that

$$\left\langle 0, 0 \middle| \hat{O}^1_{-1} \middle| 1, 1 \right\rangle = \langle 1, 1; 1, -1 \mid 1, 1; 0, 0 \rangle \frac{\left\langle 0 \middle\| \hat{O}^1 \middle\| 1 \right\rangle}{\sqrt{3}}, \tag{5}$$

where $\langle 1, 1; 1, -1 \mid 1, 1; 0, 0 \rangle$ is a known Clebsch-Gordan (CG) coefficient and $\left\langle 0 \middle\| \hat{O}^1 \middle\| 1 \right\rangle$ is irrelevant to I_z. To evaluate $\left\langle 0 \middle\| \hat{O}^1 \middle\| 1 \right\rangle$, we may consider a process without quark flavour changing which can be calculated by using sequential-source propagators. We choose $pn(^1\mathrm{S}_0) \to d$ process in this work and found that,

$$\left\langle d \middle| \hat{O}^1_{-1} \middle| pp \right\rangle = - \left\langle d \middle| \hat{O}^1_0 \middle| pn(^1\mathrm{S}_0) \right\rangle. \tag{6}$$

For the four point correlation function of $nn \to pp$ process, I_z of the initial and final states has changed two units in total due to two weak current insertions. In order to calculate the matrix element of $\left\langle pp \middle| \hat{O}^2_2 \middle| nn \right\rangle$, we need to consider two flavour conserving processes to eliminate the unwanted contribution from $\hat{O}^0_0$. Again from Wigner-Eckart theorem and the CG coefficients we found that,

$$\left\langle pp \middle| \hat{O}^2_2 \middle| nn \right\rangle = \frac{\sqrt{6}}{3} \left(\left\langle pp \middle| \hat{O}_0 \middle| pp \right\rangle - \left\langle pn(^1\mathrm{S}_0) \middle| \hat{O}_0 \middle| pn(^1\mathrm{S}_0) \right\rangle \right). \tag{7}$$

By using eq. (6) and eq. (7), we are able to get the desired nuclear matrix elements of proton-proton fusion and double beta decay through calculating some flavour-conserving matrix elements.

The flavour-conserving matrix elements can be obtained by examining the time dependence of three-point and four-point correlation functions. For three point function with one current insertion,

we found that

$$C_3(t) = \sum_{t'=0}^{T-1} \left\langle \Omega \left| T\left\{ O(t)\mathcal{J}(t')\, O^\dagger(0)\right\}\right| \Omega \right\rangle, \tag{8}$$

$$= \sum_n \frac{\mathrm{e}^{-E_n t}}{2E_n}\left((t-1)J_{nn}\frac{Z_n^{\mathbf{0}} Z_n^\dagger}{2E_n} + d_n\right) + \sum_{m\neq n}\frac{Z_n^{\mathbf{0}} Z_m^\dagger}{4E_n E_m}\frac{e^{-E_n t - \frac{\Delta_{mn}}{2}} - e^{-E_m t - \frac{\Delta_{nm}}{2}}}{e^{\frac{\Delta_{mn}}{2}} - e^{\frac{\Delta_{nm}}{2}}} J_{nm}, \tag{9}$$

where $Z_n^{\boldsymbol{p}} = \sum_{\boldsymbol{x}} \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}} \langle \Omega \,|\, O(0,\boldsymbol{x}) \,|\, n\rangle$, $Z_n^\dagger = \left\langle n \left| O^\dagger(0,\mathbf{0}) \right| \Omega \right\rangle$, $J_{nm} = \langle n \,|\, \mathcal{J} \,|\, m\rangle$, $\Delta_{mn} = E_m - E_n$. The detailed analysis has been given by [17], and we adopted their notation here. In this notation, the two-point correlation function can be written as,

$$C_2(t) = \sum_n \frac{Z_n^{\mathbf{0}} Z_n^\dagger}{2E_n}\mathrm{e}^{-E_n t}. \tag{10}$$

We noticed that the desired matrix element can be extracted from the ratio,

$$R_3(t) = \frac{C_3(t)}{C_2(t)} = \left(\frac{1}{2E_0}\langle f \,|\, J \,|\, i\rangle\right) t + (\text{time independent terms}) + (\text{expoentially suppressed terms}). \tag{11}$$

This equation tells us that we are able to extract the matrix element of proton-proton fusion $\langle d \,|\, \mathcal{J} \,|\, pp\rangle$ from the term proportional to t.

For four point function with two current insertions, there are three possible time-orderings since these two currents are identical in our case,

$$\mathrm{I}: 0 < t' < t'' < t, \tag{12}$$

$$\mathrm{II}: 0 < t' < t < t'', \tag{13}$$

$$\mathrm{III}: 0 < t < t' < t''. \tag{14}$$

We found that only case I contains polynomially time-dependent terms and we reached

$$C_4(t) = \sum_{t'=0}^{T-1}\sum_{t''=0}^{T-1}\left\langle 0 \left| T\left\{ O(t)\mathcal{J}(t')\,\mathcal{J}(t'')\, O^\dagger(0)\right\}\right| 0 \right\rangle, \tag{15}$$

$$= \sum_{n,\alpha}\frac{J_{n\alpha}J_{\alpha n} Z_n Z_n^\dagger}{8E_n^2 E_\alpha}\mathrm{e}^{-E_n t}\left(\frac{t}{\Delta} - \frac{1-\mathrm{e}^{-\Delta t}}{\Delta^2}\right) + (\text{round-world effects, expoentially suppressed}), \tag{16}$$

where $\Delta \equiv E_\alpha - E_n$ is the energy difference between initial states and intermediate states. If $\Delta \approx 0$, we have $(t/\Delta - (1-\mathrm{e}^{-\Delta t})/\Delta^2) = t^2/2 + O(\Delta)$, which is the case of the long-distance contribution in $2\nu 2\beta$ process, in which the intermediate state is a deuteron whose mass is very close to a dineutron or diproton.

Similar to the three-point function, we eliminate the coefficients by considering the ratio of $C_4(t)/C_2(t)$,

$$R_4(t) = \frac{C_4(t)}{C_2(t)} = \underbrace{\left.\frac{\langle f \,|\, J \,|\, \alpha\rangle \langle \alpha \,|\, J \,|\, i\rangle}{8E_0 E_\alpha} t^2\right|_{E_\alpha \approx E_0}}_{\text{long-distance term}} + \underbrace{\sum_{E_\alpha \neq E_0}\frac{\langle f \,|\, J \,|\, \alpha\rangle \langle \alpha \,|\, J \,|\, i\rangle}{4E_0 E_\alpha (E_\alpha - E_0)} t}_{\text{short-distance term}}$$

$$+ (\text{t-independent terms \& exp. suppressed terms}) \tag{17}$$

With this equation, we are able to extract the long-distance contribution of two-neutrino double beta decay from the t^2-dependent term of the four-point correlation function while extracting the short-distance contribution from the t-dependent term.

3. Lattice Calculation

3.1 Lattice Setup

We adopted the gauge-configuration ensembles with domain wall fermion and Iwasaki gauge action in our calculation. Detailed information is listed in table 1. Currently, 42 configurations were generated for the ensemble 24I and 235 configurations were generated for 16I. For a given configuration, at every two time slices, we calculated propagators with 8 different source locations and sequential-source propagators with 4 source locations. The random field selection method was also applied after smearing to save disk and the cost of spatial summation[18].

Table 1: Information about the lattice setup. The lattice spacing, pion mass and Z_A is taken from the work by RBC/UKQCD[1, 19–21].

	β	$L^3 \times T$	am_l	L(fm)	a^{-1}(GeV)	m_π(MeV)	Z_A
24I	2.13	$24^3 \times 64$	0.01	~ 2.7	1.7844(49)	432.2(1.4)	0.71759(56)
16I	2.13	$16^3 \times 32$	0.01	~ 1.8	1.73(3)	438(3)	0.72222(8)

3.2 Preliminary Results

In this section we will present some preliminary results to demonstrate our method and all the results in this section are obtained using the correlated fitting.

3.2.1 Results form Two-point Correlation Functions

We presented the effective mass of a diproton system in fig. 1. We found that it's almost impossible to find a plateau before the noise overwhelms the signal in the results from ensemble 16I, so the results are mainly reported using ensemble 24I. From the picture on the right panel we noticed that the excited-state contamination has been significantly reduced by introducing PSPS correlators as we expected, thus we are able to choose a relatively early time window for fitting without worrying about excited states. Since the signal-to-noise ratio problem of dibaryon system prohibited us from getting reliable data at large time slices, this feature of PSPS correlation functions allows us to fit the data at an earlier time window.

The mass difference between deuteron and $pn(1S0)$ (which has the same effective mass as dineutron or diproton in our simulation) is an example, and it will be used in the analysis of double beta decay. With the help of point-splitting operators, we are able to obtain a nonzero mass difference from our data.

3.2.2 Results form Three-point Correlation Functions

The first result from three-point correlation function is the proton axial charge g_A which is used to normalize the results of dibaryon system. The fitting may contains severe excited-state

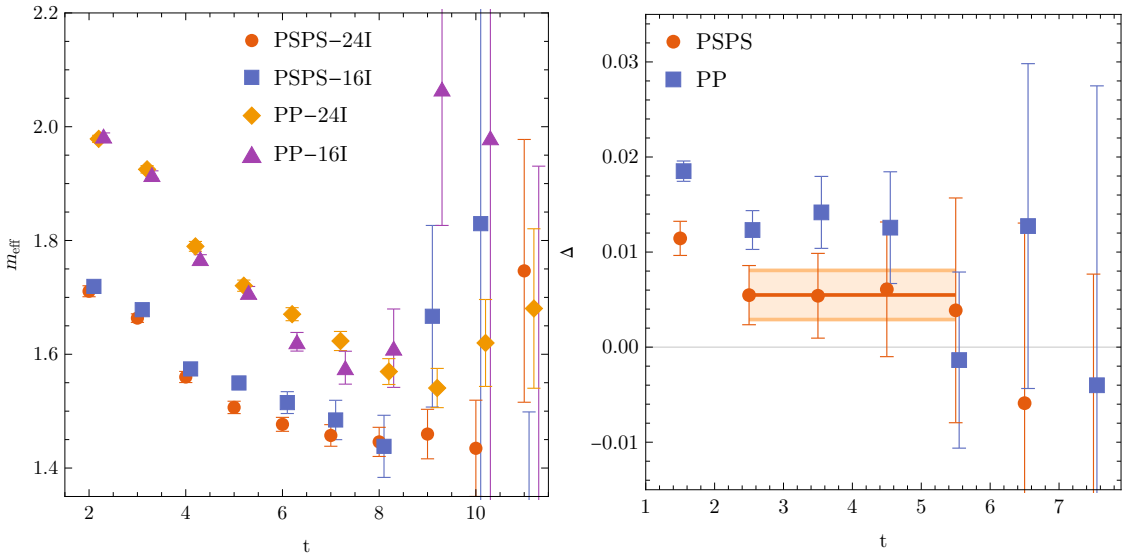

Figure 1: The left panel shows the effective mass of diproton system and the right panel shows the mass difference $\Delta \equiv E_{\text{pn}(1S0)} - E_{\text{pn}(3S1)}$. Data points with different setup are slightly offset in t for clarity. We used 235 configurations for ensemble 16I while 42 configurations for 24I.

contamination[22] so we performed a multi-state fit to check our results, which is presented in the left panel of fig. 2. Our fitting result of g_A is,

$$g_A|_{\text{single-state}} = 1.14(6),\ g_A|_{\text{multi-state}} = 1.16(6) \tag{18}$$

which is consistant with each other and it also agrees with the result from RBC/UKQCD with the same lattice setup[1].

The second result is the matrix element of proton-proton fusion, which is presented in fig. 2. We noticed that the slope of curve from PP correlators is significantly greater than that form PSPS correlators at earlier time slices and is overwhelmed by the noise at later time slices. This behavior agrees with the effective mass curve in fig. 1 that the PP correlators suffered more severe excited-state contamination than the PSPS correlators. Our fitting result using PSPS correlators is,

$$\frac{\langle pp | \tilde{J}_3 | d \rangle}{g_A} = 1.06(28), \tag{19}$$

which is consistent with [2, 3], in spite of the fact that our calculation is performed at a smaller pion mass.

3.2.3 Results from four-point correlation function

The fitting of long-distance and short-distance matrix element of double beta decay is presented in fig. 3. We noticed that the fitting results from PSPS correlators are more stable when the fitting

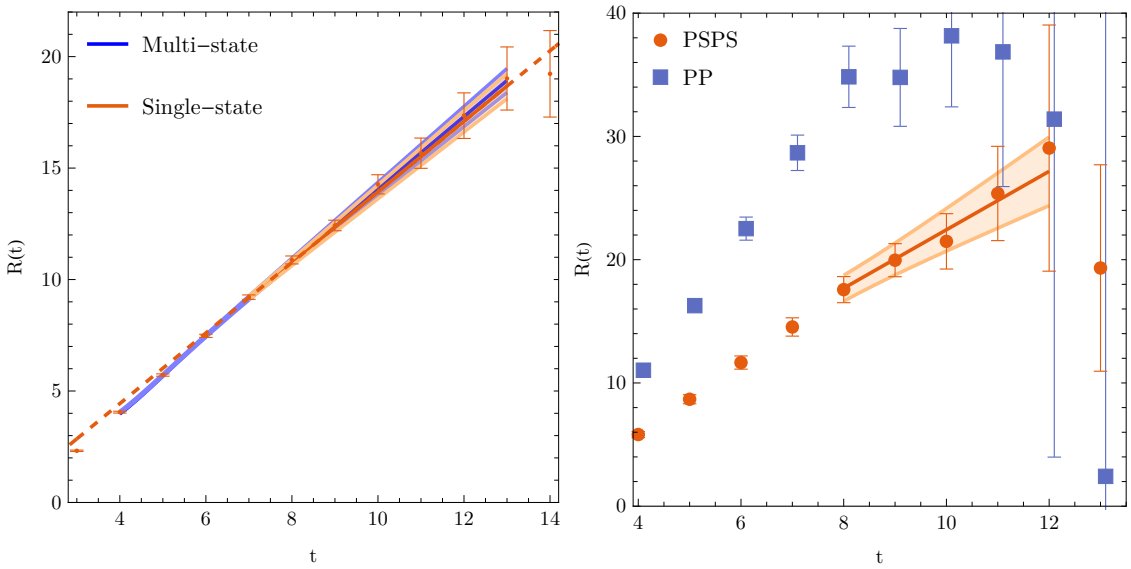

Figure 2: The left panel shows the single-state (orange) and multi-state (blue) fitting of proton axial charge. The right panel shows the fitting of matrix element of proton-proton fusion. The orange points represents data from point-splitting source point-splitting sink correlation functions and the blue points represents data from point source point sink correlation functions.

windows changes compared to the PP correlators. Our fitting result is,

$$\frac{\Delta}{g_A^2}\frac{\langle pp\,|\,J\,|\,d\rangle\,\langle d\,|\,J\,|\,nn\rangle}{\Delta} = 1.01(19), \tag{20}$$

$$\frac{\Delta}{g_A^2}\sum_{l\neq d}\frac{\langle pp\,|\,J\,|\,l\rangle\,\langle l\,|\,J\,|\,nn\rangle}{\delta_l} = -0.041(22), \tag{21}$$

which is again consistent with the result from NPLQCD[3].

4. Conclusion

We have presented our method of calculating dibaryon weak transition matrix elements and reported the preliminary results of proton-proton fusion and double beta decay matrix elements. A comparison is made between PP and PSPS correlation functions. In the next stage of our research, we plan to work for $0\nu2\beta$ matrix elements and further reduce both statistical and systematic uncertainties.

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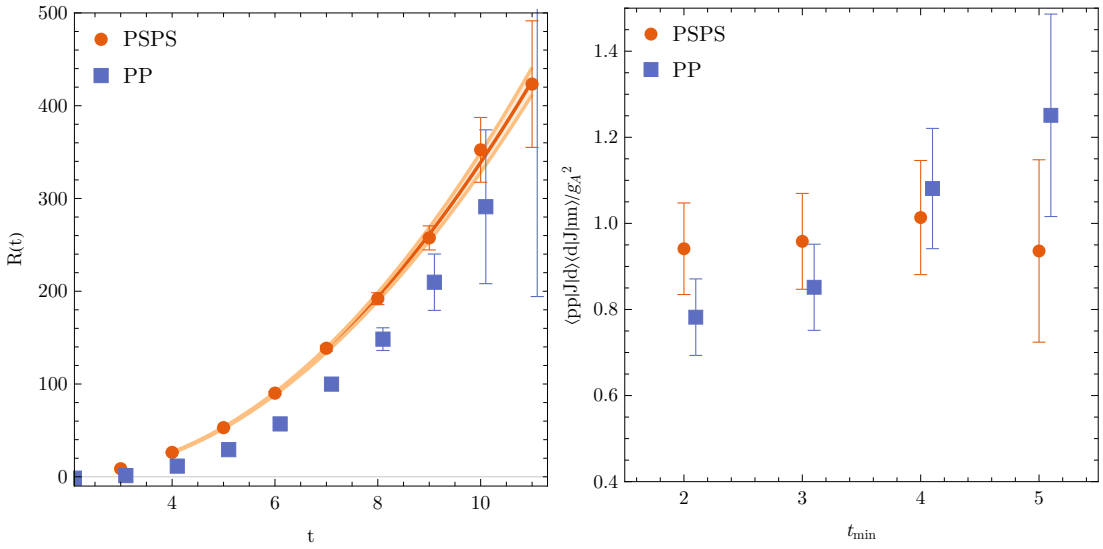

Figure 3: Fitting of the matrix element of double beta decay ($2\nu2\beta$). The left panel shows the ratio between four-point function and two-point function and right panel shows the fitting result of long-distance contribution with different fitting windows.

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