PROCEEDINGS OF SCIENCE

Baselines for higher-order cumulants of net-charge distributions from local charge conservation

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In studies of heavy-ion collisions, fluctuations of conserved quantities are considered as an important signal of the transition between the hadronic and partonic phases of nuclear matter. In this article, it is investigated how the local charge conservation affects higher-order cumulants of net-charge distributions at LHC energies. Simple expressions for the cumulants are derived under the assumption that particle-antiparticle pairs are produced in local processes from sources that are nearly uncorrelated in rapidity. For calculations with these expressions, one needs to know only the second cumulant of net-charge distribution and low-order cumulants of particle number distribution, which are directly measurable experimentally. It is argued that if one wishes to relate susceptibilities with cumulants of net-proton distributions, the developed model provides a better baseline than the conventional Skellam limit or baselines based on Monte Carlo simulations.

The International conference on Critical Point and Onset of Deconfinement - CPOD2021
15 – 19 March 2021
Online

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PoS(CPOD2021)010

1. Introduction

Heavy-ion collisions at relativistic energies allow investigating properties of nuclear matter at extreme conditions. One of the key theoretical predictions confirmed by LQCD calculations is that at high energy densities, reached at RHIC and LHC, nuclear matter transforms into a deconfined state of quarks and gluons known as Quark-Gluon Plasma (QGP). As a possible signature of the transition between the hadronic and partonic phases, it is theoretically shown that higher-order fluctuations of conserved quantities, such as net-charge, net-baryon, net-strangeness, should greatly enhance near the critical point [1]. At LHC energies, for non-zero quark masses a smooth crossover between a hadron gas and the QGP is expected [2].

Higher-orders cumulants of distributions of conserved quantities are of great interest to be precisely measured because of their direct connection to theoretically calculated susceptibilities, for example, in the lattice QCD. Cumulants and their ratios are extensively studied experimentally, in particular, the STAR collaboration reported the energy dependence of cumulants up to the sixth order [3, 4]. At LHC energies, net-proton cumulants of the second order were studied by ALICE [5], there are also preliminary results on the third and the fourth order [6, 7]. Net-proton and net-kaon fluctuations are usually considered as a proxy for the net-baryon and net-strangeness, respectively.

Comparison of the theoretically calculated susceptibilities with the experimentally measured cumulants is tricky since the cumulants are sensitive to various physical effects. For example, starting already from the second-order, they are sensitive to fluctuations in a number of particle emitting sources – the so-called "volume fluctuations" (VF) [8]. Net-charge cumulants are also significantly affected by charge conservation laws [8, 9, 10]. These two effects make interpretation of the experimental measurements very non-trivial, especially for cumulants of higher orders. Thus, one needs some solid baselines for experimentally measured values of the higher-order cumulants. Such baselines are always developed under certain assumptions about the system. The most typical example is when one assumes that distributions of particles and anti-particles are independent and Poissonian, then the net-proton multiplicity follows the Skellam distribution, with simple expression for cumulants. This assumption violated in any realistic system with the VF and charge conservation, making the Skellam baseline rather artificial. As an another extreme, calculations in event generators could be considered as baselines as well [11], however, they are obviously very model-dependent.

One may try to construct a baseline by mediating between experiment and theory. For example, it is suggested to estimate influence from the VF on the higher-order cumulants by simulating the centrality selection criteria,used in real experiments, within the Wounded Nucleon Model, with Poissonian particle production from each source [8]. This model implies that particles are produced from independent wounded nucleons that makes this approach quite model-dependent. In [12], authors consider cumulants of a conserved charge measured in a subvolume of a thermal system, with global charge conservation taken into account, which is opposed to the binomial sampling from the full volume of a system. However, the system volume is considered as fixed, which blocks a direct comparison with the experiment. Moreover, none of the models mentioned above takes into account the shape of the balance function which contains information about angular separation of opposite charges produced in some local processes.

In collisions of hadrons at LHC energies, initial baryonic and electric charges in the final state typically fly outside the mid-rapidity acceptance ($|y| \lesssim 1$), and all opposite-charge pairs at mid-rapidity are newly produced. In this article, it is investigated how the local production of particle-antiparticle pairs (e.g. from resonance decays or fragmentation of quark-gluon strings) affects the higher-order cumulants of net-charge distributions at LHC energies. The baselines for cumulants are derived under the assumption that the pairs are loosely correlated in rapidity. Derived expressions contain quantities that are easily measurable in an experiment. It is argued that such baselines for net-proton fluctuations are more meaningful than the conventional Skellam limit, and deviations from them should be studied if one wishes to relate cumulants of net-charge distribution to corresponding higher-order susceptibilities.

2. Cumulants for composition of sources

Suppose that a system produced in each event consists of sources that emit particles independently, a number of sources N_S fluctuates event-by-event, and each source is characterized by an (extensive) quantity x, such that the total sum from all the sources in each event is $X = \sum_{i=1}^{N_S} x_i$. In this case, cumulants κ_r of order r of X-distribution could be expressed through a combination of cumulants k_q ($q = 1,...,r$) of the x-distribution of a single source and cumulants[1] K_p ($p = 1,...,r$) of the distribution of the number of sources N_S [8]. In the context of net-charge fluctuations, $X \equiv \Delta N$, where net-charge $\Delta N = N^+ - N^-$ is the difference between numbers of particles of opposite charges measured within the rapidity acceptance Y in a given event. For a single source, $x \equiv \Delta n$ with $\Delta n = n^+ - n^-$, where n^+ and n^- are multiplicities from a source within Y.

At LHC energies $\langle \Delta n \rangle = 0$, and the second and the fourth cumulants of ΔN decomposes as [8]

$$\kappa_2(\Delta N) = \langle (\Delta N)^2 \rangle - \langle \Delta N \rangle^2 = k_2(\Delta n)\langle N_S \rangle + \langle \Delta n \rangle^2 K_2(N_S) = k_2(\Delta n)\langle N_S \rangle, \tag{2.1}$$

$$\kappa_4(\Delta N) = k_4(\Delta n)\langle N_S \rangle + 3k_2^2(\Delta n)K_2(N_S). \tag{2.2}$$

One can calculate also the sixth cumulant:

$$\kappa_6(\Delta N) = k_6\langle N_S \rangle + \left(10k_3^2 + 15k_2k_4\right) K_2(N_S) + 15k_2^3 K_3(N_S), \tag{2.3}$$

where the (Δn) argument for the k_q terms is omitted for clarity. Corresponding ratios of $\kappa_4(\Delta N)$ and $\kappa_6(\Delta N)$ to the second cumulant read as

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = \frac{k_4}{k_2} + 3k_2 \frac{K_2(N_S)}{\langle N_S \rangle}, \tag{2.4}$$

$$\frac{\kappa_6}{\kappa_2}(\Delta N) = \frac{k_6}{k_2} + \left(10\frac{k_3^2}{k_2} + 15k_4\right) \frac{K_2(N_S)}{\langle N_S \rangle} + 15k_2^2 \frac{K_3(N_S)}{\langle N_S \rangle}. \tag{2.5}$$

Note, that in this case the volume fluctuations do not cancel – they contribute via the scaled variance $K_2(N_S)/\langle N_S \rangle$ in (2.4) and (2.5), and additionally via $K_3(N_S)/\langle N_S \rangle$ ratio in (2.5). If net-charge distribution for each source follows Skellam, and if there are no volume fluctuations ($K_2(N_S) = K_3(N_S) = 0$), the ratios (2.4) and (2.5) become unity.

[1]Different notations for cumulants (κ, k and K) serve only for better visual distinction which distribution they are referred to. The first cumulant κ_1 is just the mean value of X, the second and third cumulants coincide with the 2nd and 3rd central moments, in particular, κ_2 is the variance of X. For higher orders, relations between cumulants and moments are more complicated.

3. Model with particle-antiparticle sources

Formulae from the previous section are valid for any type of sources. For example, it is typical to treat sources as "wounded nucleons", what is done, for instance, in [8]. In the current paper, we use the developed formalism to study effects of local charge conservation. Namely, we may consider a system, where each source is positioned at some rapidity and emits exactly one *particle-antiparticle pair*. There could be a mixture of sources of different nature (for instance, resonances of several types) – in this case it is enough to consider a "weighted averaged" source of the system, which is characterized by the balance function [13]. Assume also that rapidities of different sources are uncorrelated, and that particles produced from one source do not interact with particles from other sources. For a particle-antiparticle source, all cumulants k_q of orders $q > 2$ can be expressed via the second-order cumulant $k_2(\Delta n)$, in particluar,

$$k_4(\Delta n) = k_2 - 3k_2^2 \;\text{ and }\; k_6(\Delta n) = k_2\big(1 - 15k_2 + 30k_2^2\big). \tag{3.1}$$

Substituting k_4 and k_6 into (2.5), we get corresponding cumulant ratios for the full system:

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3k_2\left(\frac{K_2(N_S)}{\langle N_S\rangle} - 1\right), \tag{3.2}$$

$$\frac{\kappa_6}{\kappa_2}(\Delta N) = 1 - 15k_2 + 30k_2^2 + 15k_2(1-3k_2)\frac{K_2(N_S)}{\langle N_S\rangle} + 15k_2^2\frac{K_3(N_S)}{\langle N_S\rangle}. \tag{3.3}$$

In both relations (3.2) and (3.3), information about the decaying sources is now contained only in $k_2(\Delta n)$. To simplify these expressions, it is convenient to re-express the cumulants of the number of sources $K_2(N_S)$ and $K_3(N_S)$ in terms of *scaled factorial moments* (minus unity), namely,

$$R_2(N_S) = \frac{\langle N_S(N_S-1)\rangle}{\langle N_S\rangle^2} - 1, \qquad R_3(N_S) = \frac{\langle N_S(N_S-1)(N_S-2)\rangle}{\langle N_S\rangle^3} - 1. \tag{3.4}$$

The quantities R_r are "robust" in the following sense: if rapidities of the sources are independently sampled from some distribution (as it is assumed), while we observe sources only in a restricted acceptance window Y (so that we see on average only a fraction of all the sources), then R_r do not depend on Y. This means that it is irrelevant in which acceptance we calculate $R_2(N_S)$ and $R_3(N_S)$. Recall now that, in our interpretation, each source produces an oppositely charged particle pair. In this case, we can use cumulants of number distribution of one of its daughter particles as a *proxy* for cumulants of N_S: $K_r(N_S) \to K_r(N^-)$, and, correspondingly, $R_r(N_S) \to R_r(N^-)$, where N^- is a number of negative particles measured within the Y acceptance[2]. Under this assumption, the cumulant ratios (3.2) and (3.3) can be rewritten as

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N)R_2(N^-), \tag{3.5}$$

$$\frac{\kappa_6}{\kappa_2}(\Delta N) = 1 + 15\kappa_2(\Delta N)\Big[\big(1 - 3\kappa_2(\Delta N)\big)R_2(N^-) + \kappa_2(\Delta N)R_3(N^-)\Big]. \tag{3.6}$$

Thus, with assumptions and approximations done above, in order to calculate the fourth-to-second order cumulant ratio it is enough to measure within the Y acceptance the second cumulant $\kappa_2(\Delta N)$

[2] Equally, we can take $K_r(N^+)$ instead, since $K_r(N^+) = K_r(N^-)$ in mid-rapidity region at the LHC energies.

and the second-order robust quantity $R_2(N^-)$, while for the six-to-second order ratio $R_3(N^-)$ is needed in addition. All these quantities are directly measurable experimentally[3]. Model cumulant ratios (3.5) and (3.6) can be considered as baselines for experimental values (instead of, for instance, the Skellam baseline). Possible signals from critical phenomena should lead to multi-particle correlations, which should be indicated by deviations from these baselines. Applicability of this model in realistic situations is discussed in the next section.

4. Discussion on model assumptions

Creation of oppositely charged particle pairs is governed by local charge conservation. The simplest case of a pair production process is a two-body neutral resonance decay, where integer $+1$ and -1 charges are produced, and net-charge contribution to cumulants from a resonance is determined solely by its decay kinematics and resonance spectra. Another process is string fragmentation that produces fractional charges at each breaking point (quarks, diquarks), which then combine with partons from next breaking points. This may lead to a correlation between hadrons coming from several adjacent parts of a string (i.e. multi-particle correlations), and influence net-charge fluctuations in a complicated way. Yet another type of multi-particle sources are jets. Sources that produce more than two correlated charged particles violate the assumptions of the model discussed in the previous section.

Consider, however, protons and antiprotons, which are relevant for the analysis of net-proton fluctuations. There are no resonances that decay into p–$\overline{p}$ pair. In models like PYTHIA, such pairs may be produced in string breaking into a diquark-antidiquark pair (directly or via a decay of a short-lived resonance), however, a probability of production of two or more baryon pairs from adjacent parts of the same string is low. Multi-particle contribution from jets should be very low as well, since it is improbable to have more than two (anti)protons from a jet within the soft range of $p_{\rm T}$ considered here. Therefore, if there are no processes other than resonance decays and string fragmentation, the p-$\overline{p}$ pairs visible in an event may be considered as nearly uncorrelated.

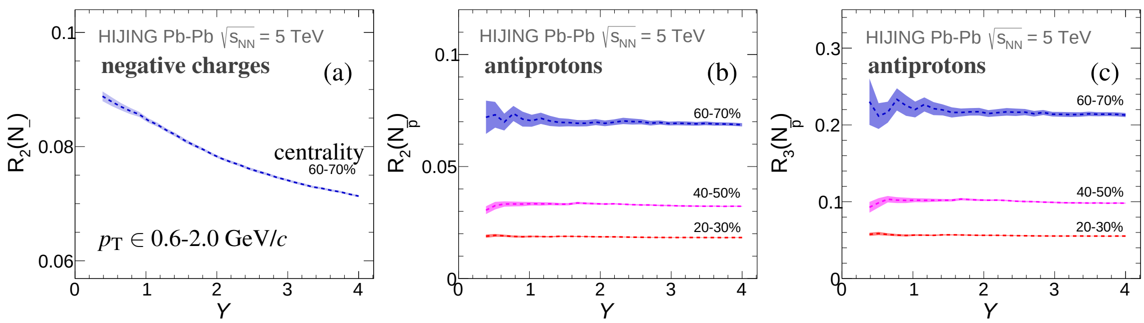

Figure 1: (a) – dependence on rapidity acceptance width Y of the robust quantity R_2 for fluctuations of negative particle number (Pb-Pb collisions at $\sqrt{s_{\rm NN}} = 5$ TeV from HIJING), (b) – R_2 of number of antiprotons, (c) – R_3 of number of antiprotons. Several centrality classes are shown, $p_{\rm T}$ range is 0.6–2.0 GeV/c. Note that point-by-point statistical uncertainties are correlated.

[3]Quatnities $R_r(N^-)$ are robust also to detection efficiency losses (provided that the efficiency is nearly flat within the acceptance), so the only quantity that should be corrected for efficiency is $\kappa_2(\Delta N)$.

Recall now that in the absence of rapidity correlations between sources the robust quantities R_r are expected to be independent on the acceptance Y where they are measured. To test this, Pb-Pb collisions simulated in HIJING event generator at $\sqrt{s_{NN}} = 5$ TeV were used. Centrality classes were selected using a sum of particle multiplicities in symmetric $3 < |\eta| < 5$ ranges, which emulates the way how the centrality is determined in real experiments. Particles were selected with cuts $|\eta| < 2$ and $p_{\mathrm{T}}\in$0.6–2.0 GeV/c[4]. Figure 1 (a) shows scaled factorial moment R_2 of the number of *negative* particles as a function of the acceptance width Y. A clear dependences on Y can be seen, manifesting significant correlations between rapidities of negative particles. Dependence of R_3 (not shown in the plot) is also non-flat (changes from ~ 0.28 at small Y to ~ 0.22 at $Y = 4$). On the contrary, R_2 and R_3 for fluctuations of the number of *antiprotons* in HIJING shown in panels (b, c) are independent of Y, indicating that rapidities of antiprotons (number of which is taken as a proxy for a number of proton-antiproton pairs) are nearly uncorrelated.

5. Comparison with direct calculations in HIJING

In Figure 2 (a), centrality dependences of the κ_4/κ_2 ratios in a wide pseudorapidity acceptance $Y = 4$ are shown for HIING Pb–Pb events in classes of 10% and 5% widths. Markers represent direct calculations of the ratios, while lines stand for the calculations in particle-antiparticle souce model with (3.5). Values are well compatible, at least in peripheral and mid-central events, where statistical uncertainties are small enough to conclude. Note that ratios in 5% centrality classes are lower due to reduced volume fluctuations. Results for κ_6/κ_2 are similar, but uncertainties are higher.

In order to suppress the impact from VF, the so-called centrality bin width correction technique (CBWC) is often used in analysis of real data [14]. It was shown in [8] that this procedure nevertheless does not completely remove effect from VF in the model with wounded nucleons. Indeed,

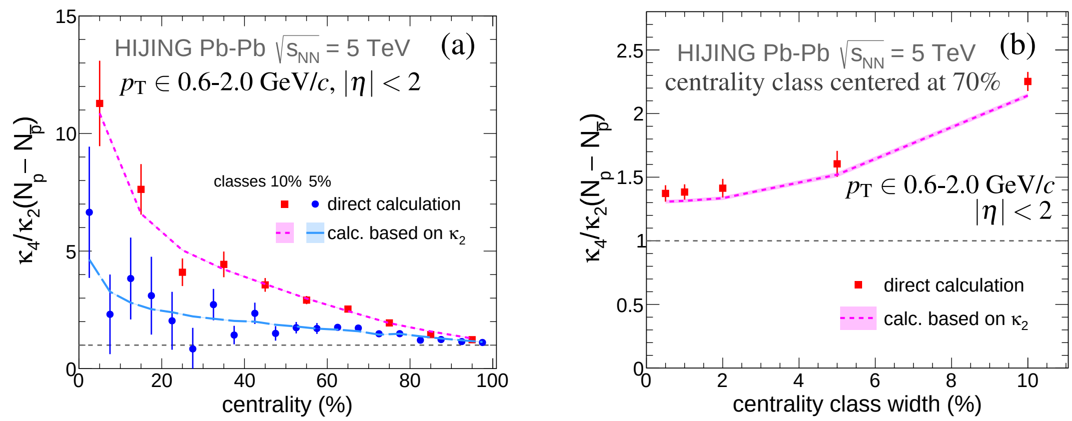

Figure 2: (a) – centrality dependence of the net-proton κ_4/κ_2 ratio in HIJING in Pb-Pb events. Direct calculations are shown by markers, analytical calculations with (3.5) – by dashed lines. Centrality class widths 10% and 5%, kinematic cuts $|\eta| < 2$ and $p_{\mathrm{T}} \in (0.6, 2.0)$ GeV/c. (b) – dependence of the net-proton κ_4/κ_2 ratio on the centrality bin width in Pb-Pb collisions in HIJING. Values for each point are averaged over several bins according to the CBWC.

[4] Results are very similar if one imposes cuts on rapidity y instead of η. p_{T} range 0.6–2.0 GeV/c is similar to what is applied in STAR and ALICE analysis of net-proton fluctuations.

the CBWC is essentially a procedure of averaging of values from several narrow bins. Figure 2 (b) shows dependence of the net-proton κ_4/κ_2 on the centrality bin width in HIJING, where, following the CBWC prescription, a 65-75% centrality interval was split into 1, 2, 5, 10 and 20 sub-intervals, and κ_4/κ_2 ratios where averaged for each splitting. It can be seen that for narrow classes the ratios "converge" to a value around 1.4. The line corresponds to the calculation with (3.5) in the model with two-particle sources, and it gives the same result. This implies that the non-unity model value is due to remaining fluctuations in a number of (anti)protons and the $\kappa_2(\Delta N)$. This demonstrates that interplay of local charge conservation and VF can produce non-trivial values of the cumulant ratios without any criticality in a system. In this respect, analysis of higher-order *factorial* cumulants may be more reliable, which give zero values for net-proton distribution in HIJING [15].

6. Summary

In this article, it was studied how the local charge conservation coupled with the volume fluctuations affects higher-order cumulants of net-charge distributions in A–A collisions. Simple expressions for cumulants ratios up the the sixth order were derived under the assumption that particle-antiparticle pairs are produced from sources that are nearly uncorrelated in rapidity. For calculations in this model, it is enough to measure the second moment of a net-charge distribution and lower-order cumulants of number of positive (or negative) particles within the experimental acceptance. It is argued and confirmed with the HIJING model that the derived expressions are especially relevant for the analysis of net-proton cumulants at LHC energies, since in the absence of critical behaviour in the system there are no significant multi-particle rapidity correlations between protons (antiprotons). If one wishes to study susceptibilities with net-proton cumulants at the LHC, the expressions derived in this paper provide a more natural data-driven baseline for cumulant ratios than the Skellam limit or baselines based on Monte Carlo simulations.

Acknowledgements

This work is supported by the Russian Science Foundation, grant 17-72-20045.

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