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A falling magnetic monopole as a local quench

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We look at the free fall of a magnetic monopole in Poincaré AdS_4 as a holographic model of local quench in a strongly coupled CFT_3 activated by the insertion of a condensing scalar operator. Comparing with the setup obtained by replacing the monopole with a black hole, we probe to what extent the physics of the quench is sensitive to the features of the falling object. As a result, we argue that in case the energy of the dual quenches is conserved, the holographic energy-momentum tensor holds the same functional form in both configurations. Instead, the spreading of entanglement driven by the quench drastically changes. We comment on the implications of such outcomes on the validity of the first law of entanglement entropy.

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1. Introduction

The AdS/CFT correspondence offers the opportunity to investigate strongly coupled conformal field theories (CFTs) by means of more accessible weakly coupled gravitational theories in asymptotically anti-de Sitter (AdS) spacetimes. An interesting issue that can be addressed in this framework is the evolution of strongly coupled quantum systems during non-equilibrium processes, an example of which is the quench, describing the thermalization of a system after the sudden injection of energy. In case such an excitation is localized, the CFT quench is holographically equivalent to the free fall of a particle-like object in higher dimensional Poincaré AdS spacetime. This raises the question of how the details of the falling object affect the physics of the dual quench. Based on [1], the discussion is organised as follows. In Section 2 we describe the gravitational system, specialising on a magnetic monopole and a black hole (BH) in 4-dimensional AdS spacetime. In Section 3 we analyse the dual CFT dynamics and, referring to the energy-momentum tensor and the entanglement entropy, we draw a comparison between the two cases. We conclude in Section 4.

2. A falling monopole in AdS

We consider a 4-dimensional theory of gravity coupled to an $SU(2)$ gauge field A_l^a and a massive adjoint scalar ϕ^a. An asymptotically global AdS solution can be modelled by the metric

$$ds^2 = L^2 \left[-h(r)g(r)(1+r^2)d\tau^2 + \frac{h(r)}{g(r)} \frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right], \tag{1}$$

with $h(r)$, $g(r) \to 1$ at the spacetime boundary $r \to +\infty$. For our purposes, it sufficies to work at first order in a backreaction parameter ε, slightly modifying global AdS_4. Under this assumption, the profile functions can be expressed as $h(r) = 1 + \varepsilon h_\varepsilon(r)$ and $g(r) = 1 + \varepsilon g_\varepsilon(r)$.
A monopole solution to this model is given by a generalisation of 't Hooft-Polyakov ansatz [2]:

$$\phi^a = \frac{1}{L} H(r) n^a, \qquad A_l^a = rF(r)\varepsilon^{aik} n^k \partial_l n^i, \qquad n^a = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta), \tag{2}$$

where $x^l = (r, \theta, \varphi)$ and $A_\tau^a = 0$. Parameterising the matter backreaction by the Newton's constant $\varepsilon \sim G/L^2$, numerical results for $h_\varepsilon(r)$ and $g_\varepsilon(r)$ can be extracted from the equations of motion [1]. For vanishing matter fields $H(r) = F(r) = 0$, a BH solution is recovered. Taking as a backreaction parameter the BH mass $\varepsilon = M$, the metric is first-order exact: $h(r) = 1$ and $g(r) = 1 - M/(r + r^3)$.

Both the monopole and the BH can be thought of as static particle-like objects located at $r = 0$ in global AdS_4. Starting from these configurations, we can readily obtain solutions describing free falling objects in Poincaré AdS_4 by performing the coordinate transformation [3]

$$(\tau, r, \theta, \varphi) = \left(\arctan\frac{t}{a_+}, \frac{\sqrt{a_-^2 + \rho^2}}{z}, \arctan\frac{\rho}{a_-}, \varphi \right), \qquad a_\pm(z, \rho, t) = \frac{z^2 + \rho^2 - t^2 \pm A^2}{2A}. \tag{3}$$

As a result, the particle center, located at $\rho = 0$, describes the trajectory $a_-(z, 0, t) = 0$. In this perspective, the scale A represents the z-coordinate position of the object at initial time $t = 0$.

In Fig. 1 we show the energy density of the falling monopole at different times t. Strictly speaking, the monopole behaves as a pointlike particle only at initial times. Then, as it falls deeper into the bulk, its energy density spreads on a spherical wavefront. As we will see shortly, this mimics the propagation of the perturbation away from the excited region in the dual CFT.

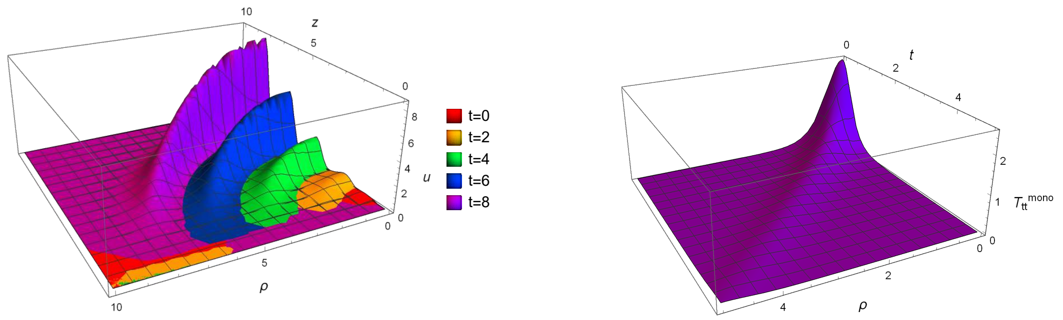

Figure 1: On the left: energy density of the falling monopole in AdS_4 at different times. On the right: energy density of the dual CFT_3 system interested by a local quench. We have set $G = 0.1$ and $A = L = M = \alpha_H = 1$.

3. A CFT local quench

In the holographic framework, adding a particle into 4-dimensional AdS translates into a perturbation of the boundary 3-dimensional CFT. In the presence of a bulk scalar triplet ϕ^a, such a perturbation is triggered by the insertion of a global $SU(2)$ triplet of operators O^a with VEV and source related to the boundary expansion of ϕ^a itself. Setting the mass of the scalar field to $m_\phi^2 = -2/L^2$, the near-boundary behaviour of (2) is given by $H(r) \sim \alpha_H\, r^{-1} + \beta_H\, r^{-2}$. Since both modes are normalisable, a possible quantisation choice is the so-called triple-trace deformation, under which O^a has no source and the CFT action is deformed by the term

$$\Delta I_{CFT} = -\frac{\beta_H}{3\alpha_H^2}\int d^3x\,(O^a O^a)^{3/2}\,, \qquad \langle O^a\rangle = \frac{\alpha_H}{\sqrt{a_-^2(0,\rho,t)+\rho^2}}\,n^a\,. \tag{4}$$

The energy of the resulting system is conserved, making this choice the most convenient. Regarding the gauge field (2), we require it to vanish at the boundary, getting a dual CFT with no explicit breaking of the global $SU(2)$ symmetry.

Energy-momentum tensor: Information on the dynamics of the CFT state is provided by the holographic energy-momentum tensor T_{ij}, which can be extracted from the Fefferman-Graham asymptotic expansion of the bulk metric. In Fig. 1 the energy density T_{tt} for the monopole case is shown. As it is clear from the picture, at time $t = 0$ energy is injected around $\rho = 0$ in a region of radius A. Then, the perturbation propagates at the speed of light while being quenched. Such a time evolution is common to both models, as revealed by an explicit computation leading to [1]:

$$T_{ij}^{mono} = \frac{16\pi G\alpha_H\beta_H - 3L^2 g_3}{3L^2 M}\,T_{ij}^{BH}\,, \tag{5}$$

where g_3 is the coefficient of the $O(r^{-3})$ term in the boundary expansion of $g(r)$. From eq. (5) we conclude that the energy-momentum tensor does not make a significant distinction between a falling monopole and BH, motivating us to look for a non-local probe of the bulk geometry.

Entanglement entropy: According to the Ryu-Takayanagi proposal, the entanglement entropy of a CFT subsystem is related to the area of an extremal codimension-2 surface in higher dimensional AdS. The $O(\varepsilon)$ variation of this area due to backreaction is thus interpreted as the difference of

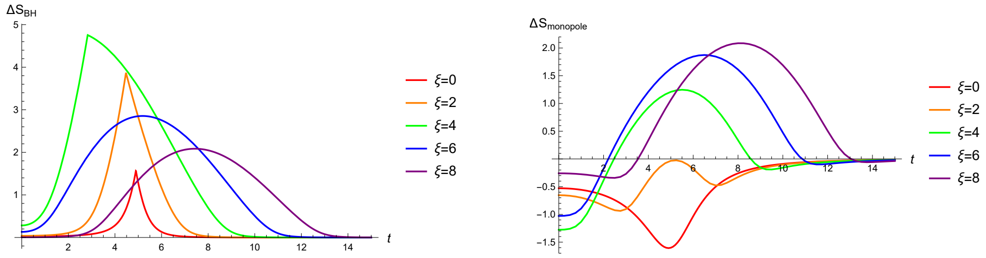

Figure 2: Entanglement entropy variation for a disk-shaped subsystem centered at $(\rho, \varphi) = (\xi, 0)$ for a CFT quench dual to a falling BH (left) and monopole (right). Plots for subsystems with radius $l = 5$ and different ξ are shown. The parameters have been set to $A = L = G = M = \alpha_H = 1$.

entanglement entropy between the CFT excited state and vacuum for small perturbations: $\Delta S = \frac{\Delta \mathcal{A}}{4G}$. In Fig. 2 we show the time evolution of ΔS for disk-shaped CFT subsystems with fixed radius and different centers. The BH result is consistent with a model of entanglement spread based on the free propagation of Einstein-Podolski-Rosen (EPR) pairs of entangled quasiparticles generated in the excited region at $t = 0$. In this pattern, when only a particle of an EPR pair is inside the subsystem, entanglement with the exterior increases compared to vacuum. Additionally, in the monopole quench the expanding condensate O^a can cause a decreasing of entanglement due to the condensation of degrees of freedom. A translation of the subsystem with respect to the quench center $\rho = 0$ emphasises the contribution of EPR pairs, enabling a positive net variation of entanglement in the monopole quench and resulting in a broadening of the ΔS peak in the BH one.

4. Conclusions

We have considered models of a falling monopole and BH in Poincaré AdS_4, which are dual to local quenches in the boundary CFT_3. The energy-momentum tensor has the same functional form in both cases, whereas entanglement entropy behaves in a radically different way. This is crucial for the validity of the first law of entanglement entropy (FLEE), relating the variations of entanglement entropy ΔS and energy ΔE between neighbouring states. While the FLEE holds in a BH quench, where for subsystems with radius $l << A$ we have $\Delta S = \frac{\pi l}{2} \Delta E$ [3], the law is violated in the monopole case. This can be understood from the fact that the monopole and vacuum belong to different topological sectors, so the dual states are not continuously connected to each other. In light of this, it would be interesting to test the validity of FLEE for other non-equilibrium systems.

References

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