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# The full electroweak $\mathcal{O}(\alpha)$ corrections to $\gamma\gamma \to \ell^-\ell^+$

**M. Demirci*** **and M. F. Mustamin**

*Department of Physics, Karadeniz Technical University,*
*Trabzon, TR61080, Turkey*
*E-mail:* mehmetdemirci@ktu.edu.tr, mfmustamin@ktu.edu.tr

We provide high-precision predictions for $\ell^-\ell^+$ production ($\ell = \mu, \tau$) at $\gamma\gamma$ collisions by considering a complete set of one-loop order scattering amplitudes, i.e., full electroweak (EW) $\mathcal{O}(\alpha)$ corrections together with soft and hard QED radiation. We decompose the one-loop EW radiative corrections into the pure QED and Weak parts. The total corrections enhance the Born cross section within ten percent of total relative correction for both processes. Our results show that the full EW corrections must be included to improve a percent level accuracy.

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## 1. Introduction

The process of $\gamma\gamma$ collisions presents an important framework to improve our understanding of the current Standard Model (SM) theory and beyond. It can be virtually investigated every aspect of the SM and beyond as photon couples directly to all fundamental fields via electromagnetic current, such as leptons, quarks, $W$'s, and even supersymmetric particles [1].

When aiming to have accurate measurements, high precision predictions from the theory are necessary. Meaning that for most processes there is a need to proceed beyond the leading order calculations, thus the full one-loop contributions become significant for analyzing phenomena at the future experiments. In the present work, we examine the charged heavy lepton pairs production via photon-photon collision for $\mu$ and $\tau$ in the framework of SM, with full one-loop EW radiative corrections [2]. We present numerical evaluations accordingly with special emphasis on the pure QED and weak corrections. We also present angular distributions for the tree-level and one-loop cross sections.

## 2. Calculation of Cross Sections

### 2.1 Lowest Order Contributions

Process of $\gamma\gamma \to \ell^-\ell^+$ in lowest order is a pure QED where the leading contribution comes from $t$ and $u$-channel diagrams of the charged lepton-exchange. The tree-level matrix elements are given by

$$\begin{aligned} \mathcal{M}_1 &= \frac{-ie^2}{\left[\hat{t}-m_\ell^2\right]}\overline{u}(k_2,m_\ell)\gamma^\nu\varepsilon^\nu(p_2)(\not{k}_2-\not{p}_2+m_\ell)\varepsilon^\mu(p_1)\gamma^\mu v(k_1,m_\ell),\\ \mathcal{M}_2 &= \frac{-ie^2}{\left[\hat{u}-m_\ell^2\right]}\overline{u}(k_2,m_\ell)\gamma^\mu\varepsilon^\nu(p_2)(\not{p}_2-\not{k}_1+m_\ell)\varepsilon^\mu(p_1)\gamma^\nu v(k_1,m_\ell), \end{aligned} \tag{2.1}$$

where $\varepsilon_\mu(p_1)$ and $\varepsilon_\nu(p_2)$ indicate to the initial photon polarization vectors, and $\alpha = \frac{e^2}{4\pi}$. Following the square of total amplitude and the summation of final particle helicities, we calculate the total cross-section with ( $\hat{t}^\pm = (m_{l^\pm}^2 - \frac{\hat{s}}{2}) \pm \frac{1}{2}\sqrt{\hat{s}^2 - 4\hat{s}m_{l^\pm}^2}$)

$$\hat{\sigma}_{\rm Born}^{\gamma\gamma\to\ell^-\ell^+} = \frac{1}{16\pi\hat{s}^2}\int_{\hat{t}^-}^{\hat{t}^+}\left(\frac{1}{4}\right)\sum_{\lambda_{1,2},\sigma_{1,2}}|\mathcal{M}_{\rm Born}|^2 d\hat{t}. \tag{2.2}$$

### 2.2 One-loop Order Contributions

Higher-order contributions (at least, next-to-leading order corrections) are required to precisely predict analysis of high-energy processes at the current and future colliders. The process on $\gamma\gamma \to \ell^-\ell^+$ includes one-loop level contributions as the next-to-leading order in $O(\alpha)$, which based on pure EW corrections. The total amplitude of this one-loop process can be taken as a linear sum of box, triangle, and bubble integrals. All possible loops of leptons, $\gamma$, $Z$, $W^\pm$, Higgs and Goldstone bosons on the propagator of charged leptons are included in self-energy diagrams, as shown in diagrams (1)-(3) of Fig. 1. Diagrams (4)-(7) of Fig. 1 are irreducible one-loop box-type diagrams. They include all possible loops of leptons, $\gamma$, $Z$, $W^\pm$, Higgs and Goldstone bosons. The diagrams with vertex-correction are shown in Fig. 2, composed of triangle corrections to $\hat{t}$-channel charged lepton exchange, triangle and bubbles vertices, attached to the final state via an intermediate $\gamma$, $Z$ or neutral Higgs and Goldstone bosons.

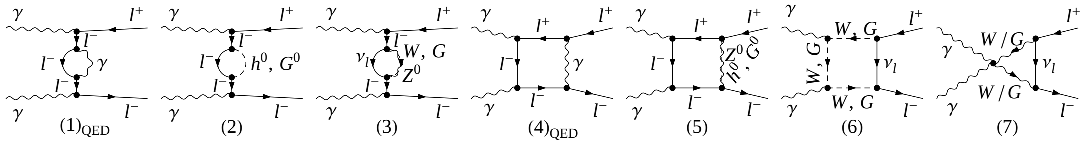

**Figure 1:** The self-energy and box-type correction diagrams contributing to $\gamma\gamma \to \ell^-\ell^+$.

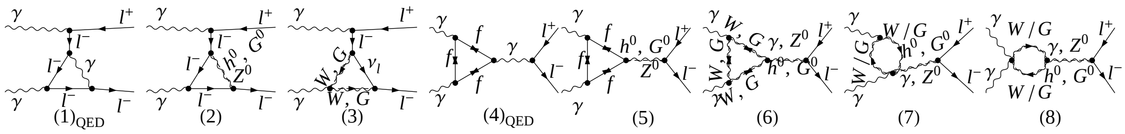

**Figure 2:** The vertex-correction diagrams contributing to process in $\gamma\gamma \to \ell^-\ell^+$.

#### 2.2.1 Virtual corrections

The differential cross section for the virtual corrections can be obtained by summation over self-energy, triangle-type and box-type diagrams, given as

$$d\hat{\sigma}_{\text{virt}}^{\gamma\gamma\to\ell^-\ell^+} = \frac{1}{16\pi\hat{s}^2}\left(\frac{1}{4}\right)\sum_{hel} 2\mathrm{Re}\big[\mathcal{M}_{\text{Born}}^{*}(\mathcal{M}_{\bigcirc} + \mathcal{M}_{\square} + \mathcal{M}_{\triangle})\big]d\hat{t}, \tag{2.3}$$

where $|\delta\mathcal{M}_{\text{virt}}|^2$ is not included because it is negligible. The one-loop Feynman diagrams, which form the virtual $O(\alpha)$ corrections $\delta\mathcal{M}_{\text{virt}}$, have been calculated in 't Hooft-Feynman gauge using the on-shell renormalization scheme described in Ref. [3]. Accordingly, the UV divergences have been treated by dimensional regularization, while the IR divergences have been cancelled by the involvements of soft and hard QED radiation.

#### 2.2.2 Real corrections

Radiation of real photon in the $\gamma\gamma \to \ell^+\ell^-$ process leads to different kinematics which can be expressed as $\gamma(p_1,\lambda_1)\gamma(p_2,\lambda_2) \to \ell^+(k_1,\sigma_1)\ell^-(k_2,\sigma_2)\gamma(k_3,\lambda_3)$. Here $k_3$ is the radiated photon four-momentum. The process is depicted in Fig. 3. Correction from the real photon radiation can

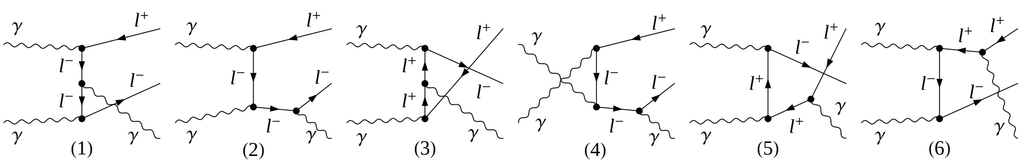

**Figure 3:** The Feynman diagrams for the real photon radiation.

be given as

$$d\hat{\sigma}_{\text{real}}^{\gamma\gamma\to\ell^-\ell^+\gamma} = d\hat{\sigma}_{\text{soft}}(\Delta_s) + d\hat{\sigma}_{\text{hard}}(\Delta_s), \tag{2.4}$$

where the soft cut-off energy parameter is given by $\Delta_s = \Delta E_\gamma/(\sqrt{\hat{s}}/2)$, with $\Delta E_\gamma$ is the soft photon cut-off energy. The radiated photon is named as soft when $k_3^0 < \Delta E_\gamma = \Delta_s\sqrt{\hat{s}}/2$, while it is hard if

$k_3^0 > \Delta E_\gamma$. Integrating the soft photon phase space in the center-of-mass system gives

$$d\hat{\sigma}_{\rm soft} = -\, d\hat{\sigma}_{\rm Born}\frac{\alpha}{\pi}Q_\ell^2\left[2\ln\left(\frac{2\Delta E_\gamma}{m_\gamma}\right)\left(1+\ln\left(\frac{m_\ell^2}{\hat{s}}\right)\right)+\frac{1}{2}\ln^2\left(\frac{m_\ell^2}{\hat{s}}\right)+\ln\left(\frac{m_\ell^2}{\hat{s}}\right)+\frac{\pi^2}{3}\right]. \tag{2.5}$$

It is clear from Fig. 4 that the total corrections for both processes are independent of the soft cut-off parameter $\Delta_s$.

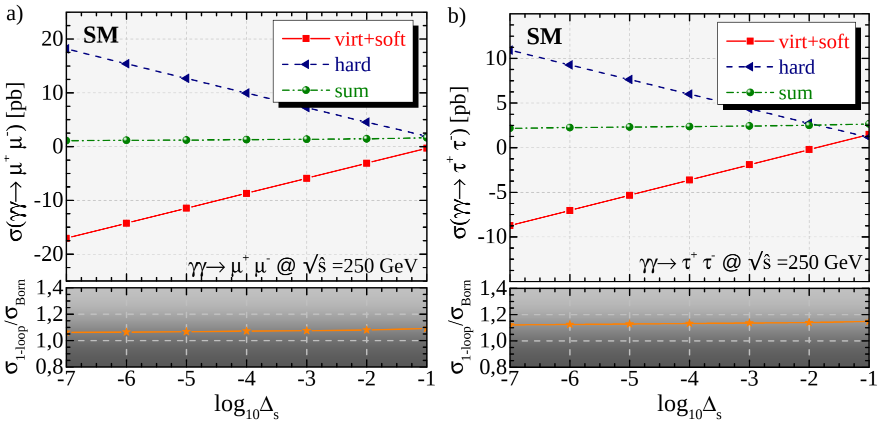


**Figure 4:** The soft cut-off parameter dependence for processes (a) $\gamma\gamma \to \mu^+\mu^-$ and (b) $\gamma\gamma \to \tau^+\tau^-$.

The virtual corrections may also be classified according to gauge-invariant subsets: the QED-corrections $\delta_{\rm QED}$ and the Weak corrections $\delta_{\rm Weak}$. Accordingly, the total relative correction can be written as follows:

$$\delta_{\rm Total} = \delta_{\rm QED} + \delta_{\rm Weak}. \tag{2.6}$$

The QED corrections consist of real-photon emission, virtual-photon exchange, and the corresponding counter-terms. All QED-like diagrams (those with only $A$ and $l$ fields as virtual lines) form a subset of gauge-invariant. The $\delta_{\rm QED}$ are considered by the sum of the soft-photon correction and the contribution of diagrams (1) and (4) of Fig. 1 and Fig. 2. The remaining non-QED correction are called as weak corrections $\delta_{\rm Weak}$ that include the $Z^0$ and $W^\pm$ gauge boson.

## 3. Numerical Results

During our numerical evaluation, we implement the $\alpha(0)$-scheme where $\alpha$ is fixed as $\alpha(0) = 1/137.03599907$ and take other input parameters as in Ref. [2]. Additionally, we take the soft cutoff parameter as $\Delta_s = 10^{-3}$, and $|\cos\theta| < 0.99$ for the range of scattering angles of the final particles. The numerical calculation of our work is conducted using: FeynArts [4] (generating Feynman diagrams and amplitudes) & FormCalc [5] (squaring amplitudes and simplifying fermion chains), LoopTools [5] (computing scalar loop integral), and CalcHEP [6] (calculating hard photon bremmsstrahlung). The total corrections provide a positive contribution to the Born cross section in the parameter regions as can be seen in Fig. 5. Figure 6 shows the angular distributions which are (symmetrically) strongly peaked in the forward and backward directions. The relative corrections modify somewhat the tree-level angular distributions because their influences are larger in the central regions.

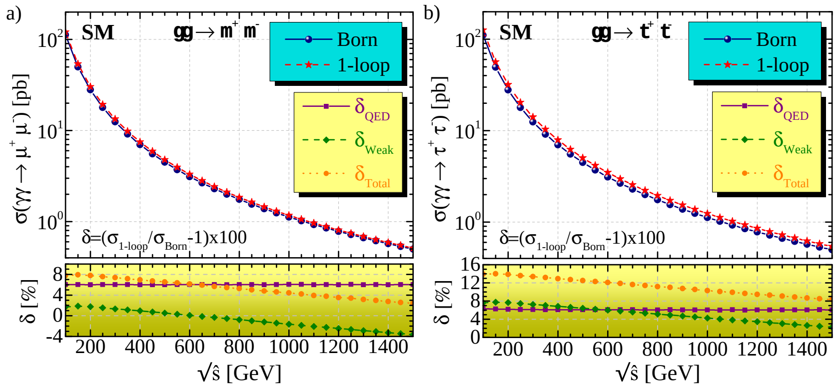


**Figure 5:** The energy dependence of the EW corrections for processes (a) $\gamma\gamma \to \mu^+\mu^-$ and (b) $\gamma\gamma \to \tau^+\tau^-$.

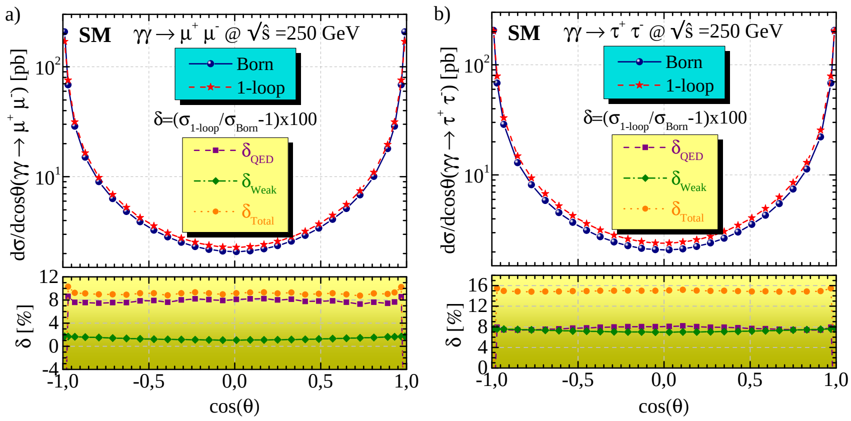


**Figure 6:** The angular dependence of EW corrections for processes (a) $\gamma\gamma \to \mu^+\mu^-$ and (b) $\gamma\gamma \to \tau^+\tau^-$.

## 4. Conclusion

We have investigated the lepton-pair production at photon-photon collisions by considering a full set of one-loop-level scattering amplitude. We have decomposed the one-loop EW radiative corrections into the pure QED and Weak corrections. The total corrections increase the tree-level cross section within ten percent of total relative correction. This implies that the full EW corrections must be included to improve a percent level accuracy.

## References

[1] M. Demirci, *Nucl. Phys. B* **961** (2020) 115235.

[2] M. Demirci and M. F. Mustamin, *Phys. Rev. D* **103** (2021) 113004.

[3] A. Denner, *Fortschr. Phys.* **41** (1993) 307.

[4] J. Küblbeck, M. Böhm, and A. Denner, *Comput. Phys. Commun.* **60** (1990) 165.

[5] T. Hahn and M. Perez-Victoria, *Comput. Phys. Commun.* **118** (1999) 153.

[6] A. Belyaev, N. D. Christensen, and A. Pukhov, *Comput. Phys. Commun.* **184** (2013) 1729.