PROCEEDINGS OF SCIENCE

Finite-Width Effects in Three-Body B Decays

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It is customary to apply the so-called narrow width approximation $\Gamma(B \to RP_3 \to P_1P_2P_3) = \Gamma(B \to RP_3)\mathcal{B}(R \to P_1P_2)$ to extract the branching fraction of the quasi-two-body decay $B \to RP_3$, with R and P_3 being an intermediate resonant state and a pseudoscalar meson, respectively. However, the above factorization is valid only in the zero width limit. We consider a correction parameter η_R from finite width effects. Our main results are: (i) We present a general framework for computing η_R and show that it can be expressed in terms of the normalized differential rate and determined by its value at the resonance. (ii) We evaluate η_R in the theoretical framework of QCD factorization (QCDF) and in the experimental parameterization (EXPP) for three-body decay amplitudes. In general, $\eta_R^{\rm QCDF}$ and $\eta_R^{\rm EXPP}$ are similar for vector mesons, but different for tensor and scalar resonances. A study of the differential rates enables us to understand the origin of their differences. (iii) Finite-width corrections to $\mathcal{B}(B^- \to RP)_{\rm NWA}$ obtained in the narrow width approximation are generally small, less than 10%, but they are prominent in $B^- \to \sigma/f_0(500)\pi^-$ and $B^- \to \overline{K}{}^{*0}_0(1430)\pi^-$ decays. The EXPP of the normalized differential rates should be contrasted with the theoretical predictions from QCDF calculation as the latter properly takes into account the energy dependence in weak decay amplitudes. (iv) It is common to use the Gounaris-Sakurai model to describe the line shape of the broad $\rho(770)$ resonance. After including finite-width effects, the PDG value of $\mathcal{B}(B^- \to \rho\pi^-) = (8.3 \pm 1.2) \times 10^{-6}$ should be corrected to $(7.9 \pm 1.1) \times 10^{-6}$ in EXPP and $(7.7 \pm 1.1) \times 10^{-6}$ in QCDF. (v) For the very broad $\sigma/f_0(500)$ scalar resonance, we use a simple pole model to describe its line shape and find a very large width effect: $\eta_\sigma^{\rm QCDF} \sim 2.15$ and $\eta_\sigma^{\rm EXPP} \sim 1.64$. Consequently, $B^- \to \sigma\pi^-$ has a large branching fraction of order 10^{-5}. (vi) We employ the Breit-Wigner line shape to describe the production of $K_0^*(1430)$ in three-body B decays and find large off-shell effects. The smallness of $\eta_{K_0^*}^{\rm QCDF}$ relative to $\eta_{K_0^*}^{\rm EXPP}$ is ascribed to the differences in the normalized differential rates off the resonance.

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In a B meson decay $B \to RP_3 \to P_1P_2P_3$ decay, where R and P_3 are an intermediate resonant state and a pseudoscalar meson, respectively, it is a common practice to apply the narrow width approximation (NWA),

$$\Gamma(B \to RP_3 \to P_1P_2P_3) = \Gamma(B \to RP_3)\mathcal{B}(R \to P_1P_2), \tag{1}$$

to extract the branching fraction of the quasi-two-body decay, $\mathcal{B}(B \to RP_3)$. For the case when R has a finite-width, Eq. (1) does not hold. Let us define a quantity [1]

$$\eta_R \equiv \frac{\Gamma(B \to RP_3)\mathcal{B}(R \to P_1P_2)}{\Gamma(B \to RP_3 \to P_1P_2P_3)}, \tag{2}$$

so that the deviation of η_R from unity measures the degree of departure from the NWA when the width is finite. After taking into account the finite-width effect η_R from the resonance, the branching fraction of the quasi-two-body decay reads

$$\mathcal{B}(B \to RP_3) = \eta_R \frac{\mathcal{B}(B \to RP_3 \to P_1P_2P_3)_{\rm expt}}{\mathcal{B}(R \to P_1P_2)_{\rm expt}}, \tag{3}$$

which is suitable for a comparison with theoretical calculations.

In [1], we calculated the parameter η_R within the framework of QCD factorization [2] (QCDF) and the experimental parametrization [3] (EXPP) for various resonances and use these examples to highlight the importance of finite-width effects. We developed a general framework for the study of η_R and showed that basically η_R can be expressed in terms of a normalized differential decay rate,

$$\eta_R = \pi m_R \Gamma_R \frac{d\Gamma(m_R^2)}{dm_{12}^2} \Bigg/ \int \frac{d\Gamma(m_{12}^2)}{dm_{12}^2}\, dm_{12}^2 = \pi m_R \Gamma_R \frac{d\tilde{\Gamma}(m_R^2)}{dm_{12}^2}, \tag{4}$$

and one can verify that η_R given in the above equation approaches unity in the narrow width limit, reproducing the well-known result of NWA. It turns out that η_R is nothing but the value of the normalized differential decay rate evaluated at the contributing resonance.

We compare between $\eta_R^{\rm QCDF}$ and $\eta_R^{\rm EXPP}$ for their width dependence in Fig. 1, while numerical results are summarized in Table 1 (with BW and GS stand for Breit-Wigner and Gounaris-Sakurai line shapes, respectively). In general, the two quantities are similar for vector mesons but different

Resonance	$B^+ \to Rh_3 \to h_1h_2h_3$	Γ_R (MeV)	Γ_R/m_R	$\eta_R^{\rm QCDF}$	$\eta_R^{\rm EXPP}$
$f_2(1270)$	$B^+ \to f_2\pi^+ \to \pi^+\pi^-\pi^+$	$186.7^{+2.2}_{-2.5}$	0.146	$1.003^{+0.001}_{-0.002}$	$0.937^{+0.006}_{-0.005}$
$K_2^*(1430)$	$B^+ \to K_2^{*0}\pi^+ \to K^+\pi^-\pi^+$	109 ± 5	0.076	0.972 ± 0.001	1.053 ± 0.002
$\rho(770)$	$B^+ \to \rho^0\pi^+ \to \pi^+\pi^-\pi^+$	149.1 ± 0.8	0.192	0.93 (GS)	0.95 (GS)
				1.11 (BW)	1.15 (BW)
$\rho(770)$	$B^+ \to K^+\rho^0 \to K^+\pi^+\pi^-$	149.1 ± 0.8	0.192	0.95 (GS)	0.93 (GS)
				1.13 (BW)	1.13 (BW)
$K^*(892)$	$B^+ \to K^{*0}\pi^+ \to K^+\pi^-\pi^+$	47.3 ± 0.5	0.053	1.067 ± 0.002	1.075
$\sigma/f_0(500)$	$B^+ \to \sigma\pi^+ \to \pi^+\pi^-\pi^+$	700 ± 26 [**?**]	≈ 1.24	2.15 ± 0.05	1.64 ± 0.03
$K_0^*(1430)$	$B^+ \to K_0^{*0}\pi^+ \to K^+\pi^-\pi^+$	270 ± 80	≈ 0.19	0.83 ± 0.04	1.11 ± 0.03

Table 1: A summary of the η_R parameter for various resonances produced in the three-body B decays. Note that $\eta_R^{\rm QCDF}$ are obtained in QCDF calculations, while $\eta_R^{\rm EXPP}$ from the experimental parameterization.

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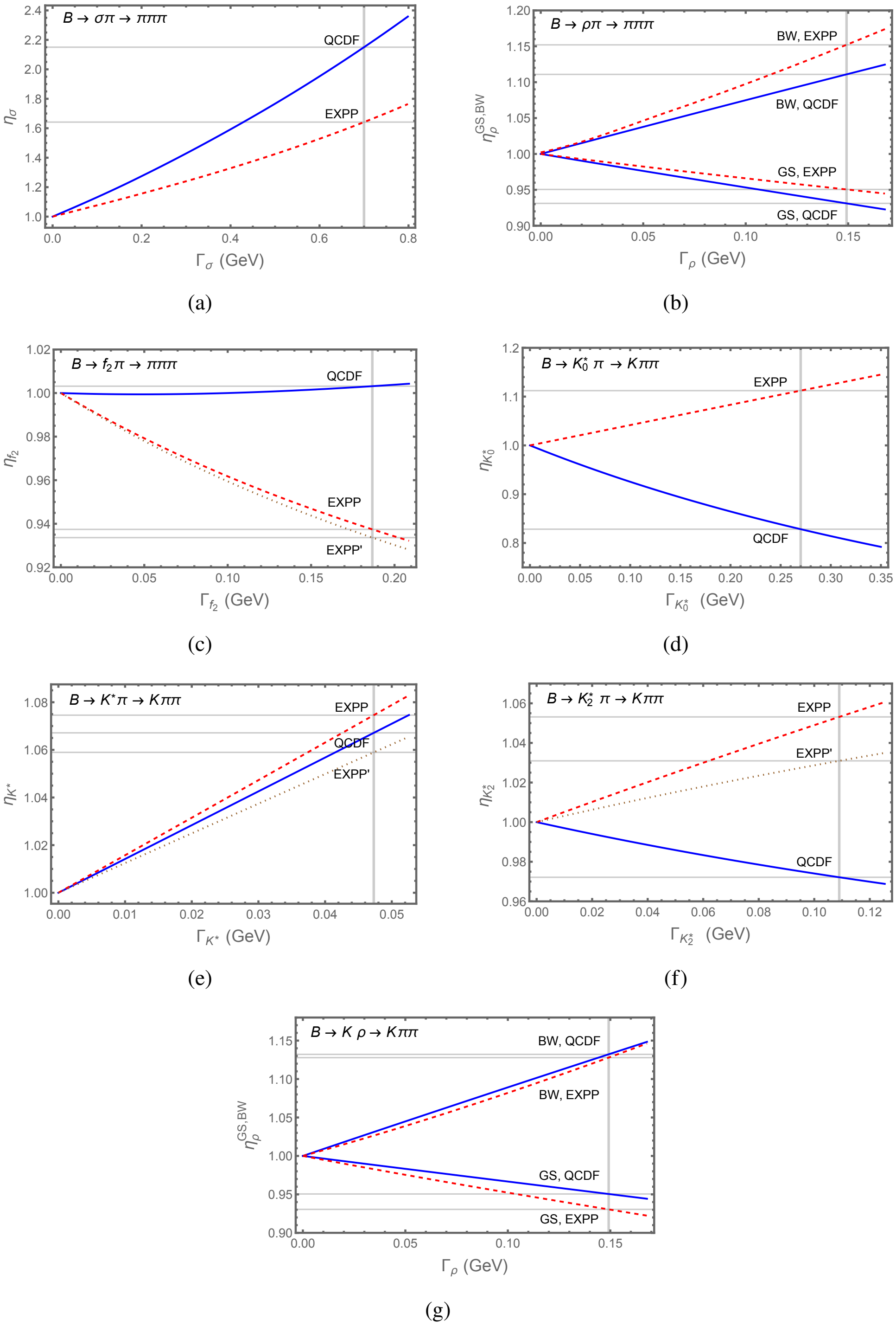

Figure 1: The parameter η_R as a function of the Γ_R in η_R of $B^- \to R\pi^- \to \pi^-\pi^+\pi^-$ and $B^- \to R\pi^- \to K^-\pi^+\pi^-$ decays, where the solid curves are derived from the QCDF calculation and the dashed (dotted) curves from the experimental parameterization (EXPP) with (without) the transversality condition imposed. The positions of the central values of the physical widths are marked by vertical lines in the plots.

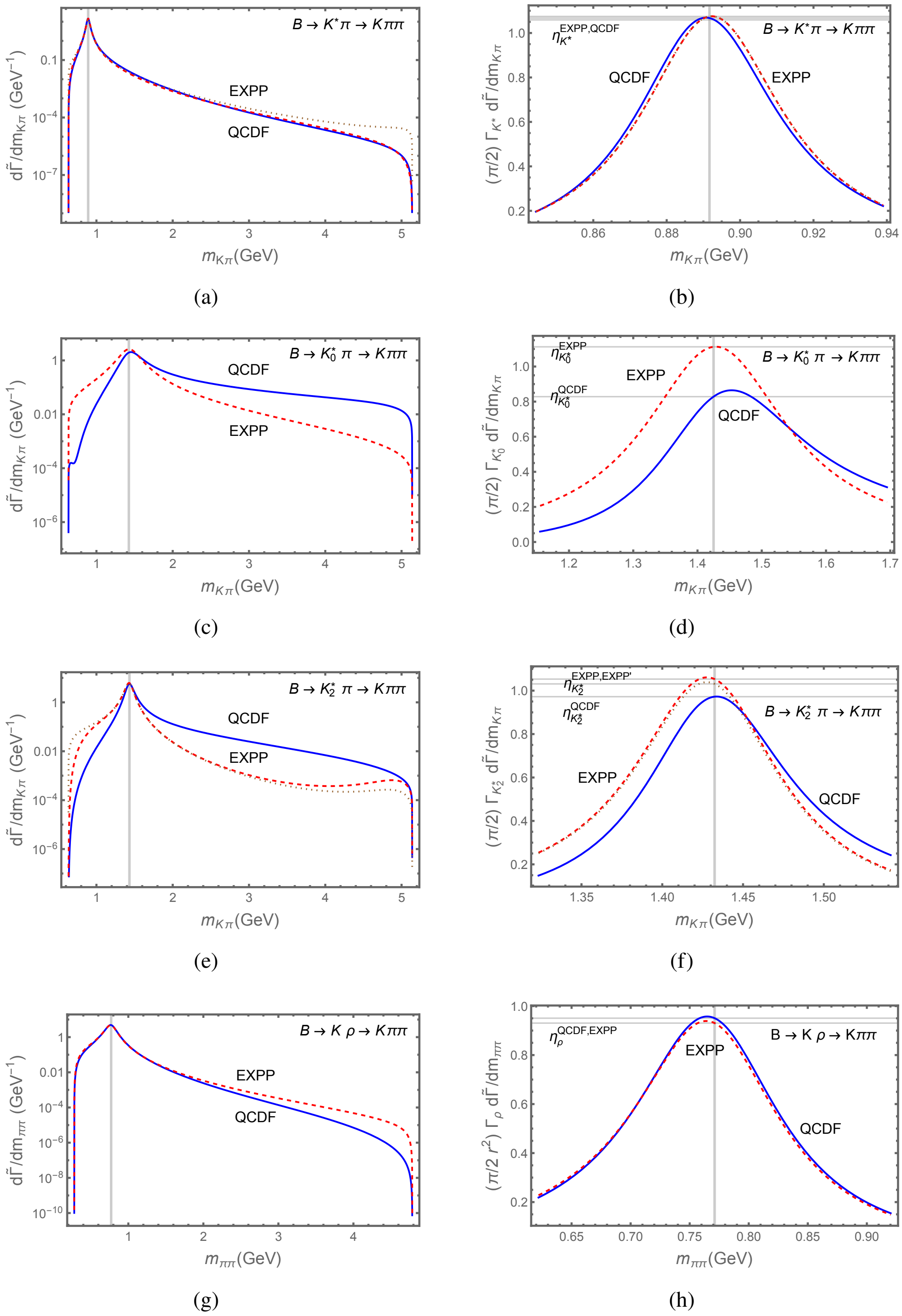

Figure 2: Left column: the normalized differential rates in $B^- \to R\pi^- \to K^-\pi^+\pi^-$ and $B^- \to K^-\rho \to K^-\pi^+\pi^-$ decays. Right column: plots scaled and blown-up in the resonance regions, where the heights at the resonances equal η_R. In plot (h), we use $r \equiv 1 + d\Gamma^0_\rho/m_\rho$. The solid curves come from the QCDF calculation and the dashed (dotted) curves from the experimental parameterization with (without) the transversality condition imposed.

Table 2: Branching fractions of quasi-two-body decays $B^+ \to RP_3$ (in units of 10^{-6}) derived from the measured $B^+ \to RP_3 \to P_1P_2P_3$ rates. $\mathcal{B}(B^+ \to RP_3)_{\rm NWA}$ denotes the branching fraction obtained in the narrow width approximation.

Mode	$\mathcal{B}(B \to RP \to PPP)_{\rm expt}$	$\mathcal{B}(B \to RP)_{\rm NWA}$	$\eta_R^{\rm QCDF}\mathcal{B}(B \to RP)_{\rm NWA}$	$\eta_R^{\rm EXPP}\mathcal{B}(B \to RP)_{\rm NWA}$
$B^+ \to f_2\pi^+ \to \pi^+\pi^-\pi^+$	1.17 ± 0.20	2.08 ± 0.36	2.09 ± 0.36	1.95 ± 0.33
$B^+ \to K_2^{*0}\pi^+ \to K^+\pi^-\pi^+$	$1.85^{+0.73}_{-0.50}$	$5.56^{+2.19}_{-1.50}$	$5.40^{+2.13}_{-1.46}$	$5.85^{+2.31}_{-1.58}$
$B^+ \to \rho^0\pi^+ \to \pi^+\pi^-\pi^+$	8.36 ± 0.77	8.36 ± 0.77	7.78 ± 0.72 (GS)	7.95 ± 0.73 (GS)
			9.28 ± 0.86 (BW)	
$B^+ \to K^+\rho^0 \to K^+\pi^+\pi^-$	3.7 ± 0.5	3.7 ± 0.5	3.5 ± 0.5 (GS)	3.4 ± 0.5 (GS)
			4.2 ± 0.6 (BW)	
$B^+ \to K^{*0}\pi^+ \to K^+\pi^-\pi^+$	6.71 ± 0.57	10.1 ± 0.8	10.7 ± 0.9	10.9 ± 0.9
$B^+ \to \sigma\pi^+ \to \pi^+\pi^-\pi^+$	3.83 ± 0.84	5.75 ± 1.26	12.36 ± 2.71	9.44 ± 2.08
$B^+ \to K_0^{*0}\pi^+ \to K^+\pi^-\pi^+$	$27.9^{+5.6}_{-4.3}$	45^{+9}_{-7}	37^{+8}_{-6}	50^{+10}_{-8}

for tensor and scalar mesons. A study of the differential rates in Fig. 2 enables us to understand the origin of their differences. For example, the similar normalized differential rates for ρ and K^* at and near the resonance account for $\eta^{\rm QCDF}_{\rho,K^*} \simeq \eta^{\rm EXPP}_{\rho,K^*}$. In contrast, the $m^2_{K\pi}$ dependence associated with the penguin Wilson coefficients $(a_6^p - a_8^p/2)$ in $B^- \to \overline{K}{}_0^*(1430)\pi^- \to K^-\pi^+\pi^+$ yields a large enhancement in the QCDF differential rate in the large $m_{K\pi}$ distribution, rendering $\eta^{\rm QCDF}_{K_0^*} < \eta^{\rm EXPP}_{K_0^*}$.

Finite-width corrections to the branching fractions of quasi-two-body decays obtained in the NWA, are summarized in Table 2 for both QCDF and EXPP schemes. In general, finite-width effects are small, less than 10%, but they are prominent in $B^+ \to \sigma/f_0(500)\pi^+$ and $B^+ \to K_0^{*0}(1430)\pi^+$ decays. In the presence of finite-width corrections, the PDG value of $\mathcal{B}(B^+ \to \rho\pi^+) = (8.3 \pm 1.2) \times 10^{-6}$ should be corrected to $(7.7 \pm 1.1) \times 10^{-6}$ in QCDF and $(7.9 \pm 1.1) \times 10^{-6}$ in EXPP. We have found very large width effects: $\eta^{\rm QCDF}_\sigma \sim 2.15$ and $\eta^{\rm EXPP}_\sigma \sim 1.64$. Consequently, $B^- \to \sigma\pi^-$ has a large branching fraction of order 10^{-5}. We have employed the Breit-Wigner line shape to describe the production of $K_0^*(1430)$ in three-body B decays and found large off-shell effects. The smallness of $\eta^{\rm QCDF}_{K_0^*}$ relative to $\eta^{\rm EXPP}_{K_0^*}$ is ascribed to the fact that the normalized differential rate obtained in the QCDF calculation is much larger than that using the EXPP scheme in the off-resonance region. The large discrepancy between QCDF estimate and experimental data of $\Gamma(B^- \to \overline{K}{}_0^{*0}\pi^- \to K^-\pi^+\pi^-)$ still remains an enigma.

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References

[1] H. Y. Cheng, C. W. Chiang and C. K. Chua, Phys. Rev. D **103** (2021) no.3, 036017 doi:10.1103/PhysRevD.103.036017 [arXiv:2011.07468 [hep-ph]]; Phys. Lett. B **813** (2021), 136058 doi:10.1016/j.physletb.2020.136058 [arXiv:2011.03201 [hep-ph]].

[2] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914-1917 (1999) [arXiv:hep-ph/9905312 [hep-ph]].

[3] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **78**, 012004 (2008) [arXiv:0803.4451 [hep-ex]].