

How an ω_0 condensate can spike the speed of sound in cold quarkyonic matter

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I describe a novel mechanism where the variation of an ω_0 condensate can generate a “spike” in the speed of sound in hadronic matter. An ω_0 condensate naturally increases the speed of sound; the real problem is how to get the speed of sound to decrease. I suggest this can happen through the appearance of a Quantum Pion Liquid.

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Consider a $SU(N_c)$ non-Abelian gauge theory, coupled to N_f flavors of quarks in the fundamental representation, at nonzero temperature and zero quark chemical potential. As the temperature is raised, theoretically there are two phase transitions possible. One is deconfinement, and for light quarks, a second for the restoration of chiral symmetry.

The details of these transitions depend upon the number colors and flavors. One particularly simple example is the case of an infinite number of colors, where the number of flavors is held fixed as $N_c \rightarrow \infty$. Then everything is dominated by the pure glue theory, and we expect (and numerical simulations on the lattice confirm) that the deconfining transition, at a temperature T_d , is of first order. This is easy to understand. In the confined phase all hadrons are color singlets, and so any hadron has a degeneracy of order one. Thus the pressure in the confined phase, below T_d , is of order one. In the deconfined phase, above T_d we can certainly have a complicated, strongly interacting phase until very high temperature. Even so, no matter how strongly the deconfined quarks and gluons interact, there are $\sim N_c N_f$ quarks and $\sim N_c^2$ gluons, with a pressure which is $\sim N_c^2$. That is, in the limit of infinite N_c the pressure itself can be used as an order parameter.

If there are also massless quarks, we can also characterize the restoration of chiral symmetry at a temperature T_χ . If the number of flavors is held finite as $N_c \rightarrow \infty$, though, the quarks are really driven by the dynamics in the pure glue theory. Thus it is very hard to imagine that $T_\chi < T_d$: why should the quark dynamics change at all?

Indeed, general arguments suggest that the confined phase is completely independent of temperature. At first sight this sounds surprising, but again it isn't. Not only are the number of hadrons of order one, but any interactions between them are suppressed by powers of $1/N_c$. Thus in the strict limit of infinite N_c , there is no way that any quantity can change with temperature. This is why a first order transition is expected: the confined phase has a pressure of one, and the deconfined phase has a pressure which is negative below T_d and positive above. Thus the derivative of the pressure is nonzero at T_d , which implies that the energy density jumps from ~ 1 below T_d to $\sim N_c^2$ above. If one works hard, one can use the Hagedorn spectrum to get a second order deconfining transition at infinite N_c [1], but as I said, the numerical evidence strongly disfavors this.

Similarly, it is possible that T_χ is greater than T_d , but again, it would be unexpected. Instead, the safest best is simply that when deconfinement occurs, chiral restoration also occurs, $T_\chi = T_d$.

Let me make another, apparently trivial, comment. At zero quark chemical potential, in the confined phase there is no condensate which affects deconfinement. The order parameter for deconfinement is zero below T_d , and is only nonzero above T_d . Of course below T_d the chiral condensate, $\langle \bar{\psi}\psi \rangle$, is nonzero. Still, any loop which contributes to a change in the chiral condensate with temperature involves a coupling to the hadronic state, and so the change will be suppressed by some power of $1/N_c$.

All of these clean results are special to holding N_f fixed as $N_c \rightarrow \infty$. If N_f is as large as N_c , then most of the above results go away. In particular, since there are $\sim N_f^2$ hadronic states, then we don't even know the order of the transition: it could be first order, but it could be second, or crossover. The pressure is then $\sim N_f^2 \sim N_c^2$ at all temperatures. Similarly, if neither N_f nor N_c is large, then we need the results of numerical simulations on the lattice.

Let us then return to the case of $N_f \ll N_c \rightarrow \infty$, and consider nonzero quark chemical potential, μ_{qk} . For the pressure, the quark contribution is no larger than $\sim N_f N_c \mu_{qk}^4$, so at nonzero

temperature, the quark contribution is only commensurate with that from gluons, $\sim N_c^2 T^4$, when $\mu_{qk} \sim N_c^{1/4} T$. This is the basic idea behind a quarkyonic regime at nonzero density, where the free energy is that of (interacting) quarks and gluons, but excitations near the Fermi surface are still confined [2–9].

However, unlike the case of nonzero temperature, the confined phase can depend upon density. This is because while couplings between mesons are suppressed at large N_c , those between mesons and baryons are not: they are large, $\sim N_c^{1/2}$. This suggests that it is possible for the chiral transition to *split* from the deconfining transition at nonzero μ_{qk} , so that as μ_{qk} increases, T_χ is *less* than the deconfining temperature. Of course μ_{qk} only matters when one can first generate a Fermi sea of baryons, which requires $\mu_{qk} > m_B/N_c$, where m_B is the mass of the lightest baryon.

Further, at nonzero density there is uniquely one condensate which plays a privileged role. The importance of this was first noted by Zeldovich [10]; it also provides the basis for the effective theory of nuclear matter, Quantum HadroDynamics [11]. At nonzero density, by definition there is a nonzero expectation value for the timelike component of the current for fermion number, $\langle \bar{\psi} \gamma^0 \psi \rangle \neq 0$. But this current couples directly to that for the ω_0 meson, as that couples to nucleons as $g_\omega \bar{\psi}^\dagger \gamma^\mu \omega_\mu \psi$. Thus if n_B is the baryon density, nonzero density generates a term linear in ω_0 , and induces an expectation value for ω_0 :

$$\mathcal{L}_\omega^B = -g_\omega n_B \omega_0 + \frac{m_\omega^2 \omega_\mu^2}{2} \Rightarrow \langle \omega_0 \rangle = \frac{g_\omega}{m_\omega^2} n_B . \quad (1)$$

Here m_ω is the mass of the ω meson. With $n_B \sim N_c^0$ and $g_\omega N_c^{1/2}$, this contributes to the free energy as $\sim N_c$, as expected for the quark contribution.

The mass of the ω meson can be nontrivial. I take the effective Lagrangian for the ω meson as

$$\mathcal{L}_\omega = \frac{\mathcal{F}_{\mu\nu}^2}{4} + \frac{1}{2} \left(\tilde{m}_\omega^2 + \kappa^2 \vec{\phi}^2 \right) \omega_\mu^2 . \quad (2)$$

$\mathcal{F}_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ is the standard, Abelian field strength for the ω_μ meson. The mass \tilde{m}_ω is a constant, but in addition I also add a quartic coupling, $\sim \kappa^2$, between ω_μ . Here $\vec{\phi}$ is the $O(4)$ chiral field for two light flavors, $\vec{\phi} = (\sigma, \vec{\pi})$. The coupling κ^2 must be positive to ensure stability for large values of the ω_μ and $\vec{\phi}$ fields.

In the vacuum, the σ field acquires a vacuum expectation value from chiral symmetry breaking, $\langle \sigma \rangle = f_\pi$, so that the ω_μ mass is $m_\omega^2 = \tilde{m}_\omega^2 + \kappa f_\pi^2$. In vacuum this is fine, as one can't really distinguish between the part of the ω_μ mass which is bare, and the part induced by chiral symmetry breaking. Similarly, vector meson dominance is used to characterize the coupling of the ρ_μ meson to the photon, but it doesn't constrain how the ρ_μ mass arises.

At the stationary point in ω_0 , the effective Lagrangian of Eq. (1) becomes

$$\mathcal{L}_\omega^B = -\frac{g_\omega}{m_\omega^2} n_B^2 . \quad (3)$$

As demonstrated first by Zeldovich [10], this gives a speed of sound equal to the speed of light. This also arises at nonzero isospin density, in the limit of asymptotically large density.

Of course this is only a leading term, and cannot be taken as exact. Instead, one should use a model of Quantum HadroDynamics (QHD) [11], where saturation arises from a balance between

attraction, due to exchange of a σ -meson, and repulsion, from exchange of the ω_μ meson. This balance tends to weaken the effect of the leading order term in Eq. (3), but still, the speed of sound tends to increase strongly.

This is important for astrophysics. Drischler *et al.* [12] use chiral effective field theory to extrapolate from the saturation density of nuclear matter, n_0 , to twice that. Given the observation of neutron stars with masses above two solar masses [13, 14], it is *imperative* to have a region of density in which the EoS is stiff, with a speed of sound significantly above that of an ideal quark gas, where $c_s^2 = 1/3$. Using the small tidal deformability observed from GW170817, though, the EoS of nuclear matter must be soft until $n_B \sim 1.5 - 1.8n_0$, and then increase sharply. That is, there is a “spike” in the speed of sound, with a relatively narrow peak at a density significantly above n_0 , Fig. (1) of Ref. [15].

As I argued above, an ω_0 condensate naturally gives an *increase* in the speed of sound. The question which I wish to stress is the following: *how can one get the ω_0 condensate to evaporate*, and thus for the speed of sound to *decrease*?

In a quarkyonic phase, which is confined, it is manifestly sensible to speak of an ω_μ meson. This is very different from the case of increasing temperature at zero density: then there is no condensate for the ω_0 , and vector mesons just fall apart into the constituent quarks. But at nonzero density, in the quarkyonic phase the ω_μ meson remains confined, and as a confined meson, can't fall apart into quarks.

This problem does not appear to have been appreciated previously. Either a QHD-type result was used uniformly, or a QHD model was matched onto constituent quarks, as in the model of Cao and Liao [16]. But the ω_μ meson doesn't go away: there must be a *dynamical* reason why the contribution of the ω_0 condensate evaporates at nonzero density.

I suggest a rather speculative argument as to how this could happen. Consider the effective mass of the ω_μ meson, including the κ coupling:

$$\langle \omega_0 \rangle = \frac{g\omega\rho_B}{\tilde{m}_\omega^2 + \kappa^2\langle \phi^2 \rangle} . \quad (4)$$

As the chiral transition is approached, this will tend to increase, as the chiral condensate decreases, $\langle \phi^2 \rangle \approx \langle \sigma \rangle^2$. This stiffens the equation of state.

The simplest way for the ω_0 condensate to become small is for the ω_μ meson to become large. This is where the κ coupling enters. In a linear σ model [17], this can arise as follows. Consider an effective chiral Lagrangian,

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_0\vec{\phi})^2 + \frac{1}{2M^2}(\partial_i^2\vec{\phi})^2 + \frac{Z}{2}(\partial_i\vec{\phi})^2 + \frac{m_0^2}{2}\vec{\phi}^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2 . \quad (5)$$

This includes higher spatial derivatives, but by causality, only two time derivatives. If the coefficient of the term with two spatial derivatives, Z , is negative, in mean field theory spatially inhomogeneous structures (“chiral spirals”) are generated. The simplest chiral spiral is one where although $\vec{\phi}$ winds along a single spatial direction, that $\vec{\phi}^2$ is constant (a “single mode”). This is no problem, *unless* there are Goldstone bosons. In that case, the Goldstone bosons have zero energy at a *non-zero* spatial momentum, k_0 :

$$\Delta_{\text{transverse}}^{-1}(\omega, \vec{k}) = \omega^2 + \frac{1}{M^2}(\vec{k}^2)^2 + Z\vec{k}^2 + m^2 + \lambda\phi_c^2 = \omega^2 + \frac{(k - k_0)^2}{M^2} , \quad (6)$$

In isospin space, the Goldstone bosons are transverse in isospin space to the field for the background chiral spiral, $\vec{\phi}_c$, whose period is $2\pi/|k_0|$, where $k_0^2 \sim -ZM^2$. However, consider the tadpole diagram for fluctuations over the Goldstone modes. For simplicity, I consider the integral at nonzero temperature, taking the static mode in energy, and the integral over the three spatial momenta [17]:

$$\delta m^2 \sim \lambda \langle \vec{\phi}^2 \rangle \sim \lambda T \int d^3k \frac{1}{(k - k_0)^2} \sim \lambda T k_0^2 \int \frac{d\delta k}{(\delta k)^2}. \quad (7)$$

Because the Goldstone boson has zero energy at nonzero momentum, there is a *linear* infrared divergence, about the momentum for the chiral spiral. (This happens in any number of spatial dimensions.) The only way to avoid this divergence is if a *dynamical* mass is generated for the Goldstone bosons, which adds a term m_{dyn}^2 to the propagator. This is a non-perturbative phenomenon, and generates a novel form of the symmetric phase, which in Ref. [7] I termed a “quantum pion liquid” (Q π L). (In Ref. [17] we used the term a pion quantum spin liquid, which isn’t as accurate, since pions don’t carry spin.) In this Q π L, one can show that the fluctuations are *very* large at large Z , $\langle \vec{\phi}^2 \rangle \sim Z^2$. This means that the ω_μ mass squared is $m_\omega^2 \sim Z^2$, so $\langle \omega_0 \rangle \sim 1/Z^2$.

At zero temperature, after integrating over ω the integral over spatial momenta gives a logarithmic instead of a linear infrared divergence [7], so that the dynamical mass is exponentially small. The magnitude of the fluctuations remains the same, $\langle \vec{\phi}^2 \rangle \sim Z^2$, because a much smaller dynamical mass compensates the weaker infrared divergence. As a non-perturbative result, to demonstrate this we considered an $O(N)$ model, and solving in the limit of infinite N . It is not clear if it is valid for $N = 4$, but it is reasonable to suspect that it is.

This leaves a raft of questions open. The analysis of the sigma model assumes that when $Z < 0$, that $\langle \vec{\phi}^2 \rangle$ is constant. Under this assumption, it is easy to show that the energy of the Goldstone bosons vanishes at a nonzero momentum, characteristic of the would be momentum of the spatially inhomogeneous condensate. This appears to be a technical assumption, but certainly one expects that the energy of the Goldstone bosons will vanish at *some* momentum. If Z is positive, then it is certainly at zero momentum. When Z is negative, however, it is natural to expect that the zero of the energy is at nonzero momentum, which naturally generates a Q π L.

There are many other avenues of investigation. Couplings similar to κ , which couple chiral and vector mesons, have been introduced by Refs. [18]. It is imperative to consider these couplings, as a neutron star is not isospin symmetric.

In the end, the most direct conclusion is the following. The standard assumption is that in the plane of temperature and chemical potential, that there is a single transition which is something like a semi-circle. Instead, the example of small N_f and large N_c suggests that cold, dense quark matter may look *nothing* like the deconfined phase at zero density. Understanding the properties of such nuclear matter brings together results from condensed matter to astrophysics.

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