Exploring straight infinite Wilson lines towards formulating a new classical theory for gluodynamics

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We develop a new classical action that in addition to MHV vertices contains also $N^k$ MHV vertices, where $1 \leq k \leq n - 4$ with $n$ the number of external legs. The lowest order vertex is the four-point MHV vertex – there is no three point vertex and thus the amplitude calculation involves fewer diagrams than in the CSW method and, obviously, considerably fewer than in the standard Yang-Mills action. The action is obtained by a canonical transformation of the Yang-Mills action in the light-cone gauge, where the field transformations are based on Wilson line functionals.
1. Introduction

The following work focuses on a description of pure gluonic scattering amplitudes in terms of a new action, currently developed at the classical level (thus suitable for tree amplitudes). Despite considered as fundamental, gluon fields are often not the most efficient degrees of freedom for computing amplitudes. Interestingly, in [1, 2], the Maximally Helicity Violating (MHV) vertices used in the Cachazo-Svrcek-Witten (CSW) method [3] were shown to be connected with straight infinite Wilson lines on the self-dual plane. Following that, in [4], we derived a new classical action for gluodynamics in which the fields are directly related to Wilson line functionals. The new action does not have the triple-gluon vertices at all. This is because the triple-gluon vertices have been effectively resummed inside the Wilson lines. Higher-point vertices include not only the MHV vertices, but also other helicity configurations. The number of diagrams needed to obtain amplitudes beyond the MHV level is thus greatly reduced. We performed explicit calculations within the new formulation of several higher multiplicity amplitudes, to verify the consistency of the results.

2. Self-Dual Yang-Mills theory and scattering amplitudes

The starting point is the full Yang-Mills action on the constant light-cone time \(x^+\) in the light-cone gauge \(\hat{A}^+ = 0\) (we denote \(\hat{A} = A_M t^a\)). Integrating out the \(\hat{A}^-\) fields (appearing quadratically) from the partition function, leaves only two complex fields \(\hat{A}^*, \hat{A}^\bullet\) that correspond to plus-helicity and minus-helicity gluon fields. The action in this setup has \((++-), (-+-)\) and \((+-+)\) vertices. The fully covariant form of the self dual equations is \(\hat{F}^\mu\nu = \ast\hat{F}^\mu\nu\) where the Hodge dual is defined as \(\ast\hat{F}^\mu\nu = -i\varepsilon_{\mu\nu\alpha\beta}\hat{F}^{\alpha\beta}\). The corresponding self dual equation in light-cone gauge, can be obtained from the truncated action consisting of just the kinetic term and the \((++-\)) vertex.

We are interested in the classical solution to the self-dual EOM that give us information about scattering amplitudes. The tree-level Green functions can be extracted by coupling the classical action to an external current \(\hat{j}\) and postulating the power series solution \(A^*_a [\hat{j}] (x)\). Assuming the currents \(\hat{j}\) are supported on the light-cone, the momentum space off-shell currents generated by the solution correspond to the off-shell currents similar to the Berends-Giele currents, i.e. to the amplitude of an off-shell gluon scattering into on-shell gluons.

3. Straight infinite Wilson lines in Self-Dual and MHV Lagrangians

It is very interesting, how the self-dual equation encodes an infinite Wilson line spanning over the transverse complex plane [2]. This can be simply demonstrated as follows. First assume a power series expansion for the inverse functional \(j_a [A^*]\). The kernels for this expansion can be found by substituting it to \(A^*_a [\hat{j}] (x)\). Then, it is straightforward to realise that the kernels are nothing but the momentum space expansion coefficients of the following straight infinite Wilson line (in the light-cone gauge) [1]:

\[
j_a [A] (x) = \int_{-\infty}^{\infty} da \; \textrm{Tr} \left\{ \frac{1}{2\pi i} t^a \partial_- \mathcal{P} \exp \left[ ig \int_{-\infty}^{\infty} ds \; e^+_a \cdot \hat{A} (x + s e^+_a) \right] \right\},
\]

where \(e^+_a = e^+_1 - \alpha \eta\). Notice, that this four vector has the form of a gluon polarization vector. Indeed for \(\alpha = p \cdot e^+_1 / p^+\), it is the transverse polarization vector for a gluon with momentum \(p\). Thus, in

The Mansfield’s transformation [5] eliminates one of the triple gluon vertex (++−) while the other triple gluon vertex (+−−) still exists in the MHV action. Triple point vertices are not very effective building blocks for calculating amplitudes, and, actually they do not constitute any physical amplitude themselves – in the on-shell limit they are zero (for real momenta). Motivated by the geometric considerations mentioned earlier and the above arguments, we proposed in [4] a more general transformation \{\hat{A}^*, \hat{\hat{A}}^*\} \rightarrow \{\hat{Z}^*[A^*, A^*], \hat{Z}^*[A^*, \hat{A}^*]\} leading to a new action. These transformations are based on path ordered exponentials of the gauge fields, extending over both the self-dual and anti-self-dual planes. It maps the kinetic term and both the triple-gluon vertices of the Yang-Mills action into a free term in the new action. In order to preserve the functional measure in the partition function, up to a field independent factor, it is necessary that the transformation is
canonical. The transformation can be solved to obtain the explicit solutions for $\hat{Z}^*[A^*, A^*]$ and $\hat{Z}^*[A^*, A^*]$ fields [4]. Substituting their inverse in the Yang-Mills action results in the new action. For convenience, we shall call the new action as $Z$-field action hereafter. It has the following generic structure:

$$S^{(LC)}_{Y-M}[Z^*, Z^*] = \int dx^* \left[ L^{(LC)}_{-+} + L^{(LC)}_{--+} + L^{(LC)}_{----} + \cdots + L^{(LC)}_{---++} + \cdots \right]$$

The key properties of the action are:

i) There are no three point interaction vertices. The reason is that the triple-gluon vertices have been effectively resummed inside the Wilson lines.

ii) At the classical level, there are no all-plus, all-minus, as well as $(-+\cdots+), (-\cdots-)$ vertices.

iii) There are MHV vertices, $(-+\cdots+)$, corresponding to MHV amplitudes in the on-shell limit.

iv) There are $\overline{\text{MHV}}$ vertices, $(-\cdots++)$, corresponding to $\overline{\text{MHV}}$ amplitudes in the on-shell limit.

v) All vertices have the form which can be easily calculated. In fact, we reported a compact general form for any vertex in the $Z$-field action in [T].

Using this new action we computed several tree-level amplitudes. For example, for the 6-point Next-To-MHV (NMHV) amplitude with helicity configuration $(---+++)$ we have just three contributing diagrams. The sum of these diagrams reproduce in the on-shell limit the known result [7]. For 7-point NNMHV amplitude $(-+---++)$ we had just five contributing diagrams depicted in Fig. 2. Furthermore, the higher multiplicity amplitudes, up to 8-point NNMHV $(-+---+++)$,

Figure 2: Diagrams contributing to the 7-point NNMHV amplitude $(-+---++)$.

were calculated and shown to be in agreement with the standard methods [8]. The maximum number of diagrams we encountered in that case was 13. The absence of triple-gluon vertices resulted in fewer diagrams required to compute amplitudes, when compared to the CSW method and, obviously, considerably fewer than in the standard Yang-Mills action.

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