One-Loop Correlators of Charged Fermionic Currents Modified by Electromagnetic Fields and Their Applications

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The non-diagonal correlators of tensor fermionic current with the vector and axial-vector ones are considered as examples of the two-point one-loop amplitudes modified by a constant homogeneous magnetic field. The tensor current is a fermionic part of the Pauli Lagrangian density relevant for a fermion electromagnetic interaction through its anomalous magnetic moment. Under an assumption that this interaction enters the effective QED Lagrangian, the contribution to the photon polarization operator linear in the magnetic moment is calculated. The other example where a photon interacts with a neutral pseudoscalar particle like an axion is also discussed.

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1. Two-Point Correlators

The Lagrangian density of the local fermion interaction has the form:

$$\mathcal{L}_{\text{int}}(x) = \left[\bar{f}(x)\Gamma_A f(x)\right] J^A(x), \tag{1}$$

where J^A is a generalized current (like a photon, neutrino current, etc.) which includes a corresponding coupling, Γ_A is any of γ-matrices from the set $\{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2\}$ or their linear combination. After generalized currents are cut, a correlation function of fermionic currents can be constructed. The two-point correlation function of general form is as follows [1]:

$$\Pi_{AB} = \int d^4 X \, e^{-i(qX)} \, \text{Sp} \left\{ S_{\text{F}}(-X) \, \Gamma_A \, S_{\text{F}}(X) \, \Gamma_B \right\}, \tag{2}$$

where $S_{\text{F}}(X)$ is the Lorentz-invariant part of an exact fermion propagator modified by an external electromagnetic field. In calculations, the Fock-Schwinger representation for fermionic propagators is used. Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied [1]. Here, as examples, correlators of the tensor current with the vector and axial-vector ones are presented.

2. Tensors in Presence of Magnetic Field

Let us consider an external constant magnetic field with the strength **B** as the electromagnetic background. Minkowski space filled with such a field is divided into two subspaces. Its metric tensor can be written as follows: $g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$, where $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$ and $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$. It includes the dimensionless tensor of the external magnetic field $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and its dual $\tilde{\varphi}_{\alpha\beta} = \varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}/2$. An arbitrary four-vector $q^\mu = (q_0, q_1, q_2, q_3)$ can be decomposed into two orthogonal components: $q_\mu = \tilde{\Lambda}_{\mu\nu}q^\nu - \Lambda_{\mu\nu}q^\nu = q_{\parallel\mu} - q_{\perp\mu}$. Tensors $\varphi_{\alpha\beta}$, $\tilde{\varphi}_{\alpha\beta}$, $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta}$, and the vector above allow to determine an orthogonal basis in Minkowski space: $b_\mu^{(1)} = (q\varphi)_\mu$, $b_\mu^{(2)} = (q\tilde{\varphi})_\mu$, $b_\mu^{(3)} = q^2(\Lambda q)_\mu - (q\Lambda q)q_\mu$, and $b_\mu^{(4)} = q_\mu$. Correlators having rank non-equal to zero, should be decomposed in this orthogonal set of vectors, in particular, a third-rank tensor $T_{\mu\nu\rho}$ can be presented as follows:

$$T_{\mu\nu\rho} = \sum_{i,j,k=1}^{4} T_{ijk} \frac{b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}}{(b^{(i)}b^{(i)})(b^{(j)}b^{(j)})(b^{(k)}b^{(k)})}, \tag{3}$$

where $T_{ijk} = T^{\mu\nu\rho} b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}$ are decomposition coefficients.

3. Vector—Tensor Correlator

Vector-current conservation and antisymmetry of the tensor current reduce the number of independent coefficients $Y_{ijk}^{(\text{VT})}(q^2, q_\perp^2, \beta; t, u)$ in the basis decomposition of the vector-tensor correlator

$$\Pi_{ijk}^{(\text{VT})}(q^2, q_\perp^2, \beta) = \frac{1}{4\pi^2} \int_0^\infty \frac{dt}{t} \frac{\beta t}{\sin(\beta t)} \int_0^1 du \, Y_{ijk}^{(\text{VT})}(q^2, q_\perp^2, \beta; t, u) \, e^{-i\Omega(q^2, q_\perp^2, \beta; t, u)}, \tag{4}$$

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where $\Omega(q^2, q_\perp^2, \beta; t, u) = m_f^2 t - q_\parallel^2 t\,(1-u^2)/4 + q_\perp^2\,[\cos(\beta tu) - \cos(\beta t)]\,/[2\beta\sin(\beta t)]$ is the phase of the correlator, to 18. Of them, four coefficients only are non-trivial $\left(Y_{ijk}^{(\mathrm{VT})} = -Y_{ikj}^{(\mathrm{VT})}\right)$:

$$Y_{114}^{(\mathrm{VT})} = -m_f\,q_\perp^2\,q^2\,\cos(\beta tu), \quad Y_{223}^{(\mathrm{VT})} = m_f\,q_\perp^2\,(q_\parallel^2)^2\,[\cos(\beta t) - \cos(\beta tu)], \tag{5}$$

$$Y_{224}^{(\mathrm{VT})} = m_f\,q_\parallel^2\left[q_\perp^2\cos(\beta t) - q_\parallel^2\cos(\beta tu)\right], \quad Y_{334}^{(\mathrm{VT})} = -m_f\,q_\perp^2\,q_\parallel^2\,(q^2)^2\,\cos(\beta tu),$$

where $q_\parallel^2 = q^2 + q_\perp^2$ and $\beta = eB|Q_f|$. All $Y_{4jk}^{(\mathrm{VT})}$ vanish in the basis chosen.

4. Axial-Vector—Tensor Correlator

The correlator of the axial-vector and tensor currents is also the rank-3 tensor. Six coefficients in the basis expansion are non-trivial $\left(Y_{ijk}^{(\mathrm{AT})} = -Y_{ikj}^{(\mathrm{AT})}\right)$:

$$Y_{112}^{(\mathrm{AT})} = q_\perp^2\,q_\parallel^2\,u\sin(\beta tu), \qquad Y_{234}^{(\mathrm{AT})} = q^2\,q_\perp^2\,q_\parallel^2\,\frac{\cos(\beta t)\cos(\beta tu) - 1}{\sin(\beta t)}, \tag{6}$$

$$Y_{323}^{(\mathrm{AT})} = \frac{-q_\perp^2\,(q_\parallel^2)^2}{\sin(\beta t)}\left\{u\sin(\beta t)\sin(\beta tu)\,q_\parallel^2 - \left[2 - 2\cos(\beta t)\cos(\beta tu) - \sin^2(\beta t)\right]q_\perp^2\right\},$$

$$Y_{324}^{(\mathrm{AT})} = -q_\perp^2\,q_\parallel^2\left\{\left[\frac{1 - \cos(\beta t)\cos(\beta tu)}{\sin(\beta t)}\right]q_\perp^2 + \left[\cot(\beta t)\,[\cos(\beta t) - \cos(\beta tu)] - u\sin(\beta tu)\right]q_\parallel^2\right\},$$

$$Y_{424}^{(\mathrm{AT})} = -q_\parallel^2\left\{\sin(\beta t)\,q_\parallel^2 - \left[\frac{2 - 2\cos(\beta t)\cos(\beta tu)}{\sin(\beta t)} - u\sin(\beta tu)\right]q_\perp^2\right\}.$$

and $Y_{423}^{(\mathrm{AT})} = Y_{324}^{(\mathrm{AT})}$.

5. Photon Polarization Operator

The photon polarization operator $\mathcal{P}^{\mu\nu}(q)$ is related with $\gamma \to \gamma$ transition matrix element and, consequently, with the correlator of two vector currents: $\mathcal{M}_{\gamma\to\gamma} = -i\,\varepsilon_\mu'^*(q)\,\mathcal{P}^{\mu\nu}(q)\,\varepsilon_\nu(q)$. The photon dispersion relations follow from the equations $q^2 - \Pi^{(\lambda)}(q) = 0$, where $\lambda = 1, 2, 3$. $\Pi^{(\lambda)}(q)$ are eigenvalues of the photon polarization operator (the electron loop is considered only)

$$\mathcal{P}_{\mu\nu}(q) = \sum_{\lambda=1}^{3} \frac{b_\mu^{(\lambda)}\,b_\nu^{(\lambda)}}{(b^{(\lambda)})^2}\,\Pi^{(\lambda)}(q). \tag{7}$$

As in the vacuum, there are two physical eigenstates of $\mathcal{P}_{\mu\nu}(q)$ in the external constant homogeneous magnetic field: $\varepsilon_\mu^{(1)} = b_\mu^{(1)}/\sqrt{q_\perp^2}$ and $\varepsilon_\mu^{(2)} = b_\mu^{(2)}/\sqrt{q_\parallel^2}$. In the magnetic field, $\Pi^{(\lambda)}(q)$ contains both the vacuum and field-induced parts:

$$\Pi^{(\lambda)}(q) = -i\,\mathcal{P}(q^2) - \frac{\alpha}{\pi}\,Y_{VV}^{(\lambda)}, \tag{8}$$

where $\alpha = e^2/(4\pi)$ is the fine structure constant. Details about $Y_{VV}^{(\lambda)}$ can be found in [2].

Models beyond the Standard Model can give rise effectively to the Pauli Lagrangian density:

$$\mathcal{L}_{\mathrm{AMM}}(x) = -\frac{\mu_B a_e}{2}\left[\bar\psi(x)\sigma_{\mu\nu}\psi(x)\right]F^{\mu\nu}(x), \tag{9}$$

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where $\mu_B = e/(2m_e)$ is the Bohr magneton and a_e is the electron anomalous magnetic moment (AMM). Due to $\mathcal{L}_{\text{AMM}}$, an additional terms in (7) are produced. The contribution linear in the electron AMM is related with the correlator of the vector and tensor currents, $\Pi^{(\text{VT})}_{\mu\nu\rho}$. The field-induced part of $\Pi^{(\lambda)}(q)$ is modified by the additional term $\alpha\, a_e\, Y^{(\lambda)}_{VT}/\pi$, where

$$Y^{(\lambda)}_{VT} = \int_0^\infty \frac{dt}{t} \int_0^1 du \left\{ \frac{\beta t}{\sin(\beta t)}\, y^{(\lambda)}_{VT}\, e^{-i\Omega} - q^2\, e^{-i\Omega_0} \right\}, \quad \Omega_0 = m_f^2 t - q^2 t\,(1-u^2)/4.$$

$$y^{(1)}_{VT} = y^{(3)}_{VT} = q^2 \cos(\beta t u), \qquad y^{(2)}_{VT} = q_\parallel^2 \cos(\beta t u) - q_\perp^2 \cos(\beta t), \tag{10}$$

Notations are taken from [2]. The part independent on the field is subtracted. For the electron, $a_e \sim \alpha$ and the AMM correction is small.

6. Axion-Photon Mixing

Axion-photon mixing is arising from a local interaction of an axion with two photons when bosons are propagating in an external magnetic background [3]. The local axial-vector interaction of an axion with fermions:

$$\mathcal{L}_{af}(x) = -\frac{C_a}{2f_a} \left[\bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x) \right] \partial^\mu a(x), \tag{11}$$

modifies this mixing. An existence of the Pauli Lagrangian $\mathcal{L}_{\text{AMM}}(x)$ gives an extra term into the $a \to \gamma$ transition matrix element which is related with the correlator of the axial-vector and tensor currents, $\Pi^{(\text{AT})}_{\mu\nu\rho}$. The one with $Y^{(\text{AT})}_{424}$ contributes into the mixing of axions with photons of the polarization $\epsilon_\mu = b^{(2)}_\mu$. More details are presented in a forthcoming publication.

7. Conclusions

Two-point correlators in the presence of the constant homogeneous external magnetic field are considered. Study of correlators of the tensor fermionic current with the others allows to investigate effects of the fermion AMM in the one-loop approximation. The field-induced contributions to the photon polarization operator and axion-photon mixing are discussed.

8. Acknowledgements

References

[1] M. Y. Borovkov, A. V. Kuznetsov and N. V. Mikheev, *Phys. Atom. Nucl.* **62** (1999) 1601.

[2] A. Kuznetsov and N. Mikheev, *Electroweak Processes in External Active Media*, Springer-Verlag, New York 2013.

[3] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics: The astrophysics of neutrinos, axions, and other weakly interacting particles*, University of Chicago Press, Chicago & London 1996.