PROCEEDINGS OF SCIENCE

Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab

PoS(EPS-HEP2021)861

E. Bottalico[a,b] **and G. Venanzoni**[b,†]

[a] *University of Pisa, INFN Sez. Pisa, Largo Bruno Pontecorvo 3, Pisa, Italy*
[b] *INFN Sez. Pisa, Largo Bruno Pontecorvo 3, Pisa, Italy*
†*For the Muon g-2 Collaboration*

E-mail: elia.bottalico@pi.infn.it, Graziano.Venanzoni@pi.infn.it

The Muon g-2 Experiment at Fermilab (E989) aims to measure the muon magnetic anomaly (a_μ) with a final accuracy of 140 parts per billion (ppb). The first result on the Run-1 dataset was unveiled on April 7, 2021, showing a very good agreement with the previous result from the Brookhaven National Laboratory (BNL) experiment, with a slightly better uncertainty. The corresponding experimental average increases the significance of the discrepancy between the measured and Standard Model predicted a_μ to 4.2 σ. Four different beam dynamics corrections must be applied to obtain the final value of the anomalous precession frequency. In the following contribution, this high precision measurement is presented, focusing on the beam dynamics corrections to ω_a.

**** The European Physical Society Conference on High Energy Physics (EPS-HEP2021), ****
**** 26-30 July 2021 ****
**** Online conference, jointly organized by Universität Hamburg and the research center DESY ****

1. Introduction

The magnetic moment of a lepton with spin s, charge q, mass m and gyromagnetic ratio g is defined as:

$$\vec{\mu}_l = g_l \left(\frac{q}{2m_l} \right) \vec{s}. \tag{1}$$

Dirac predicted $g_e = 2$ [2] for the electron (and, consequently, any spin $\frac{1}{2}$ elementary particle). Schwinger proposed an additional contribution to the electron magnetic moment from a radiative correction, predicting the anomaly $a_e = \frac{g_e - 2}{2} = \alpha/2\pi \simeq 0.00116$ in agreement with experiment [3]. Afterwards, a series of experiments confirmed that the muon behaves as a heavy electron, confirming $g_\mu \approx 2$. Subsequently, the theoretical calculation showed how each sector of the Standard Model contributes, through the vacuum polarization, to the estimate of a_μ. At the Muon g-2 Experiment at Fermilab, polarized muons, with a momentum of 3.1 Gev/c, are injected into a superconducting storage ring of 14 meters in diameter. In the rest frame, the muon's spin rotates with a frequency proportional to the g-factor according to the Larmor precession formula:

$$\vec{\omega}_S = g \frac{e}{2m} \vec{B}. \tag{2}$$

In the same time, the relativistic muons are collected in the storage ring and orbit with a frequency defined by the cyclotron frequency:

$$\vec{\omega}_C = \frac{e\vec{B}}{m\gamma}. \tag{3}$$

By computing the two quantities ω_C and ω_S in the lab frame, we can define a new quantity obtained from their difference that represents the rotation frequency of the muon spin relative to its momentum, called $\vec{\omega}_a$. This frequency (called "anomalous precession frequency") is the "g-2 frequency" and, together with the measurement of the storage ring magnetic field, represent the most important observable of the g-2 experiment. The most general expression of $\vec{\omega}_a$ is:

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = \frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) (\vec{\beta} \times \vec{E}) - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right]. \tag{4}$$

We can simplify this expression and cancel the effect of the E-field by choosing a specific value for the Lorentz boost. Computing γ from Equation 4, we find that $\gamma = \sqrt{1 + \frac{1}{a_\mu}} \approx 29.3$ that corresponds to a momentum $p_\mu = 3.094$ GeV/c called "*magic* momentum". The experiments from CERN III ([5]) to E989 use the magic momentum to reduce at minimum the influence of the E-field on the muon beam in the storage ring. In this configuration, the relationship between a_μ and B is reduced to:

$$\vec{\omega}_a = a_\mu \frac{e\vec{B}}{m}, \tag{5}$$

from which we need to know the ratio between $\vec{\omega}_a$ and the B-field to obtain a_μ. In E989, the muon anomalous magnetic moment is computed as:

$$a_\mu = \frac{g_e}{2} \frac{\omega_a}{\tilde{\omega}_p{}'} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e}, \tag{6}$$

where $\frac{g_e}{2} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e}$ are external quantities measured with very high precision. $R = \frac{\omega_a}{\tilde{\omega}_p{}'}$ is the quantity measured by the Muon g-2 Collaboration, where $\tilde{\omega}_p{}'$ represents the equivalent precession frequency

PoS(EPS-HEP2021)861

of a proton shielded in a spherical sample of water.

1.1 ω_a measurement

In the Muon g-2 experiment at Fermilab, polarized muons are produced by pions decay. Due to the parity violation in the weak muon decay, high energy positrons produced are emitted preferably along the muon's spin direction. The emitted positrons are detected by 24 electromagnetic calorimeters, which measure the energy and the arrival time of the positrons, each made up of 54 (1296 in total) crystals of lead fluoride (PbF_2) read by silicon photomultipliers (SiPM). By counting the number of positrons over a certain energy range over the time, the precession frequency of the muon spin is measured; together with the measurement of the magnetic field, this allows us to extract a_μ. The simplest description of the positron time modulation is:

$$N(t) = N_0 \cdot e^{-t/\tau} \cdot (1 + Acos(\omega_a \cdot t + \varphi_a)). \tag{7}$$

Due to the beam's motions around the ring, a 22 parameters fit function is needed to account for vertical and radial oscillations that affect the determination of a_μ. Such non-fitted frequencies can be seen by the Fast Fourier Trasform (FFT) of the fit function residuals as shown in Fig. 1, where the FFTs of the 5 and 22 parameters fit functions are shown.

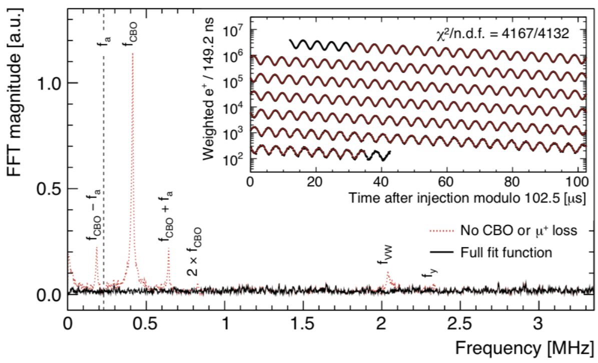

Figure 1: The red dashed curve shows the FFTs for the 5-par fit function, in which peaks of the main frequencies are shown ($\omega_{CBO}, \omega_{VW}, \omega_y$). The black solid line shows the FFT for 22 parameters fit function [1].

2. ω_a beam dynamics corrections

Due to the extremely high precision of the experiment, four additional beam dynamics corrections must be applied at the central values extracted by the fit to obtain the final value of the anomalous precession frequency. Two corrections are associated with the use of electrostatic quadrupole (ESQ) vertical focusing on the storage ring, C_E and C_p, respectively *E-field* and *Pitch* corrections. Two other corrections are caused by a time-dependent phase shift introduced by muons that escape the ring during the storage time (*Loss Muon* correction, C_{lm}) and a vertical/horizontal stored muon beam drift that occurs during the fill (*Phase acceptance* correction, C_{pa}). Thanks to a tracker system consisting of two tracker stations, the beam profile is measured at multiple locations.

2.1 E-field correction: C_e

The E-field correction C_e comes from the second term in Eq. 4. It depends on the distribution of equilibrium radii $x_e = x - R_0$, which translates to the muon beam momentum distribution via $\Delta p/p_0 \simeq x_e(1-n)/R_0$, where n is the field index determined by the ESQ voltage. By performing a Fast Fourier analysis of the incoming bunched beam, see Fig. 2, we determine the momentum distribution, the mean equilibrium radius $< x_e >\approx$ 6mm, and the width $\sigma_{x_e} \approx 9$ mm. The final correction to a_μ becomes $C_e = 2n(1-n)\beta^2 < x_e^2 > /R_0^2$, where $< x_e^2 >= \sigma_{x_e}^2 + < x_e >^2$ and the value of the correction is $C_e = (489 \pm 53)$ ppb.

2.2 Pitch correction: C_p

Since the muons inside the ring have a non-zero vertical momentum component, a pitch correction is required to account for it. In particular, the vertical betatron oscillations lead to a non-zero average value of the $\vec{\beta} \cdot \vec{B}$ term in Eq. 4. In Fig. 2, a visual explanation of the effect. The expression $C_p = n < A_y^2 > /4R_0^2$ determines the pitch correction to a_μ. The vertical amplitude A_y distribution is measured by trackers. The correction applied to ω_a is $C_p = (180 \pm 13)$ ppb.

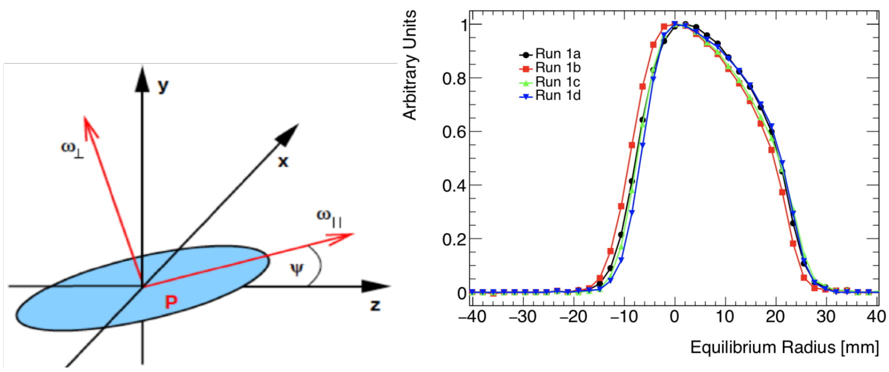

Figure 2: *Left:* Visual explanation of the pitch correction that arises from the vertical motion of the beam. There is a slight shift between the planes defined by the spin precession and cyclotron motions. *Right:* The radial distribution for the four Run-1 datasets as determined by the Fourier method. [4]

2.3 Loss muon correction: C_{lm}

Any variation as a function of time that affects the muon initial phase, φ_a in Eq. 7, biases the ω_a value. Beamline simulations predict a phase-momentum correlation $d\varphi_a/dp = (-10.0 \pm 1.6)$ mrad/$\Delta p/p_0$ and losses are known to be momentum-dependent, as shown in Fig. 3.
We verified this relationship by fitting precession data from short runs. During these runs, the storage ring magnetic field, and thus the central stored momentum p_0, varied by ± 0.67‰ compared to its nominal setting. Next, we investigated the relative rates of muon loss versus momentum in special runs, in which muon distributions were heavily biased toward high or low momenta using

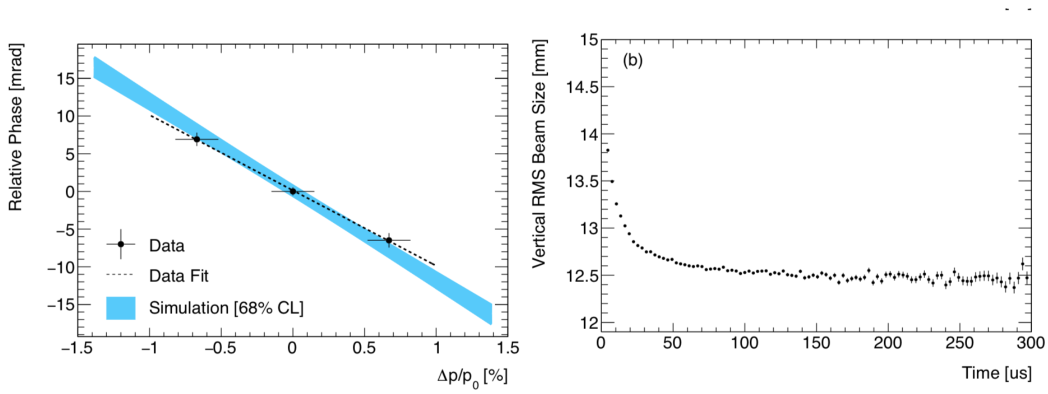

Figure 3: *Left:* Phase-momentum correlation from an end-to-end simulation (blue band) and from a data-driven approach (black). *Right:* Muon beam vertical width variation as a function of time. [4]

upstream collimators. Coupling the measured rate of lost muons in Run-1 to these two correlation factors determines the correction C_{ml} to a_μ. The final correction applied is: $C_{lm} = (-11 \pm 5)$ ppb.

2.4 Phase-acceptance correction: C_{pa}

The phase for a given (x, y) decay coordinate depends on the orientation of the muon's spin that maximizes the acceptance. Its orientation into the detector system is not parallel to its momentum but rotated slightly radially inward. This rotation causes an effective phase shift φ_{pa}, which is a function of transverse decay coordinates due to acceptance effects. It is mainly caused by the vertical width variation, shown in Fig. 3, over the muon fill, and the phase-acceptance effect was exaggerated in Run-1 by damaged ESQ resistors that induced slow changes in the muon distribution over the first 100 μs. The net effect on ω_a is computed via toy MC simulation, obtaining a final correction of $C_{pa} = (-158 \pm 75)$ ppb.

3. Acknowledgement

This work was supported by the EU Horizon 2020 Research and Innovation Program under the Marie Sklodowska-Curie Grant Agreement No.734303.

References

[1] B. Abi *et al.* "*Measurement of the positive muon anomalous magnetic moment to 0.46 ppm*", PRL 126, 141801 (2021);

[2] P. A. M. Dirac, "*The quantum theory of electron. Part II*", Proc. R. Soc. A 118, 351 (1928).

[3] J. S. Schwinger, "*On quantum electrodynamics and the magnetic moment of the electron*", Phys. Rev. 73, 416 (1948).

[4] T. Albahri *et al.* "*Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab*", Phys. Rev. Accel. Beams 24, 044002

[5] J. Bailey et al. "*New Measurement of (G-2) of the Muon*", Phys.Lett. 55B (1975) 420-424;