Dineutrino modes probing lepton flavor violation

Rigo Bause,\textsuperscript{a} Hector Gisbert,\textsuperscript{a,*} Marcel Golz\textsuperscript{a} and Gudrun Hiller\textsuperscript{a}

\textsuperscript{a}Fakultät für Physik, TU Dortmund, Otto-Hahn-Str. 4, D-44221 Dortmund, Germany
E-mail: rigo.bause@tu-dortmund.de, hector.gisbert@tu-dortmund.de, marcel.golz@tu-dortmund.de, ghiller@physik.uni-dortmund.de

\textsuperscript{*}Speaker

\textit{SU(2)}_L–invariance links charged dilepton $\bar{q}' q^* \ell^+ \ell^−$ and dineutrino $\bar{q}' q^* \nu \nu$ couplings. This connection can be established using the Standard Model Effective Field Theory framework, and allows to perform complementary experimental tests of lepton universality and charged lepton flavor conservation with flavor-summed dineutrino observables. We present its phenomenological implications for the branching ratios of rare charm decays $c \rightarrow u \nu \bar{\nu}$ and rare $B$ decays $b \rightarrow s \nu \bar{\nu}$ decays.

\textit{The European Physical Society Conference on High Energy Physics (EPS-HEP2021), 26-30 July 2021}

\textit{Online conference, jointly organized by Universität Hamburg and the research center DESY}
1. Introduction

Flavor-changing neutral currents (FCNCs) of $q^a$ and $q^B$ quarks induced by $|\Delta q^a| = |\Delta q^B| = 1$ processes represent excellent probes of New Physics (NP) beyond the Standard Model (SM). Their weak loop suppression triggered by the Glashow-Iliopoulos-Maiani (GIM) mechanism and Cabibbo-Kobayashi-Maskawa (CKM) hierarchies, not necessarily present in SM extensions, can result in large experimental deviations from the SM predictions alluding to a breakdown of SM symmetries. In addition, its environment is enriched with further tests if leptons are involved, that is $q_\alpha q_\beta \ell_i^\alpha \bar{\ell}_j^\beta$. We exploit the $SU(2)_L$-link between left-handed charged lepton and neutrino couplings, which may be used to assess charged lepton flavor conservation (cLFC) and lepton universality (LU) quantitatively using flavor-summed dineutrino observables [1]. This link (3) is presented for $|\Delta q^a| = |\Delta q^B| = 1$ processes, but we stress that it holds analogously for other conserved quark transitions, both in the up- and down-sector.

These proceedings are organized as follows: In Section 2, we present the effective theory framework where the $SU(2)_L$-link between neutrino and charged lepton couplings is derived. In Sections 3 and 4, we work out its phenomenological implications for charm and beauty, respectively. The conclusions are drawn in Section 5. The results are based on Refs. [1–3], we refer there for further details.

2. $SU(2)_L$-link between dineutrino and charged dilepton couplings

At lowest order in the SM effective field theory (SMEFT), the Lagrangian accounting for semileptonic (axial-)vector four-fermion operators is given by [4],

$$\mathcal{L}_{\text{eff}} \supset \frac{C^{(1)}_{\ell q}}{v^2} \bar{Q} \gamma_\mu Q \bar{L} \gamma^\mu L + \frac{C^{(3)}_{\ell q}}{v^2} \bar{Q} \gamma_\mu \gamma^5 Q \bar{L} \gamma^\mu \gamma^5 L + \frac{C_{\ell u}}{v^2} \bar{U} \gamma_\mu U \bar{L} \gamma^\mu L + \frac{C_{\ell d}}{v^2} \bar{D} \gamma_\mu D \bar{L} \gamma^\mu L. \quad (1)$$

Reading off couplings to dineutrinos ($C^N_A$) and charged dileptons ($K^N_A$) by writing the operators (1) into $SU(2)_L$-components, one obtains

$$C_{L}^{U} = K_{L}^{D} = \frac{2\pi}{\alpha} \left( C_{\ell q}^{(1)} + C_{\ell q}^{(3)} \right), \quad C_{R}^{U} = K_{R}^{D} = \frac{2\pi}{\alpha} C_{\ell u}, \quad (2)$$

where $N = U \ (N = D)$ represents the up-quark sector (down-quark sector), and $A = L(R)$ denotes left- (right-) handed quark currents. Interestingly, $C^N_R = K^N_R$ holds model-independently, while $C^N_L$ is not fixed by $K^N_L$ in general due to the different relative signs of $C_{\ell q}^{(1)}$ and $C_{\ell q}^{(3)}$. Expressing Eqs. (2) in the mass basis, that is $C^N_L = W^\dagger K^M_L W + O(\lambda)$, $C^N_R = W^\dagger K^R_L W$ where $W$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and $\lambda \sim 0.2$ the Wolfenstein parameter, and summing lepton flavors $i, j$ incoherently, one obtains the following identity [1]

$$\sum_{\ell = i, j} \left( |C_{L}^{N(ij)}|^2 + |C_{R}^{N(ij)}|^2 \right) = \sum_{\ell = i, j} \left( |K_{L}^{M(ij)}|^2 + |K_{R}^{N(ij)}|^2 \right) + O(\lambda), \quad (3)$$

between charged lepton couplings $K_{L,R}$ and neutrino ones $C_{L,R}$.1 Here, we use $N, M = U, D$ when the link is exploited for neutrino couplings in the up-quark sector, while $N, M = D, U$ for

---

1Wilson coefficients in calligraphic style denote those for mass eigenstates.
the down-quark sector. Eq. (3) allows the prediction of dineutrino rates for different leptonic flavor structures \( \mathcal{K}_{L,R}^{N_{ij}} \),

i) \( \mathcal{K}_{L,R}^{N_{ij}} \propto \delta_{ij} \), i.e. lepton-universality (LU),

ii) \( \mathcal{K}_{L,R}^{N_{ij}} \) diagonal, i.e. charged lepton flavor conservation (cLFC),

iii) \( \mathcal{K}_{L,R}^{N_{ij}} \) arbitrary,

which can be probed with lepton-specific measurements. In the following sections, we use the following notation i.e. \( \mathcal{K}_{L,R}^{h_{ij}} = \mathcal{K}_{L,R}^{D_{2ij}} \), \( \mathcal{C}_{L,R}^{h_{ij}} = \mathcal{C}_{L,R}^{D_{2ij}} \), etc., to improve the readability.

<table>
<thead>
<tr>
<th></th>
<th>( ee )</th>
<th>( \mu\mu )</th>
<th>( \tau\tau )</th>
<th>( ee )</th>
<th>( e\tau )</th>
<th>( \mu\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^\ell\ell )</td>
<td>21</td>
<td>6.0</td>
<td>77</td>
<td>6.6</td>
<td>59</td>
<td>70</td>
</tr>
<tr>
<td>( \delta R^\ell\ell )</td>
<td>19</td>
<td>5.4</td>
<td>69</td>
<td>5.7</td>
<td>55</td>
<td>63</td>
</tr>
<tr>
<td>( r^\ell\ell )</td>
<td>39</td>
<td>11</td>
<td>145</td>
<td>12</td>
<td>115</td>
<td>133</td>
</tr>
</tbody>
</table>

Table 1: Bounds on \( |\Delta c| = |\Delta u| = 1 \) parameters \( R^\ell\ell \) and \( \delta R^\ell\ell \) from Eqs. (4), as well as their sum, \( r^\ell\ell = R^\ell\ell + \delta R^\ell\ell \). Table taken from Ref. [2].

3. Predictions for charm

In this section, we study the implications of (3) for \( c \to u \nu \bar{\nu} \) dineutrino transitions, where the situation is exceptional as the SM amplitude is fully negligible due to an efficient GIM-suppression [5] and the current lack of experimental constraints. We use upper limits on \( \mathcal{K}_A^{N_{\ell\ell}} \) from high-\( p_T \) [6, 7], which allow to set constraints on

\[
R^\ell\ell = |\mathcal{K}_L^{d_{\ell\ell}}|^2 + |\mathcal{K}_R^{u_{\ell\ell}}|^2, \quad \delta R^\ell\ell = |\mathcal{K}_L^{d_{\ell\ell}} \pm \mathcal{K}_R^{u_{\ell\ell}}|^2,
\]

\[
\delta R^\ell\ell = 2 \lambda \text{Re} \left\{ |\mathcal{K}_L^{d_{\ell\ell}}|^2 |\mathcal{K}_R^{u_{\ell\ell}}|^2 - |\mathcal{K}_L^{d_{\ell\ell}}|^2 |\mathcal{K}_R^{u_{\ell\ell}}|^2 \right\},
\]

which directly enter in \( c \to u \nu \bar{\nu} \) branching ratios. Upper limits on \( R^\ell\ell \), \( \delta R^\ell\ell \) and their sum \( r^\ell\ell = R^\ell\ell + \delta R^\ell\ell \) are provided in Table 1. Since the neutrino flavors are not tagged, the branching ratio is obtained by an incoherent sum

\[
\mathcal{B}(c \to u \nu \bar{\nu}) = \sum_{i,j} \mathcal{B}(c \to u \nu_i \bar{\nu}_j) \propto x_{uc},
\]

where \( x_{uc} = \sum_{i,j} \left( |c_{UL}^{ij}|^2 + |c_{UR}^{ij}|^2 \right) \). Using (3) with \( N, M = U, D \) and Table 1, we obtain upper limits for the different benchmarks i)-iii):

\[
x_{uc} = 3 \, r^{\mu\mu} \leq 34, \quad \text{(LU)}
\]

\[
x_{uc} = r^{ee} + r^{\mu\mu} + r^{\tau\tau} \leq 196, \quad \text{(cLFC)}
\]

\[
x_{uc} = r^{ee} + r^{\mu\mu} + r^{\tau\tau} + 2 \left( r^{ee} + r^{e\tau} + r^{\mu\tau} \right) \leq 716. \quad \text{(8)}
\]
Since dimuon bounds are the most stringent ones, see Table 1, they set the LU-limit (6). Experimental measurements above the upper limit in (6) would indicate a breakdown of LU, while values above the limit in (7) would imply a violation of cLFC. Corresponding upper limits on branching ratios of dineutrino modes of a charmed hadron $h_c$ into a final hadronic state $F$,

$$\mathcal{B}(h_c \to F \nu \bar{\nu}) = A_{h_c}^{h_c,F} x_{\nu F}^+ + A_{h_c}^{h_c,F} x_{\bar{\nu} F}^- ,$$  

(9)

are provided in Table 2 for several decays modes. The $A_{h_c}^{h_c,F}$ coefficients in Eq. (9) are given in the second column of Table 2. Using the limits (6), (7), (8), together with Eq. (9) and the values of $A_{h_c}^{h_c,F}$, we obtain upper limits on branching ratios for the three flavor scenarios $\mathcal{B}_{LU}^{\max}$, $\mathcal{B}_{\text{cLFC}}^{\max}$, and $\mathcal{B}_{\text{max}}$. A branching ratio measurement $\mathcal{B}_{\exp}$ within $\mathcal{B}_{LU}^{\max} < \mathcal{B}_{\exp} < \mathcal{B}_{\text{cLFC}}^{\max}$ would be a clear signal of LU violation. In contrast, a branching ratio above $\mathcal{B}_{\text{cLFC}}^{\max}$ would imply a breakdown of cLFC.

<table>
<thead>
<tr>
<th>$h_c \to F$</th>
<th>$A_{h_c}^{h_h,F}$</th>
<th>$A_{h_c}^{h_h,F}$</th>
<th>$\mathcal{B}_{LU}^{\max}$</th>
<th>$\mathcal{B}_{\text{cLFC}}^{\max}$</th>
<th>$\mathcal{B}_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \to \pi^0$</td>
<td>0.9</td>
<td>-</td>
<td>6.1</td>
<td>3.5</td>
<td>13</td>
</tr>
<tr>
<td>$D^+ \to \pi^+$</td>
<td>3.6</td>
<td>-</td>
<td>25</td>
<td>14</td>
<td>52</td>
</tr>
<tr>
<td>$D_s^+ \to K^+$</td>
<td>0.7</td>
<td>-</td>
<td>4.6</td>
<td>2.6</td>
<td>9.6</td>
</tr>
<tr>
<td>$D^0 \to \pi^0$</td>
<td>$O(10^{-3})$</td>
<td>-</td>
<td>0.21</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>$D^0 \to \pi^+ \pi^-$</td>
<td>$O(10^{-4})$</td>
<td>-</td>
<td>0.41</td>
<td>2.8</td>
<td>1.6</td>
</tr>
<tr>
<td>$D^0 \to K^+ K^-$</td>
<td>$O(10^{-4})$</td>
<td>-</td>
<td>0.004</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Lambda_c^+ \to \rho^+$</td>
<td>1.0</td>
<td>-</td>
<td>1.7</td>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>$\Xi_c^+ \to \Sigma^+$</td>
<td>1.8</td>
<td>-</td>
<td>3.5</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>$D^0 \to \chi$</td>
<td>2.2</td>
<td>-</td>
<td>2.2</td>
<td>15</td>
<td>8.7</td>
</tr>
<tr>
<td>$D^+ \to \chi$</td>
<td>5.6</td>
<td>-</td>
<td>5.6</td>
<td>38</td>
<td>22</td>
</tr>
<tr>
<td>$D_s^+ \to \chi$</td>
<td>2.7</td>
<td>-</td>
<td>2.7</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h_c \to F$</th>
<th>$A_{h_c}^{h_h,F}$</th>
<th>$A_{h_c}^{h_h,F}$</th>
<th>$\mathcal{B}_{LU}^{\max}$</th>
<th>$\mathcal{B}_{\text{cLFC}}^{\max}$</th>
<th>$\mathcal{B}_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \to \pi^0$</td>
<td>0.9</td>
<td>-</td>
<td>6.1</td>
<td>3.5</td>
<td>13</td>
</tr>
<tr>
<td>$D^+ \to \pi^+$</td>
<td>3.6</td>
<td>-</td>
<td>25</td>
<td>14</td>
<td>52</td>
</tr>
<tr>
<td>$D_s^+ \to K^+$</td>
<td>0.7</td>
<td>-</td>
<td>4.6</td>
<td>2.6</td>
<td>9.6</td>
</tr>
<tr>
<td>$D^0 \to \pi^0$</td>
<td>$O(10^{-3})$</td>
<td>-</td>
<td>0.21</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>$D^0 \to \pi^+ \pi^-$</td>
<td>$O(10^{-4})$</td>
<td>-</td>
<td>0.41</td>
<td>2.8</td>
<td>1.6</td>
</tr>
<tr>
<td>$D^0 \to K^+ K^-$</td>
<td>$O(10^{-4})$</td>
<td>-</td>
<td>0.004</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Lambda_c^+ \to \rho^+$</td>
<td>1.0</td>
<td>-</td>
<td>1.7</td>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>$\Xi_c^+ \to \Sigma^+$</td>
<td>1.8</td>
<td>-</td>
<td>3.5</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>$D^0 \to \chi$</td>
<td>2.2</td>
<td>-</td>
<td>2.2</td>
<td>15</td>
<td>8.7</td>
</tr>
<tr>
<td>$D^+ \to \chi$</td>
<td>5.6</td>
<td>-</td>
<td>5.6</td>
<td>38</td>
<td>22</td>
</tr>
<tr>
<td>$D_s^+ \to \chi$</td>
<td>2.7</td>
<td>-</td>
<td>2.7</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2: Coefficients $A_{h_c}^{h_h,F}$, as defined in (9), and model-independent upper limits on $\mathcal{B}_{LU}^{\max}$, $\mathcal{B}_{\text{cLFC}}^{\max}$, $\mathcal{B}_{\text{max}}$ from (6), (7) and (8), respectively, corresponding to the lepton flavor symmetry benchmarks i)-iii). Table taken from Ref. [2].

4. Testing lepton universality with $b \to s \nu \bar{\nu}$

In this section we study $b \to s \nu \bar{\nu}$ transitions and their interplay with $b \to s \ell^+ \ell^-$ transitions routed by (3). The branching ratios for $B \to V \nu \bar{\nu}$ and $B \to P \nu \bar{\nu}$ decays in the LU limit are given by

$$\mathcal{B}(B \to V \nu \bar{\nu})_{LU} = A_{b}^{BV} x_{b_{LS},LU}^+ + A_{-}^{BV} x_{b_{LS},LU}^- , \quad \mathcal{B}(B \to P \nu \bar{\nu})_{LU} = A_{b}^{BP} x_{b_{LS},LU}^+ ,$$

(10)

where $x_{b_{LS},LU}^+ = 3 |c_{SM}^{bs\ell\bar{\ell}} + K_{L}^{\ell\bar{\ell}} \bar{\ell} \bar{\ell} R^{-1} |^2$, and the values of $A_{b}^{BV}$ and $A_{b}^{BP}$ for different modes can be found in Ref. [3]. We obtain two solutions for the coupling $K_{L}^{\ell\bar{\ell}}$ when we solve $\mathcal{B}(B \to P \nu \bar{\nu})_{LU}$ in Eq. (10). Plugging them into Eq. (10) results in a correlation between both LU branching ratios [3]

$$\mathcal{B}(B \to V \nu \bar{\nu})_{LU} = \frac{A_{b}^{BV}}{A_{b}^{BP}} \mathcal{B}(B \to P \nu \bar{\nu})_{LU} + 3 A_{-}^{BV} \left[ \mathcal{B}(B \to P \nu \bar{\nu})_{LU} \right]^{2} 2 A_{b}^{BP} + 2 K_{L}^{\ell\bar{\ell}} R^{-1} \left( \mathcal{B}(B \to P \nu \bar{\nu})_{LU} \right)^{2} .$$

(11)
The most stringent limits on $\mathcal{K}_R^{b\ell\ell}$ are given for $\ell\ell = \mu\mu$. Performing a 6D global fit of the semileptonic Wilson coefficients $C^{(\ell)}_{(7,9,10),\mu}$ to the current experimental data on $b \to s\mu^+\mu^-$ data (excluding $K_{\ell\ell}$ which can be polluted by NP effects in electron couplings), we obtain the following $1\sigma$ fit value [3]

$$\mathcal{K}_R^{b\ell\ell} = V_{tb}V_{ts}^*(-0.46 \pm 0.26). \quad (12)$$

Fig. 1 displays the correlation between $\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$ and $\mathcal{B}(B^0 \to K^{0}\nu\bar{\nu})$, cf. Eq. (11). The SM predictions $\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{SM} = (0.02 \pm 0.005) \cdot 10^{-6}$, $\mathcal{B}(B^0 \to K^{0}\nu\bar{\nu})_{SM} = (0.02 \pm 0.005) \cdot 10^{-6}$ [3] are depicted as a blue diamond with their $1\sigma$ uncertainties (blue bars). We have scanned $\mathcal{K}_R^{b\mu\mu}$, $A_{\pm}^{b\ell\ell}$, and $A_{\pm}^{b\ell\ell}K_{\ell\ell}$ within their $1\sigma$ ($2\sigma$) regions in Eq. (10), resulting in the dark red region (dashed red lines) which represents the LU region, numerically [3]

$$\frac{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{0}\nu\bar{\nu})} = 1.7 \ldots 2.6 \ (1.3 \ldots 2.9). \quad (13)$$

Interestingly, a branching ratio measurement outside the red region would clearly signal evidence for LU violation, but if a future measurement is instead inside this region, this may not necessarily imply LU conservation. Outside the light green region the validity of our effective field theory (EFT) framework gets broken [3]. More stringent limits for specific LU SM extensions are depicted as benchmarks, resulting in best fit values (markers) and $1\sigma$ regions (ellipses) for $Z'$ (red star), LQ representations $S_3$ (pink pentagon) and $V_3$ (celeste triangle) from $b \to s\mu^+\mu^-$ global fits, see Ref. [3] for details. The current experimental 90% CL upper limits, $\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{exp} < 1.8 \cdot 10^{-5}$ [8] and $\mathcal{B}(B^0 \to K^{0}\nu\bar{\nu})_{exp} < 2.6 \cdot 10^{-5}$ [8], are displayed by hatched bands. The gray bands represent the derived EFT limits, $\mathcal{B}(B^0 \to K^{0}\nu\bar{\nu})_{derived} < 1.5 \cdot 10^{-5}$, from Ref. [3]. A measurement between gray and hatched area would infer a clear hint of NP not covered by our EFT framework, i.e. light particles. The projected experimental sensitivity (10% at the chosen point) of Belle II with 50 ab$^{-1}$ is illustrated by the yellow boxes [9]. Similar conclusions can be drawn in $b \to d\nu\bar{\nu}$ decay [3].

Figure 1: Correlation between $\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$ and $\mathcal{B}(B^0 \to K^{0}\nu\bar{\nu})$. Details are given in the main text. Figure taken from Ref. [3].
5. Conclusions

$SU(2)_L$-invariance relates dineutrinos $\bar{q} q' \bar{\nu} \nu$ and charged dilepton couplings $\bar{q} q' \ell^+ \ell^-$ in a model-independent way. This link (3) allows probing lepton flavor structure in dineutrino observables in three benchmarks: lepton universality, charged lepton flavor conservation and lepton flavor violation. The link has been exploited for the rare charm and $B$ decays, resulting in novel tests of the aforementioned symmetries, see Table 2 and Eq. (13), respectively. Our predictions are well-suited for the experiments Belle II [9], BES III [10], and future $e^+e^-$-colliders, such as an FCC-ee running at the $Z$ [11], and could offer some insight on the persistent anomalies in $B$ decays.

Acknowledgments

We want to thank the organizers for their effort to make this conference such a successful event. This work is supported by the Studienstiftung des Deutschen Volkes (MG) and the Bundesministerium für Bildung und Forschung – BMBF (HG).

References


[8] P.A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020)

