

## Exploring environmentally induced decoherence effect on neutrino oscillation probabilities

---

**Arnab Sarker,\* Harshita Bhuyan and Moon Moon Devi**

*Department of Physics, Tezpur University,  
Napaam, Assam, India, Pin-784028*

*E-mail: [arnabs@tezu.ernet.in](mailto:arnabs@tezu.ernet.in), [harshitabhuyan987@gmail.com](mailto:harshitabhuyan987@gmail.com),  
[devimm@tezu.ernet.in](mailto:devimm@tezu.ernet.in)*

We discuss the effect of environmental decoherence on matter-effective neutrino oscillation probabilities. Decoherence is a phenomenon observed in systems interacting with the environment. We treat the neutrinos as an open quantum system and by using the Lindblad Master equation we study the evolution of neutrino states. The matter effect is incorporated for neutrinos passing through matter with the help of the Cayley-Hamilton formalism.

In this work, we have developed a general algorithm that attempts to solve the Lindblad Master Equation to compute the neutrino oscillation probabilities in presence of environmental decoherence. We extensively validate the algorithm and explore how environmentally induced decoherence can potentially affect the oscillation probabilities, particularly in the long-baseline sector.

\*\*\* *The 22nd International Workshop on Neutrinos from Accelerators (NuFact2021)* \*\*\*  
\*\*\* *6–11 Sep 2021* \*\*\*  
\*\*\* *Cagliari, Italy* \*\*\*

---

\*Speaker

## 1. Introduction

Neutrinos are particles which interact weakly with matter. They come in three distinct flavors i.e. electron  $\nu_e$ , muon  $\nu_\mu$  and tau  $\nu_\tau$ . These neutrinos change from one flavor to another as they travel through a distance and this phenomenon is known as Neutrino oscillation. The formalism to study the oscillation of neutrinos in vacuum and matter is well developed. Each neutrino flavor can be expressed as a superposition of three mass eigenstates with masses  $m_1, m_2$  and  $m_3$ . It can be written as,

$$|\nu_f\rangle = \sum_i |\nu_i\rangle \quad (1)$$

In nature, no system is completely isolated. Generally, the studies conducted on neutrino oscillations treat neutrinos as a closed system. But even the weakly interacting neutrinos cannot be treated as an isolated system. Only in an ideal case can we neglect the potential coupling of neutrinos with the environment. Thus, in our work we treat neutrinos as an open quantum system [4].

There exists a possibility that the propagating neutrinos may undergo some dissipative interactions with the environment. This will lead to a loss of coherence between different neutrino mass eigenstates. Such interactions can also introduce damping factors in the oscillation probabilities.

In this work, we study how appearance and disappearance probabilities get affected due to the presence of decoherence in the long-baseline sector.

## 2. Standard formalism for neutrino oscillation

The PMNS mixing matrix which describes the mixing between different mass eigenstates is given by

$$U_{PMNS} = U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \quad (2)$$

The Hamiltonian in mass basis for neutrinos in vacuum is given by

$$H_{vacuum} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \quad (3)$$

The Hamiltonian for neutrinos propagating through matter in mass basis can be written as

$$H_{matter} = H_{vacuum} + V_{matter} = H_{vacuum} + U^{-1}V_{flavor}U \quad (4)$$

where,  $V_{flavor}$  represents the matter potential and  $A = \pm\sqrt{2}G_f n_e$ ,  $+$ ( $-$ ) is for neutrinos (anti-neutrinos). The symbol  $G_f$  represents Fermi's coupling constant and  $n_e$  is electron density in matter.

$$V_{flavor} = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

We use the Cayley-Hamilton formalism to obtain a relation between the mixing matrix in mass basis with matter effective basis [9].

We incorporate the above equations in the density matrix formalism along with the Lindblad Master equation to study the oscillation behaviour of the neutrinos with decoherence in-play.

### 3. Phenomenological model

The density matrix formalism is a different approach to describing a quantum state. It is most useful for time-dependent problems and also very handy for describing an open quantum system. The density matrix for neutrinos can be defined as

$$\rho_\nu = \sum_i \rho_{ij} |v_i\rangle \langle v_j| \quad (6)$$

where  $|v_i\rangle$  represents the mass eigenstates of neutrinos.

The diagonal elements of the density matrix give information about the probability of obtaining a specific quantum state, whereas the non-diagonal matrix elements explain the evolution of coherence between different quantum states. The time evolution of the density matrix can be studied by the well-known **Liouville-Von Neumann** equation, given by

$$\frac{d}{dt}\rho_\nu(t) = -i[H, \rho_\nu(t)] \quad (7)$$

In our study, we model propagating neutrinos as an open quantum system and use the well-developed formalism of the density matrix [1]. We can use equation (7) to study the time evolution of neutrinos, in the absence of any dissipative effects.

Now to incorporate the effects of decoherence into the neutrino system, we use the **Lindblad Master equation** which allows for an additional term  $D[\rho_\nu(t)]$ .

$$\frac{d}{dt}\rho_\nu(t) = -i[H, \rho_\nu(t)] - D[\rho_\nu(t)] \quad (8)$$

The term  $D[\rho_\nu(t)]$  in the above equation is known as 'Dissipator term' which takes into account the dissipative interaction of the neutrinos with the environment. The dissipator term in equation (8) appears in the form given below.

$$D[\rho_\nu(t)] = \sum_n [\{\rho_\nu(t), D_n^\dagger D_n\} - 2D_n \rho_\nu(t) D_n^\dagger] \quad (9)$$

Here,  $n = N^2 - 1$  and  $N$  is the number of neutrinos or the dimensions of the density matrix.

The dissipator term can be reduced to a matrix of simpler form by imposing some constraints on  $D_n$ , one of which is the increase of entropy with time and the other is the conservation of average energy of the system [2]. This reduces equation (9) to a form given by

$$D[\rho_\nu(t)] = \begin{pmatrix} 0 & \Gamma_{21}\rho_{12}(t) & \Gamma_{31}\rho_{13}(t) \\ \Gamma_{21}\rho_{21}(t) & 0 & \Gamma_{32}\rho_{23}(t) \\ \Gamma_{31}\rho_{31}(t) & \Gamma_{32}\rho_{32}(t) & 0 \end{pmatrix} \quad (10)$$

where  $\Gamma$  is the decoherence coupling parameter.

To draw a similarity between the range of mass-squared splittings and the decoherence parameters, we use the approximation,  $\Gamma_{12} = 0$  and  $\Gamma_{32} \approx \Gamma_{31} = \Gamma$ .

The general probability equation calculated from the density matrix is given by

$$P(\nu_i \rightarrow \nu_f) = \text{Tr} [\alpha_f \rho_i(t)] \quad (11)$$

where,  $\alpha_f$  represents the final flavor state and  $\rho_i(t)$  represents the time evolved density matrix for the initial state  $i$ .

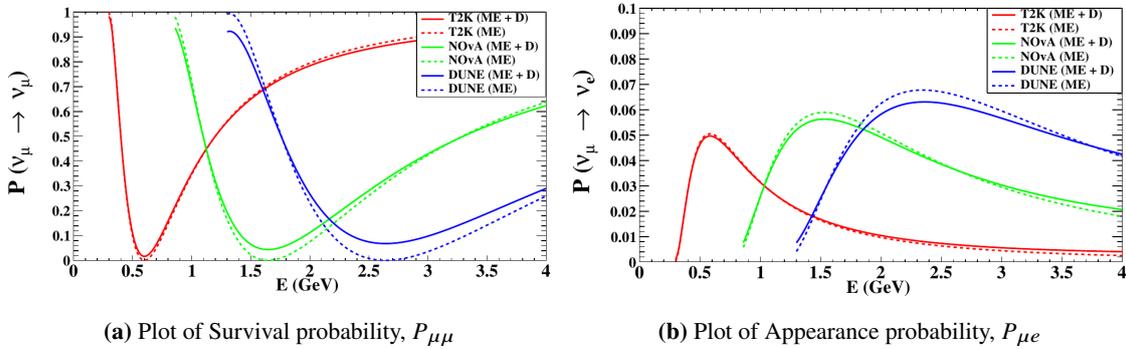
The appearance and disappearance probability expression for  $\nu_\mu$  is calculated to be,

$$P_{\mu\mu} = 1 - 2 \sum_{i>j}^3 |U'_{\mu i}|^2 |U'_{\mu j}|^2 \left[ 1 - e^{-\Gamma L} \cos\left(\frac{\Delta m_{ij}^2 L}{2E}\right) \right] \quad (12)$$

$$P_{\mu e} = \sum_{i=j}^3 |U'_{ei}|^2 |U'_{\mu j}|^2 + 2 \sum_{i>j}^3 U'_{ei}{}^* U'_{ej} U'_{\mu i} U'_{\mu j}{}^* e^{-\Gamma L} \cos\left(\frac{\Delta m_{ij}^2 L}{2E}\right) \quad (13)$$

#### 4. Results and conclusion

Using the survival and appearance probability expressions shown above, we plot for three different baselines i.e. 295km for T2K, 810km for NovA and 1300km for DUNE. The dotted line represents the curve with only matter effects and the solid line represents the curve with both matter effects and decoherence,  $\Gamma = 2.3 \times 10^{-23} \text{ GeV}$ .



**Figure 1:** Plots of neutrino oscillation probabilities

In Fig 1(a), we observe the effects of decoherence to increase with the increase in baseline. At minima, we have non-zero values and the value appears to increase with baseline. We can also observe in Fig 1(b) that the suppression of probability peaks is higher for longer baselines.

#### Acknowledgement

AS would like to acknowledge CSIR, India for the financial support received in the form of JRF-fellowship. MMD would like to acknowledge the DST SERB grant EMR/2017/001436.

**References**

- [1] J. A. B. Coelho, W. A. Mann and S. S. Bashar, *Phys. Rev. Lett.* **118**, no.22, 221801 (2017) doi:10.1103/PhysRevLett.118.221801 [arXiv:1702.04738 [hep-ph]].
- [2] J. A. B. Coelho and W. A. Mann, *Phys. Rev. D* **96**, no.9, 093009 (2017) doi:10.1103/PhysRevD.96.093009 [arXiv:1708.05495 [hep-ph]].
- [3] T. Ohlsson and H. Snellman, *Phys. Lett. B* **474**, 153-162 (2000) [erratum: *Phys. Lett. B* **480**, 419-419 (2000)] doi:10.1016/S0370-2693(00)00008-3 [arXiv:hep-ph/9912295 [hep-ph]].
- [4] Manzano, D. A short introduction to the Lindblad master equation. *AIP Advances*. **10**, 025106 (2020),
- [5] E. Lisi, A. Marrone and D. Montanino, *Phys. Rev. Lett.* **85**, 1166-1169 (2000) doi:10.1103/PhysRevLett.85.1166 [arXiv:hep-ph/0002053 [hep-ph]].
- [6] G. Barenboim, N. E. Mavromatos, S. Sarkar and A. Waldron-Lauda, *Nucl. Phys. B* **758**, 90-111 (2006) doi:10.1016/j.nuclphysb.2006.09.012 [arXiv:hep-ph/0603028 [hep-ph]].
- [7] T. Ohlsson, *Phys. Lett. B* **502**, 159-166 (2001) doi:10.1016/S0370-2693(01)00178-2 [arXiv:hep-ph/0012272 [hep-ph]].
- [8] A. M. Gago, E. M. Santos, W. J. C. Teves and R. Zukanovich Funchal, [arXiv:hep-ph/0208166 [hep-ph]].
- [9] Ohlsson, T. & Snellman, H. Three flavor neutrino oscillations in matter. *Journal Of Mathematical Physics*. **41**, 2768-2788 (2000),