$B - \bar{B}$ mixing: decay matrix at high precision

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I review the status of the Standard-Model prediction of the width difference $\Delta \Gamma_s$ among the two $B_s$ meson eigenstates. Ongoing effort addresses three-loop QCD corrections, corresponding to the next-to-next-to-leading order of QCD. With an improved theoretical precision of the ratio $\Delta \Gamma_s/\Delta M_s$, where $\Delta M_s$ denotes the mass difference in the $B_s - \bar{B}_s$ system, one can probe new physics in $\Delta M_s$ without sensitivity to $|V_{cb}|$, whose value is currently controversial.
The flavoured neutral mesons $M = K, D, B_d, B_s$ mix with their antiparticles $\bar{M} = \bar{K}, \bar{D}, \bar{B}_d, \bar{B}_s$, with two important consequences: First, the mass eigenstates do not coincide with the flavour eigenstates. Second, a meson produced in the state $|M\rangle$ evolves into a linear superposition of $|\bar{M}\rangle$ and $|\bar{\bar{M}}\rangle$. The corresponding time dependence features an oscillatory behaviour in addition to the usual exponential decay law. In this talk I discuss new calculations for $B_s - \bar{B}_s$ mixing, but the results equally apply to $B_d - \bar{B}_d$ mixing with trivial replacements of the corresponding elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The time evolution of the two-state system $(|B_s\rangle, |\bar{B}_s\rangle)$ is governed by two hermitian $2 \times 2$ matrices, the mass matrix $M^s$ and the decay matrix $\Gamma^s$. Upon diagonalisation of $M^s - i\Gamma^s/2$ one finds the expression linking the mass eigenstates $|B_L\rangle$ and $|B_H\rangle$ to the flavour eigenstates $|B_s\rangle, |\bar{B}_s\rangle$. The mass eigenstates differ in their masses $M_{H,L}$ and decay widths $\Gamma_{H,L}$ with “L” and “H” standing for “light” and “heavy”. The mass and width differences $\Delta M_s = M_H - M_L$ and $\Delta \Gamma_s = \Gamma_L - \Gamma_H$ are related to the off-diagonal elements $M^s_{12}$ and $\Gamma^s_{12}$ as

$$\Delta M_s \approx 2|M^s_{12}|, \quad \frac{\Delta \Gamma_s}{\Delta M_s} = -\text{Re} \frac{\Gamma^s_{12}}{M^s_{12}}. \quad (1)$$

The Standard Model (SM) predictions for $M^s_{12}$ and $\Gamma^s_{12}$ are calculated from the dispersive and absorptive parts of the box diagram in Fig. 1, respectively. To find $\Gamma^s_{12}$ one must therefore only consider diagrams with the light $u, c$ quarks, while box diagrams with one or two internal $t$ quarks will only contribute to $M^s_{12}$. To properly accommodate strong interaction effects one employs operator product expansions (OPE) to separate the physics from different energy scales. In the first step one matches the SM to an effective theory with $|\Delta B| = 1$ operators [1], where $\bar{B}$ is the beauty quantum number. The dependence of the SM $b$ decay amplitudes on the masses $M_W$ and $m_t$ is contained in the Wilson coefficients multiplying these operators. The most important operators, i.e. those with the largest coefficients, are the current-current operators $Q_1$ and $Q_2$ pictorially found by contracting the $W$ boson line connecting the $b_L\gamma^\mu c$ and $\bar{c}_L\gamma^\mu s$ currents to a point. $Q_1$ and $Q_2$ differ in their colour indices; both operators are needed to properly accommodate QCD corrections. The $B_s - \bar{B}_s$ mixing diagrams (to leading order (LO) in QCD) in the effective $|\Delta B| = 1$ theory are also shown in Fig. 1. The second OPE employed in the calculation is the Heavy Quark Expansion (HQE) [2], which expresses the $B_s - \bar{B}_s$ transition amplitude as an expansion in $\Lambda_{QCD}/m_b$, where $\Lambda_{QCD} \sim 400$ MeV is the fundamental scale of QCD and $m_b$ is the $b$ quark mass. The latter enters the problem as a hard momentum flowing through diagrams in Fig. 1. The HQE involves local $\Delta B = 2$ operators, now found by contracting the hard loop to a point. In the leading order of $\Lambda_{QCD}/m_b$
where the quoted number for $\Delta \Gamma_s^{\exp}$ is derived from data of LHCb [10], CMS [11], ATLAS [12], CDF [13], and DØ [14]. The CKM factor $|V_{tb} V_{ts}|^2$ drops out from the ratio $\Delta \Gamma_s/\Delta M_s$ and also the
hadronic matrix elements largely cancel from this quantity. Thus by confronting $\Delta \Gamma_s^{\text{exp}}/\Delta M_s^{\text{exp}}$ with a precise theory prediction for $\Delta \Gamma_s/\Delta M_s$ we can both bypass the controversy on $|V_{cb}|$ and eliminate a source of hadronic uncertainty.

Using the NLO results of Refs. [15–18] and state-of-the-art lattice-QCD computations of $\langle B_s|Q|\overline{B}_s\rangle$ and $\langle B_s|\overline{Q}_S|\overline{B}_s\rangle$ one finds [19]

$$\Delta \Gamma_s = \left(0.077 \pm 0.015_{\text{pert}} \pm 0.002_{\text{had}} \pm 0.017_{\Lambda_{\text{QCD}}/m_b}\right) \text{GeV (pole)}$$

$$\Delta \Gamma_s = \left(0.088 \pm 0.011_{\text{pert}} \pm 0.002_{\text{had}} \pm 0.014_{\Lambda_{\text{QCD}}/m_b}\right) \text{GeV (MS).} \quad (7)$$

Here “pole” and “MS” refers to different renormalisation schemes. The three errors denote the perturbative uncertainty, the errors from $\langle B_s|Q|\overline{B}_s\rangle$ and $\langle B_s|\overline{Q}_S|\overline{B}_s\rangle$, as well as the sub-leading power corrections. The predictions use the calculated $\Delta \Gamma_s/\Delta M_s$ multiplied by $\Delta M_s^{\text{exp}}$ in Eq. (5).

Both the perturbative error and the scheme dependence indicate that an NNLO calculation is mandatory to match the accuracy of the measurement. Furthermore, better lattice calculations [20] and a NLO calculation of the sub-leading-power corrections are needed to decrease the uncertainty in Eq. (7).

The progress since Refs. [15–18] comprises NNLO corrections enhanced by the number $N_f$ of active quark flavours [19, 21] and two-loop results with one current-current and one penguin operator [22]. The four-quark penguin operators $Q_{3–6}$ have Wilson coefficients which are much smaller than those of $Q_{1–2}$ and the chromomagnetic penguin operator contributes with a suppression factor of $a_s$. After the conference the remaining missing two-loop contributions, with two insertions of penguin operators [23], and the full NNLO corrections with two current-current operators have been completed [24]. The NNLO corrections of Refs. [19, 21, 24] involve three-loop diagrams which have been calculated in an expansion in $z = (m_c/m_b)^2$ to orders $z^0$ and $z^1$. This expansion is also used in Refs. [22, 23].

In conclusion, $B_s - \overline{B}_s$ mixing is highly sensitive to virtual effects of new physics, with a reach to particle masses of 100 TeV and more. The theory prediction for $\Delta M_s$ is currently limited by the uncertainties of $|V_{cb}|$ and, to less extent, of the hadronic matrix element $\langle B_s|Q|\overline{B}_s\rangle$. The precise measurement of $\Delta \Gamma_s$ calls for a better theory prediction; with the results of Refs. [19, 21, 24] one of the two major uncertainties is pushed below the experimental error. The other one stems from the power-suppressed contributions and will be reduced once the lattice calculations [20] become more accurate and the Wilson coefficients of power-suppressed operators are calculated to NLO. With precise experimental and theoretical values for $\Delta \Gamma_s$, the ratio $\Delta \Gamma_s/\Delta M_s$ will be an excellent quantity to probe new physics in $\Delta M_s$, because this ratio is not affected by the exclusive-vs-inclusive controversy on the value of $|V_{cb}|$. For the latest numerical theory predictions for $\Delta \Gamma_s/\Delta M_s$ I refer the reader to Refs. [23, 24].

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4
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