PROCEEDINGS OF SCIENCE

Possible studies on generalized parton distributions and gravitational form factors in neutrino reactions

S. Kumano[a,b,*] **and R. Petti**[c]

[a] *KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, Oho 1-1, Tsukuba, Ibaraki, 305-0801, Japan*

[b] *J-PARC Branch, KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, and Theory Group, Particle and Nuclear Physics Division, J-PARC Center, 203-1, Shirakata, Tokai, Ibaraki, 319-1106, Japan*

[c] *Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina, 29208, USA*

E-mail: shunzo.kumano@kek.jp, Roberto.Petti@cern.ch

Spacelike and timelike generalized parton distributions (GPDs) have been investigated in charged-lepton scattering and electron-positron collisions via deeply virtual Compton scattering and two-photon processes, respectively. Furthermore, we expect that hadron-accelerator-facility measurements will be performed in future. The GPDs will play a crucial role in clarifying the origins of hadron spins and masses in terms of quarks and gluons. It is also possible to probe internal pressure within hadrons for understanding their stability. Gravitational form factors of hadrons used to be considered as a purely academic subject because gravitational interactions are too weak to be measured in microscopic systems. However, due to the development of hadron-tomography field, it became possible to extract the gravitational form factors from the actual GPD measurements without relying on direct gravitational interactions. Neutrino reactions can also be used for GPD studies in future, for example, by using the Long-Baseline Neutrino Facility at Fermilab. The neutrino GPD measurements are valuable especially for finding the flavor dependence of the GPDs in a complementary way to the charged-lepton experiments. We give an overview of the GPDs and discuss possible neutrino GPD measurements using the single-pion production processes $\nu + N \to \ell^- + N' + \pi$ and $\bar{\nu} + N \to \ell^+ + N' + \pi$.

*Speaker

PoS(NuFact2021)092

1. Introduction

Unpolarized structure functions F_2 and F_3 of the nucleon were measured by neutrino deep inelastic inelastic scattering (DIS) from heavy nuclei with appropriate nuclear corrections [1]. These structure functions are expressed by collinear parton distribution functions (PDFs), which indicate longitudinal momentum distributions of partons. In recent years, three-dimensional structure functions have been investigated extensively for clarifying the transverse structure of the nucleon in addition to the longitudinal distributions and for understanding the origin of the nucleon spin including the partonic orbital-angular-momentum (OAM) contribution. The OAM contribution should be determined by generalized parton distributions (GPDs) [2], which have been measured by deeply virtual Compton scattering (DVCS) and meson productions at charged-lepton accelerator facilities in the spacelike region. The DVCS has been investigated by the HERMES and COMPASS collaborations and also at the Thomas Jefferson National Accelerator Facility (JLab). In 2030's, it will be investigated at electron-ion colliders in US and China (EIC, EicC) [3]. There are also possibilities of measuring the GPDs at hadron facilities by using high-energy exclusive reactions such as at the Japan Proton Accelerator Research Complex (J-PARC) [4]. All of these are spacelike GPD studies, whereas it is possible to investigate the timelike GPDs [5], which are also called generalized distribution amplitudes (GDAs), by two-photon processes in e^+e^- annihilation, for example, at the KEK-B factory.

There is another important purpose to investigate the GPDs for understanding hadron masses and their internal pressures in terms of quark and gluon degrees of freedom [5]. The studies on the origin of hadron masses by hadron-mass decomposition are now becoming one of major purposes for building the future EICs. The GPD measurements have been done mainly at charged-lepton accelerator facilities, and there is no GPD measurement in neutrino reactions at this stage. However, we may recollect that the neutrino DIS experiments have been important in determining the unpolarized PDFs, especially on the strange-quark distribution via the opposite-sign dimuon events and valence-quark distributions via the structure function F_3. Considering these past experiences, we expect that future neutrino experiments could provide valuable information on the GPDs in a complementary way to the charged-lepton and hadron-facility measurements [3]. The Long-Baseline Neutrino Facility (LBNF) at Fermilab can supply (anti)neutrino beams in the energy region of 2-15 GeV [6] allowing to measure the GPDs, for example, by the pion-production reaction $\nu_\mu + N \longrightarrow \mu + \pi + N'$ [7–9]. In general, (anti)neutrino Charge Current (CC) interactions are sensitive to the quark flavor, offering a valuable tool to study the flavor dependence of the GPDs together with charged-lepton data, and to investigate the origin of hadron spins and masses. In this article, we discuss such a possibility.

2. Generalized parton distribution functions

The spacelike GPDs of the nucleon are measured, for example, by the deeply virtual Compton scattering (DVCS) at charged-lepton accelerator facilities as shown in Fig. 1. The photon momenta are q and q', and the nucleon momenta are p and p'. We define average momenta ($\bar{P}$, $\bar{q}$) and momentum transfer Δ as $\bar{P} = (p + p')/2$, $\bar{q} = (q + q')/2$, and $\Delta = p' - p = q - q'$. Three variables for expressing the GPDs are the Bjorken variable x, the skewness parameter ξ, and the momentum-transfer squared t are defined by $x = Q^2/(2p \cdot q)$, $\xi = \bar{Q}^2/(2\bar{P} \cdot \bar{q})$, and $t = \Delta^2$. By the lightcone momentum notations, x and ξ are expressed as $x = k^+/P^+$ and $\xi = -\Delta^+/(2P^+)$ with $P = p + p'$. If the kinematical condition $Q^2 \gg |t|$, $\Lambda^2_{\rm QCD}$, where $\Lambda_{\rm QCD}$is the QCD scale parameter, is satisfied, the DVCS process is factorized into the hard part and the soft one by the GPDs H^q and E^q defined in the matrix element

$$\int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle p' \left| \bar{q}(-y/2)\gamma^+ q(y/2) \right| p \rangle_{y^+=\vec{y}_\perp=0} = \frac{1}{2P^+} \bar{u}(p') \left[H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t) \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} \right] u(p). \tag{1}$$

In pion-production and neutrino cross sections, there are other GPDs $\tilde{H}^q$ and $\tilde{E}^q$ associated with the matrix element of the axial-vector current as

$$\int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle p' \left| \bar{q}(-y/2)\gamma^+\gamma_5 q(y/2) \right| p \rangle_{y^+=\vec{y}_\perp=0} = \frac{1}{2P^+} \bar{u}(p') \left[\tilde{H}^q(x,\xi,t)\gamma^+\gamma_5 + \tilde{E}^q(x,\xi,t) \frac{\gamma_5 \Delta^+}{2m_N} \right] u(p). \tag{2}$$

The unique features of the GPDs are that they become the unpolarized and longitudinally-polarized PDFs in the forward limit: $H^q(x,0,0) = q(x)$, $\tilde{H}^q(x,0,0) = \Delta q(x)$, that their first moments are the corresponding form factors: $\int_{-1}^{1} dx H^q(x,\xi,t) = F_1^q(t)$, $\int_{-1}^{1} dx E^q(x,\xi,t) = F_2^q(t)$, $\int_{-1}^{1} dx \tilde{H}^q(x,\xi,t) = g_A^q(t)$, $\int_{-1}^{1} dx \tilde{E}^q(x,\xi,t) = g_P^q(t)$, and that the second moment is the quark contribution to the nucleon spin: $J_q = \int dx\, x\, [H^q(x,\xi,t=0) + E^q(x,\xi,t=0)]/2 = \Delta q^+/2 + L_q$. Here, L_q is a quark orbital-angular-momentum contribution (L_q) to the nucleon spin. Since we know the quark-spin contribution Δq^+ from experimental measurements, it is possible to determine L_q from the GPD measurements.

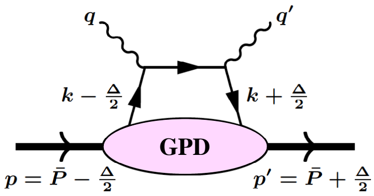

Figure 1: Spacelike GPDs in virtual Compton scattering.

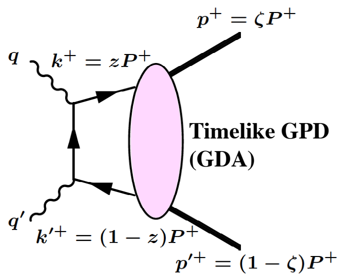

Figure 2: Timelike GPDs (GDAs) in two-photon process.

The timelike GPDs are often called the GDAs, and they are measured by the s-t crossed process of the DVCS, so called the two-photon process, as shown in Fig. 2. The GDAs or timelike GPDs are defined by the matrix element similar to Eqs. (1) and (2) between the vacuum and the final hadron pair $h\bar{h}$ [5]. For example, they are defined for the π^0 pair as

$$\Phi_q^{\pi^0\pi^0}(z,\zeta,W^2) = \int \frac{dy^-}{2\pi} e^{i(2z-1)\,P^+y^-/2} \langle\, \pi^0(p)\, \pi^0(p')\, |\, \bar{q}(-y/2)\gamma^+ q(y/2)\, |\, 0\, \rangle \Big|_{y^+=\vec{y}_\perp=0}. \tag{3}$$

The GDAs are expressed by three variables, the momentum fractions z and ζ in Fig. 2 and the invariant-mass squared W^2 as $z = k^+/P^+$, $\zeta = p^+/P^+ = (1+\beta\cos\theta)/2$, and $W^2 = s$, where β is defined by $\beta = |\vec{p}\,|/p^0 = \sqrt{1 - 4m_\pi^2/W^2}$, and θ is the scattering angle in the center-of-mass frame of the final pions. The two-photon process is factorized if the condition $Q^2 \gg W^2$, $\Lambda^2_{\rm QCD}$ is satisfied to express it in terms of the GDAs. The corresponding spacelike GPDs for the pion are given as

$$H_q^{\pi^0}(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ix\bar{P}^+y^-} \langle \pi^0(p') \left| \bar{q}(-y/2)\gamma^+ q(y/2) \right| \pi^0(p) \rangle \Big|_{y^+=\vec{y}_\perp=0}. \tag{4}$$

The spacelike and timelike GPDs are related with each other by the s-t crossing as $\Phi_q^{\pi^0\pi^0}(z',\zeta,W^2)$ $\leftrightarrow H_q^{\pi^0}\big(x = (1-2z')/(1-2\zeta),\ \xi = 1/(1-2\zeta),\ t = W^2\big)$.

3. Gravitational form factors of hadrons from spacelike and timelike GPDs

Electromagnetic and weak form factors of hadrons and nuclei have been measured in lepton scatterings, whereas gravitational form factors used to be considered purely theoretical quantities until recently, because gravitational interactions are too weak to be used for measuring interactions with microscopic particles. However, it became possible to measure them without direct gravitational interactions by hadron tomography techniques as illustrated in Fig. 3. In order to understand why the gravitational form factors can be obtained from electromagnetic and weak interactions, we consider moments of the nonlocal operators in Eqs. (1), (2), (3), and (4) as

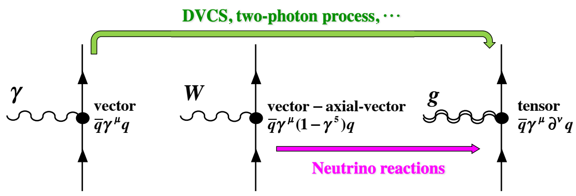

Figure 3: Gravitational form factors from electromagnetic and weak interactions.

$$\left(\frac{P^+}{2}\right)^n \int dx\, x^{n-1} \int \frac{dy^-}{2\pi} e^{ixP^+y^-/2}\, \bar{q}(-y/2)\gamma^+ q(y/2)\Big|_{y^+=\vec{y}_\perp=0} = \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+\right)^{n-1} q(0), \tag{5}$$

where $\overleftrightarrow{\partial}$ is defined by $f_1 \overleftrightarrow{\partial} f_2 = [f_1(\partial f_2) - (\partial f_1) f_2]/2$. For $n = 1$, it is the ordinary vector-type electromagnetic current; however, we notice that the operator is the energy-momentum tensor of a quark for $n = 2$ [5]. It indicates that the GPDs contain the information on the gravitational form factors. Therefore, the second moments of the spacelike and timelike GPDs are given by the matrix elements of the energy-momentum tensor $T_q^{\mu\nu}$, and they are expressed by the spacelike and timelike gravitational form factors Θ_1 and Θ_2 as

$$\begin{aligned}\int_{-1}^{1} dx\, x\, H_q^{\pi^0}(x,\xi,t) &= \frac{1}{(P^+)^2} \langle \pi^0(p') \,|\, T_q^{++}(0) \,|\, \pi^0(p) \rangle \\ &= \frac{1}{2\,(P^+)^2} \left[\left(t\, g^{++} - q^+ q^+\right) \Theta_{1,q}(t) + P^+ P^+\, \Theta_{2,q}(t) \right],\end{aligned} \tag{6}$$

$$\begin{aligned}\int_0^1 dz\, (2z-1)\, \Phi_q^{\pi^0\pi^0}(z,\,\zeta,\,W^2) &= \frac{2}{(P^+)^2} \langle \pi^0(p)\, \pi^0(p') \,|\, T_q^{++}(0) \,|\, 0 \rangle \\ &= \frac{1}{(P^+)^2} \left[\left(t\, g^{++} - q^+ q^+\right) \Theta_{1,q}(t) + P^+ P^+\, \Theta_{2,q}(t) \right].\end{aligned} \tag{7}$$

Here, the energy-momentum tensor is given by $T_q^{\mu\nu}(x) = \bar{q}(x)\, \gamma^{(\mu} i \overleftrightarrow{D}^{\nu)}\, q(x)$, with the convention $A^{(\mu}B^{\nu)} = (A^\mu B^\nu + A^\nu B^\mu)/2$ and the covariant derivative $D^\mu = \partial^\mu - ig\lambda^a A^{a,\mu}/2$ with the QCD coupling constant g and the SU(3) Gell-Mann matrix λ^a. For the spin-1/2 nucleons, the spacelike GPDs are related to the gravitational form factors A_q, B_q, C_q, and $\bar{C}_q$ in the same way as

$$\begin{aligned}&\bar{u}(p') \left[\int_{-1}^{1} dx\, x\, H_q(x,\xi,t)\, \gamma^+ + \int_{-1}^{1} dx\, x\, E_q(x,\xi,t)\, \frac{i\sigma^{+\sigma}\Delta_\sigma}{2M_N} \right] u(p) = \frac{1}{\bar{P}^+} \langle N(p') \,|\, T_q^{++}(0) \,|\, N(p) \rangle \\ &= \frac{1}{\bar{P}^+} \bar{u}(p') \left[A_q(t)\gamma^+ \bar{P}^+ + B_q(t) \frac{\bar{P}^+ i\sigma^{+\sigma}\Delta_\sigma}{2M_N} + D_q(t) \frac{\Delta^+\Delta^+ - g^{++}\Delta^2}{M_N} + \bar{C}_q(t) M_N g^{++} \right] u(p).\end{aligned} \tag{8}$$

Therefore, these various GPD measurements enable extraction of the gravitational form factors of hadrons [5] and also clarifications of the origins of hadron spins and masses.

4. GPDs in neutrino reactions

4.1 Pion-production cross section and GPDs in neutrino reactions

There have been a number of works on the GPDs in neutrino reactions [7, 8]. Instead of introducing all of these results, we explain the formalism and numerical results by Pire, Szymanowski, and Wagner [8] as a recent work in the following. Instead of the virtual-Compton-like process, it is appropriate to rely on larger meson-production cross sections in neutrino reactions. For example, the pion production process $\nu N \to \ell^- N' \pi$ is expressed by a typical subprocess with the GPDs in Fig. 4. In the pion production, quark and gluon GPD contributions to the amplitude are generally given as [8]

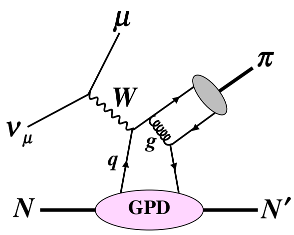

Figure 4: GPDs in neutrino scattering.

$$T^q = \frac{-iC_q}{2Q}\bar{N}(p')\left[\mathcal{H}^\nu \not{n} - \tilde{\mathcal{H}}^\nu \not{n}\gamma^5 + \mathcal{E}^\nu \frac{i\sigma^{n\Delta}}{2m_N} - \tilde{\mathcal{E}}^\nu \frac{\gamma^5 \Delta\cdot n}{2m_N}\right] N(p),$$
$$T^g = \frac{-iC_g}{2Q}\bar{N}(p')\left[\mathcal{H}^g \not{n} + \mathcal{E}^g \frac{i\sigma^{n\Delta}}{2m_N}\right] N(p). \tag{9}$$

Here, charged-current reactions probe the difference between the u and d-quark GPDs: $F^\nu(x,\xi,t) = F^d(x,\xi,t) - F^u(-x,\xi,t)$, where $F = H,\ \tilde{H},\ E,\ \tilde{E}$. The C_q and C_g are coupling constants with color and flavor factors defined as $C_q = 2\pi C_F \alpha_s V_{dc}/3$ with $C_F = 4/3$, and $C_g = \pi T_f \alpha_s V_{du}/3$ with $T_f = 1/2$. The momentum factor Q is defined as $Q^2 = -q^2$ by the momentum transfer q, $\not{n}$ is given by $\not{n} \equiv n_\mu \gamma^\mu = \gamma^+$ with $n^\mu = (1, 0, 0, -1)/\sqrt{2}$, $\sigma^{n\Delta}$ is $\sigma^{n\Delta} = \sigma^{\mu\nu} n_\mu \Delta_\nu$, and m_N is the nucleon mass. The functions $\mathcal{F}^\nu$ and $\mathcal{F}^g$ are defined by including the pion distribution amplitude ϕ_π as

$$\mathcal{F}^\nu = 2f_\pi \int_0^1 \frac{\phi_\pi(z)dz}{1-z}\int_{-1}^1 dx \frac{F^\nu(x,\xi,t)}{x-\xi+i\epsilon}, \quad \mathcal{F}^g = \frac{8f_\pi}{\xi}\int_0^1 \frac{\phi_\pi(z)dz}{z(1-z)}\int_{-1}^1 dx \frac{F^g(x,\xi,t)}{x-\xi+i\epsilon}, \tag{10}$$

where f_π is the pion decay constant.

In terms of these GPDs, the cross section is written as

$$\frac{d^4\sigma_{\nu N\to l^- N'\pi}}{dy\, dQ^2\, dt\, d\varphi} = \frac{G_F^2 Q^2\, \varepsilon\sigma_L}{32(2\pi)^4(1-\epsilon)(s-m_N^2)^2 y\sqrt{1+4x_B^2 m_N^2/Q^2}},$$
$$\sigma_L = \frac{1}{Q^2}\Big\{[\,|C_q\mathcal{H}^{\bar{q}} + C_g\mathcal{H}^g|^2 + |C_q\tilde{\mathcal{H}}^{\bar{q}}|^2](1-\xi^2) + \frac{\xi^4}{1-\xi^2}[\,|C_q\mathcal{E}^{\bar{q}} + C_g\mathcal{E}^g|^2 + |C_q\tilde{\mathcal{E}}^{\bar{q}}|^2]$$
$$- 2\xi^2 \mathcal{R}e[C_q\mathcal{H}^{\bar{q}} + C_g\mathcal{H}^g][C_q\mathcal{E}^{\bar{q}} + C_g\mathcal{E}^g]^* - 2\xi^2\mathcal{R}e[C_q\tilde{\mathcal{H}}^{\bar{q}}][C_q\tilde{\mathcal{E}}^{\bar{q}}]^*\Big\}, \tag{11}$$

where $y = p\cdot q/p\cdot k$, $Q^2 = x_B y(s-m_N^2)$, and $\varepsilon \simeq (1-y)/(1-y+y^2/2)$. The longitudinal cross-section σ_L is defined by the hadron tensor and the photon-polarization vector as $\sigma_L = \epsilon_L^{*\mu} W_{\mu\nu}\epsilon_L^\nu$. The obtained cross sections are shown in Figs. 5 and 6 for the π^+ and π^0 productions, respectively, at s = 20 GeV^2. As shown in these figures, both quark and gluon processes contribute to the π^+ production and the gluon contribution is much larger, whereas there is no contribution to the π^0 production from the gluon GPD. Therefore, the neutrino GPD measurement is valuable for clarifying the quark and gluon GPDs and the flavor dependence in the quark GPDs.

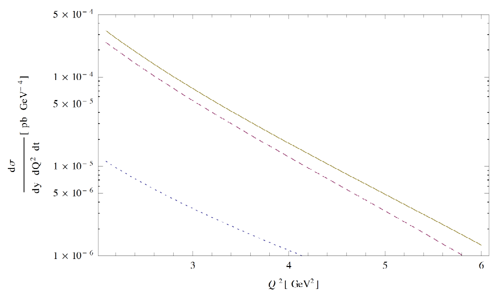

Figure 5: $\nu p \to \ell^- \pi^+ p$ cross section at $y = 0.7$ and $s = 20$ GeV^2 [8]. The dashed and dotted curves indicate gluon and quark contributions, respectively. The solid curve is their summation.

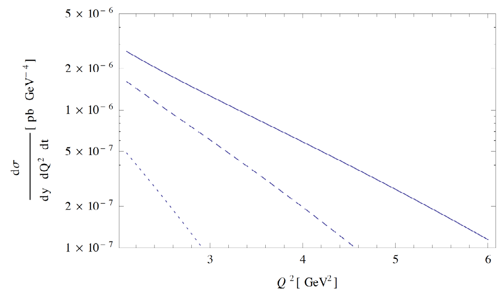

Figure 6: $\nu n \to \ell^- \pi^0 p$ cross section at $s = 20$ GeV^2 [8]. The solid, dashed, and dotted curves indicate the corss sections at $y = 0.7$, 0.5, and 0.3, respectively. There is no gluon contribution for the π^0 production.

4.2 Opportunities for GPD Measurements at LBNF

The future LBNF at the Fermi National Laboratory will deliver neutrino and antineutrino beams of unprecedented intensity with broad energy spectra. In addition to the default beam optimized for long-baseline oscillation measurements in the energy range 0.5-5 GeV, a higher energy option (mostly in the 2-15GeV range) is possible for precision measurements and searches for new physics beyond the Standard Model. The near detector complex of the Deep Underground Neutrino Experiment (DUNE) will include a high resolution on-axis detector which can address some of main limitations of (anti)neutrino experiments providing an accurate control of the targets and fluxes [10]. In particular, it will allow precision measurements of ν and $\bar{\nu}$ interactions on both hydrogen (H) and various nuclear targets (A) in combination with the high intensity and the energy spectra of the LBNF beams. The kinematic coverage is dominated by inelastic interactions – more than 54% of the events with the default low energy beam and most of the events with the high energy option have $W > 1.4$ GeV – offering a good sensitivity to the GPD measurements via the pion production processes $\nu(\bar{\nu})N \to lN'\pi$. The availability of a free proton target H will give access to high statistics measurements of the following channels: (a) $\nu_\mu p \to \mu^- \pi^+ p$; (b) $\bar{\nu}_\mu p \to \mu^+ \pi^- p$; (c) $\bar{\nu}_\mu p \to \mu^+ \pi^0 n$. While both quark and gluon GPDs contribute to the $\pi^\pm$ production, no gluon contribution is present for the π^0 production. The last two measurements with also provide information about the GPDs in a free neutron target since it is expected $\sigma(\bar{\nu}_\mu p \to \mu^+ \pi^- p) = \sigma(\nu_\mu n \to \mu^- \pi^+ n)$ and $\sigma(\bar{\nu}_\mu p \to \mu^+ \pi^0 n) = \sigma(\nu_\mu n \to \mu^- \pi^0 p)$ [8]. The study of the flavor dependence of the GPDs in free nucleons will be complemented by similar measurements performed simultaneously on a variety of nuclear targets (C, Ar, etc.) within the same detector. A comparison between measurements on H and on the nuclear targets can provide valuable information about the nuclear modifications of the GPDs. The nuclear targets will also extend the study of the flavor dependence of the GPDs by giving access to additional channels: (a) $\nu_\mu n \to \mu^- \pi^+ n$; (b) $\nu_\mu n \to \mu^- \pi^0 p$; (c) $\bar{\nu}_\mu n \to \mu^+ \pi^- n$. A sizable statistics for the various single pion production channels is expected to be collected from both H and nuclear targets with the default low energy LBNF beam [10, 11]. The high energy beam options will significantly enhance the sensitivity to the GPD measurements increasing the kinematic overlap with complementary EIC measurements.

5. Summary

The GPD studies will be crucial in understanding the origins of hadron spins and masses in terms of quarks and gluons. The spacelike GPDs are measured in charged-lepton scattering processes, deeply virtual Compton scattering and meson productions, and timelike GPDs are investigated by two-photon processes. Using the LBNF neutrino beam, we can access the spacelike GPDs in neutrino reactions. As the neutrino DIS measurements played an important role in establishing the flavor-dependence of the PDFs and the valence-quark distribution functions, the neutrino GPD measurements should be complementary to the charged-lepton ones. The high resolution on-axis detector in the DUNE near detector complex will be capable of detailed GPD studies on both free protons and nuclei in future, providing insights on the hadron spins and masses.

Acknowledgments

Figures 5 and 6 are used with the copyright permission of American Physical Society and authors. S.K. was partially supported by Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) Grant Number 19K03830. R.P. was supported by grant DE-SC0010073 from the Department of Energy, USA.

References

[1] For example, see M. Tzanov *et al.* (NuTeV collaboration), Phys. Rev. D **74**, 012008 (2006).
[2] For an introductory review, see M. Diehl, Phys. Rept. **388**, 41 (2003).
[3] R. Abdul Khalek *et al.*, arXiv:2103.05419, see Sec. 7.5.2 Neutrino physics for GPD studies in neutrino reactions.
[4] S. Kumano, M. Strikman, and K. Sudoh, Phys. Rev. D **80**, 074003 (2009); T. Sawada *et al.*, Phys. Rev. D **93**, 114034 (2016).
[5] S. Kumano, Qin-Tao Song, and O. V. Teryaev, Phys. Rev. D **97**, 014020 (2018).
[6] J. Rout *et al.*, Phys. Rev. D **102**, 116018 (2020).
[7] B. Lehmann-Dronke and A. Schafer, Phys. Lett. B **521**, 55 (2001); P. Amore, C. Coriano, and M. Guzzi, J. High Energy Phys. **02**, 038 (2005); C. Coriano and M. Guzzi, Phys. Rev. D **71**, 053002 (2005); A. Psaker, W. Melnitchouk, and A. V. Radyushkin, Phys. Rev. D **75**, 054001 (2007); G. R. Goldstein, O. G. Hernandez, S. Liuti, and T. McAskill, AIP Conf. Proc. **1222**, 248 (2010); B. Z. Kopeliovich, I. Schmidt, and M. Siddikov, Phys. Rev. D **86**, 113018 (2012); **89**, 053001 (2014); B. Pire and L. Szymanowski, Phys. Rev. Lett. **115**, 092001 (2015); M. Siddikov and I. Schmidt, Phys. Rev. D **95**, 013004 (2017); B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D **95**, 094001 (2017).
[8] B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D **95**, 114029 (2017).
[9] S. Kumano, EPJ Web Conf. **208**, 07003 (2019).
[10] R. Petti, PoS (DIS2019) 235, Proceedings of of the 27th International Workshop on Deep Inelastic Scattering and Related Subjects, Torino, Italy, April 8-12, 2019.
[11] H. Duyang, B. Guo, S. R. Mishra, and R. Petti, arXiv:1809.08752 [hep-ph].