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Colour and logarithmic accuracy in final-state parton showers

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Standard final-state dipole showers are typically derived within an approximation of a large number of colours. Efficient and widely used algorithms in the literature to go beyond this approximation are known to yield incorrect sub-leading colour contributions for a variety of observables at leading logarithmic accuracy. In this review we introduce two new algorithms, which are based on colour coherence, that allow showers to correctly account for the full colour structure of global observables at next-to-leading logarithmic accuracy. One of the two introduced schemes also enables showers to reproduce the correct full-colour matrix element for any number of commensurate-angle energy-ordered pairs of emissions.

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1. Introduction

Parton showers are indispensable tools in high energy physics. In recent years, however, their accuracy has come under intense scrutiny as their uncertainties can dominate in certain collider applications.

Showers of the dipole family [1] are typically derived within the approximation of a large number of colours (N_{C}), also called the leading-colour (LC) approximation. There has been much effort in the literature to go beyond the LC approximation, for example Refs. [2–10]. However, implementing the full colour structure is computationally challenging because of the rapidly growing number of possible colour configurations at large particle multiplicities. In this review of Ref. [11] we present a complementary, and computationally efficient approach for incorporating full colour information into a shower in terms of its logarithmic accuracy.

We judge a shower's logarithmic accuracy by whether it is able to reproduce known analytic resummations for a wide range of collider observables at a given logarithmic order. I.e. for an exponentiating property of an event $P(\alpha_s, L)$, where α_s is the strong coupling at the hard scale Q, and L the logarithm of a ratio of two scales, one may organise its resummation as follows [12]:

$$P(\alpha_s, L) = P(\alpha_s, 0)\exp\left(\underbrace{\alpha_s^{-1} g_1(\alpha_s L)}_{\mathrm{LL}} + \underbrace{g_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathrm{NNLL}} + \cdots\right) + O\left(e^{-|L|}\right). \quad (1)$$

Here, LL stands for leading-logarithmic accuracy, NLL for next-to-leading-logarithmic accuracy, and so on. Concretely, at N^{k}LL accuracy we sum terms of the form $\alpha_s^n L^{n+1-k}$.

For the purposes of controlling the achieved accuracy with respect to colour, we sometimes make the number of colours explicit. Thus the leading colour (LC) accurate part of the N^{k}LL resummation involves terms $\alpha_s^n N_{\mathrm{C}}^n L^{n+1-k}$, while the next-to-leading colour (NLC) accurate part involves terms $\alpha_s^n N_{\mathrm{C}}^{n-2} L^{n+1-k}$ and so on. We refer to this counting as N^{k}LL-LC, N^{k}LL-NLC, etc. In the case where no approximation is made to the colour structure, we call this a N^{k}LL full-colour (or N^{k}LL-FC for short) accurate result.

Up to now we could expect a shower to be at most NLL accurate at leading colour [13]. Recently it has been noted that dipole-based parton showers, equipped with established algorithms for introducing sub-leading colour corrections, fail to achieve LL-NLC accuracy [14]. The objective of the presented work [11] is to introduce two algorithms which consistently incorporate colour into final-state dipole showers, enabling NLL-FC accurate resummations for a wide class of collider observables.

2. Algorithms for achieving full colour at NLL accuracy

The basis of our solutions is colour coherence (or equivalently angular ordering), which provides insight into the colour structure of emissions at widely separated angles - the kinematic configuration which contributes at NLL accuracy.

The first of these schemes, nicknamed the Segments scheme, uses an ordered list of transition points in rapidity defined with respect to dipole ends to assign a colour factor for further emissions

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in accordance with colour coherence.[1] This list is built iteratively from the initial $q\bar{q}$ or gg system. For each dipole in an event it describes angular regions where emissions should be assigned a $C_A/2 = N_c/2$ or $C_F = (N_c^2 - 1)/2N_c$ colour factor. Consequently this scheme captures the full structure of angular ordering for any configuration of emissions widely separated in rapidity.

The second of these schemes utilises a local matrix-element correction factor. It uses an approximate double-soft gluon emission matrix element which assures that the correct full-colour emission pattern is reproduced when any number of pairs of emissions are at commensurate rapidities, but disparate energies. Consequently we call this scheme the Nested-Ordered-Double-Soft scheme, or NODS scheme for short.

For full details on the Segments (NODS) scheme we refer the reader to the original paper [11], section 3.4 (4.3). By design, they both enable showers to reproduce NLL-FC resummation results for global observables. Sub-leading colour issues still remain in the case of non-global observables.

Below, we will compare the behaviour of our two new schemes to that of typical colour-assignment algorithms used in standard dipole showers, such as Pythia8 [16, 17] and Dire v1 [18]. In these, each dipole is split into an "emitter" and a "spectator" end. Effectively, any new emission from a dipole receives its colour factor based on the identity of the emitter. E.g. if the emitter is a quark (gluon), the emission is assigned a colour factor C_F ($C_A/2$). Accordingly, we refer to this scheme as "colour-factor-from-emitter" (CFFE).

3. Numeric validation of algorithms

We evaluate the validity of the introduced algorithms on two fronts: First, by understanding how faithfully a shower reproduces analytic matrix elements when equipped with a given colour scheme introduced in sec. 2. And second, how well a given shower with an associated colour scheme reproduces analytic resummations, as outlined in sec. 1.

Beginning with the first of these criteria, we consider a $q\bar{q}g_1$ system with predefined kinematics, and ask the shower (in this case the k_t ordered shower PanGlobal $\beta = 0$ - see ref. [13] for details) to emit a further gluon g. For example, we pick g_1 to be emitted at $\eta_{qg_1} = -\ln(\tan(\theta_{qg_1}/2)) = 5$, $z_{g_1} = 10^{-8}$, and $\phi_{g_1} = \pi$. Furthermore, we equip the shower with each of the colour schemes discussed in sec. 2 and differentially compare the resulting emission probability $d\sigma_{\mathrm{FC}}/d\phi d\eta$ (σ_{FC} for short) to a corresponding analytic matrix element $d\sigma_{\mathrm{an}}/d\phi d\eta = d\Phi_g |\mathcal{M}_{\bar{q}g_1q+g}|^2/|\mathcal{M}_{\bar{q}g_1q}|^2$ (σ_{an} for short).

We analyse the resulting relative deviation between the two in Fig. 1a in terms of the pseudorapidity η_{gq} and azimuthal angle ϕ defined w.r.t. the quark. In order to avoid issues related to numerical instabilities near g_1, we do not populate phase-space points near it.[2]

The top row of Fig. 1a shows the relative deviation between the shower handling colour with CFFE from the analytic matrix element. When the gluon is emitted at large pseudo-rapidities (in proximity to the parent quark), a correct C_F colour factor is applied. However, far away from g_1, as $\eta_{qg} \to 0$, the shower mis-assigns a $C_A/2$ (instead of C_F) colour factor in a logarithmically enhanced region which spoils the LL-NLC accuracy of the shower. The middle row of Fig. 1a shows the same

[1] The scheme bears resemblance to the scheme introduced in Ref. [15] (sec. 4.2).

[2] More precisely, the phase space in Fig. 1a portrays the primary Lund plane (equiv. q Lund leaf) as determined by the Lund declustering algorithm [19].

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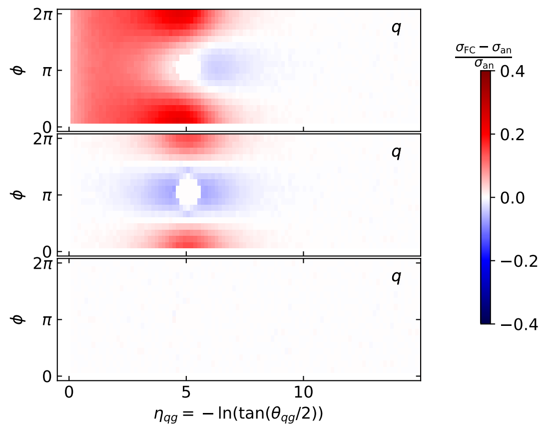
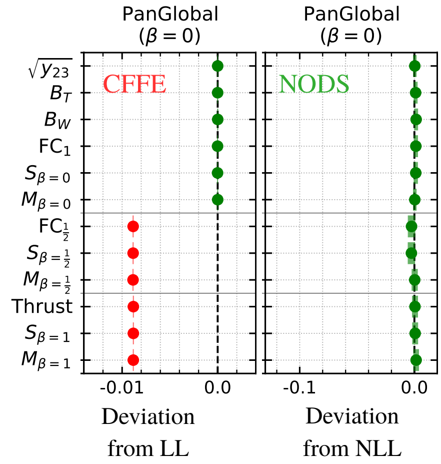


(a) Differential comparison between shower-generated and analytic matrix element for CFFE (top), Segments scheme (middle), and NODS scheme (bottom). See text for more details.



(b) Relative deviation of shower (PanGlobal $\beta = 0$) from known LL analytic resummation when equipped with CFFE (left), and deviation from NLL when equipped with NODS (right).

Figure 1: Evaluating the correctness of the colour schemes introduced in sec. 2 by how well they reproduce known matrix elements (Fig. 1a), and known analytic resummations (Fig. 1b).

analysis for a shower equipped with the Segments scheme. Here the correct colour factor C_F is assigned if the gluon is emitted close to and away from the quark q. There still, however, remains a residual deviation from the exact matrix element when $\eta_{qg} \sim \eta_{qg_1}$. Since this discrepancy happens in an $O(1)$ region, this does not impact the full-colour accuracy of the shower. Finally, the bottom row of Fig. 1a shows the deviation from a shower equipped with the NODS scheme. We note that, within statistical fluctuations, the analytic matrix element is reproduced faithfully. Tests with different initial configurations, more precisely $q\bar{q}g_1g_2$ and $q\bar{q}q'\bar{q}'$, yield similar results and are summarised in sec. 6 for the source material.

The above observations contribute to whether a shower is able to reproduce NLL full colour resummations. Fig. 1b shows the relative deviation of PanGlobal $\beta = 0$ equipped with CFFE (NODS) from LL (NLL) accurate analytic resummations of common event shapes. It is thus clear that showers equipped with CFFE are unable to reproduce full-colour resummation results even at LL. However, using the NODS (or Segments) scheme, the shower achieves NLL-FC accuracy.

We also tested the energy flow in a rapidity slice [20], which is a non-global observable. In sec. 7.3 of the source material we compared PanGlobal(antenna, $\beta = 1/2$) to a dedicated numeric resummation [21] and found good agreement when running both with our Segments and NODS schemes.

4. Conclusions

In these proceedings we reviewed two simple algorithms to recover full-colour information in modern final-state dipole showers. We introduced two schemes in sec. 2, named the Segments, and NODS scheme, and showed in sec. 3 that they allow showers to reproduce full-colour resummations for a wide class of shower observables.

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