PROCEEDINGS OF SCIENCE

In-medium masses and S-matrix

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I present an S-matrix interpretation of an in-medium modified mass in a simple contact model.

The International conference on Critical Point and Onset of Deconfinement - CPOD2021
15 – 19 March 2021
Online - zoom

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PoS(CPOD2021)037

1. thermal self-energy and S-matrix

Starting with the known expression [1–6] of the thermal self-energy of a particle (A):

$$\Sigma_T(E_A) = \int \frac{d^3 k_B}{(2\pi)^3} \frac{1}{2E_B} n_{\rm th}(E_B)\, T(AB \to AB). \tag{1}$$

where $E_B, n_{\rm th}(E_B)$ are the energy and the thermal distribution of particle species (B) in the medium, and $T(AB \to AB)$ is related to the (forward) scattering amplitude f by [6]

$$T(AB \to AB) = -8\pi \sqrt{s} f. \tag{2}$$

Eq. (1) has a clear kinetic theory interpretation: The thermal shift of the particle A, in a medium of particles B, is connected to a scattering amplitude $T(AB \to AB)$, folded with a corresponding thermal distribution of B. The important point here is that the scattering amplitudes are "vacuum" amplitudes, and if the experimental data are employed, the result becomes model independent, as far as the leading order effect is concerned.

One can consider various improvements of this leading order result, e.g., by including further diagrams and / or dressing $T(AB \to AB)$ with particles in the medium, resulting in a "temperature dependent scattering amplitude". However this usually means implicitly imposing some speculation on the $N > 2$-body scattering amplitudes, which are not necessarily consistent with those inferred from analyzing scattering experiments. The problem is already present in the perturbative theory: some high order diagrams may not be generated by iterating a restricted set of low order ones [7]. Further complications come from coupled channel dynamics. See Refs. [8–10] for some attempts in implementing coupled-channel S-matrix to understanding a thermal system, e.g. the hyperon yields from heavy ion collisions.

In this proceeding, I will focus on a particular simple case of the lowest order result (1): a structureless, momentum and energy independent, and real-valued f. This means that the interaction leads to a mass shift Δm_A for particle A:

$$\begin{aligned}\Delta m_A &= \frac{1}{2E_A} \,\mathrm{Re}\, \Sigma_T(p) \\ &\approx N_{\rm th}^B \times \frac{-4\pi f}{2 m_{\rm red}}.\end{aligned} \tag{3}$$

This is a familiar result of Fermi formula for pseudopotential. In Eq. (3) $N_{\rm th}^B$ is the integrated thermal weight of B, and the last line makes a further non-relativistic approximation in the Center-of-Mass (CoM) frame, with $m_{\rm red}$ the reduced mass.

What would be the change in thermal pressure due to such an interaction? One can consider the difference in pressure of a free gas of A with and without the shift, i.e.

$$\begin{aligned}\Delta P &\approx T \int \frac{d^3 p_A}{(2\pi)^3}\, e^{-\beta(m_A + \frac{p_A^2}{2m_A})}\, (-\beta \Delta m_A) \\ &= -\Delta m_A N_{\rm th}^A \\ &= N_{\rm th}^A N_{\rm th}^B \times \frac{4\pi f}{2 m_{\rm red}}.\end{aligned} \tag{4}$$

Eq. (4) makes it clear that it is a 2nd order virial expansion of the pressure. I now show that the same result can be obtained within an S-matrix formulation of statistical mechanics [11, 12].

2. S-matrix formulation

In the S-matrix approach, an effective spectral function relevant for describing an interacting system is defined as

$$B(E) = 2\frac{\partial}{\partial E}Q(E), \tag{5}$$

where $Q(E)$ is the scattering phase shift. The change in thermal pressure due to interaction is given by [7, 13]

$$\Delta P \approx T \int \frac{d^3P}{(2\pi)^3}\,\frac{dE'}{(2\pi)}\,e^{-\beta(m_{\text{tot}}+\frac{P^2}{2m_{\text{tot}}}+E')}\,B(E'). \tag{6}$$

For a structureless scattering one gets [7]

$$2Q(E) \approx 2\,q(E)\,f \approx -\phi\,T_{\text{NR}} \tag{7}$$

where $q(E) = \sqrt{2m_{\text{red}}E}$ is the relative momentum in the CoM frame, ϕ is the (non-relativistic) phase space

$$\begin{aligned} \phi(E) &= \int \frac{d^3q}{(2\pi)^3}\,2\pi\delta(E - \frac{q^2}{2m_{\text{red}}}) \\ &= \frac{m_{\text{red}}q(E)}{\pi}. \end{aligned} \tag{8}$$

This correctly identifies the non-relativistic T-matrix, T_{NR}, as

$$T_{\text{NR}} \approx -\frac{4\pi f}{2m_{\text{red}}}. \tag{9}$$

The thermal pressure in Eq. (6) reads

$$\begin{aligned} \Delta P &\approx \int \frac{d^3P}{(2\pi)^3}\,\frac{dE'}{(2\pi)}\,e^{-\beta(m_{\text{tot}}+\frac{P^2}{2m_{\text{tot}}}+E')}2Q(E') \\ &= \int \frac{d^3P}{(2\pi)^3}\,\frac{d^3q}{(2\pi)^3}\,e^{-\beta(m_{\text{tot}}+\frac{P^2}{2m_{\text{tot}}}+\frac{q^2}{2m_{\text{red}}})}\,(-T_{\text{NR}}) \\ &\approx N^A_{\text{th}}\,N^B_{\text{th}} \times (-T_{\text{NR}}). \end{aligned} \tag{10}$$

This agrees with Eq. (4). In fact, the change in mass in Eq. (3) is simply given by

$$\Delta m \approx \int \frac{d^3k_B}{(2\pi)^3} n_{\text{th}}(E_B)\,T_{\text{NR}}, \tag{11}$$

which can also be obtained from a non-relativistic reduction of Eq. (1).

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3. going further

The results discussed here can of course be trivially derived from the lowest order perturbation theory. The important point is that Eq. (6) is based on a density expansion and is not controlled by the order in couplings. Also the application is not restricted to non-resonant scattering. The resonant contribution can also be included via the phase shift. In addition, with a suitable generalization of Q in Eq. (5), even a coupled-channel [7, 9, 10] system can be treated by the S-matrix approach.

It is also important to realize how in-medium effects are described by "vacuum" scatterings. In fact, it is not correct to claim that in-medium properties can not be described by vacuum processes. After all, it is the same Hamiltonian that we are interested in studying [14], and the S-matrix formulation of statistical mechanics simply expresses the partition function in terms of S-matrix elements. Conceptually separating the two issues: dynamics and thermo-statistical information, could potentially be useful to understanding in-medium effects from (vacuum) $N > 2$-body processes [7]. For example, Eq. (1) may be applied to obtain an in-medium modified mass of Δ from a quasi-elastic scattering of Δ's and pions. The latter needs to be obtained from a well-constrained coupled-channel model involving the various $\pi\pi N$ channels. This also underlines an important method to scrutinize many existing in-medium models based on the known results of multi-body scatterings. These issues will be explored in the future.

4. acknowledgments

I thank the organizer for the kind invitation to the exciting meeting. I am also grateful to Edward Shuryak, Ismail Zahed, Dmitri Kharzeev, and Derek Teaney for many inspiring discussions. I also benefit greatly from the lectures on phase space from Hans Feldmeier. This work was supported by the Polish National Science Center (NCN) under the Opus grant no. 2018/31/B/ST2/01663.

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