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A model for cobimaximal neutrino mixing

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Here we explore a flavor model to understand the cobimaximal lepton mixing ansatz based on A_4 non-Abelain discrete symmetry. Guided by the symmetry, we explicitly construct the relevant lepton mass matrices with the involvement of A_4 flavons. Tiny neutrino mass and mixing are analyzed here relying on the type-I seesaw mechanism and we study the associated phenomenology. Subsequently, we draw interesting correlations between the neutrino mixing angles and absolute neutrino masses.

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1. Introduction

Historically, several mixing patterns have been proposed to explain the observed mixing scheme for the leptons and non-Abelian discrete flavor symmetries are widely used to reconstruct such mixing schemes [1, 2]. With the measurement of non-zero value of the reactor mixing angle θ_{13} most of the fixed pattern mixing schemes such as tri-bimaximal, golden-ratio, bimaximal or hexagonal mixing are now ruled out by current data [3]. The mixing scheme known as cobimaximal lepton mixing [4, 5] which predicts specific values for the atmospheric mixing angle $\theta_{23} = 45°$ and Dirac CP phase $\delta = \pm 90°$ still remains a good approximation for the observed neutrino oscillation data where θ_{13} and θ_{12} are not restricted. Under this mixing scheme the lepton mixing matrix satisfy the condition

$$|U_{\mu i}| = |U_{\tau i}| \text{ with } i = 1, 2, 3. \tag{1}$$

The mass matrix leading to the mixing matrix given in Eq. (1) can be written as [5, 6]

$$\mathcal{M} = \begin{pmatrix} a & b & b^\star \\ b & c & d \\ b^\star & d & c^\star \end{pmatrix}, \tag{2}$$

where b and c are in general complex while c and d remain real. Therefore, the main challenge is now to theoretically reproduce the cobimaximal mixing scheme. Non-Abelian discrete symmetries such as $S_3, A_4, S_4, A_5, \Delta(27)$ etc widely used to understand neutrino masses and mixing, see [1, 2] and references therein. All the discrete groups used in this regards, the group A_4 (the symmetry group of a tetrahedron) turns out to be most popular one.

Here, in this work, we attempt to realize this experimentally viable cobimaximal neutrino mixing scheme within a framework of A_4 flavor symmetry which consists of three inequivalent one-dimensional representations (1, $1'$ and $1''$) and one three-dimensional representation (3). The details of this non-abelian discrete group are given in Appendix A. Here all the left-handed lepton doublets and right-handed charged leptons are singlets under the A_4 flavor symmetry which yields a diagonal charged lepton mass matrix at tree level. The construction is such that the Dirac Yukawa coupling is diagonal at leading order. Here the structure of the heavy right-handed Majorana neutrino mass matrix essentially dictates the features of the cobimaximal neutrino mass matrix through the type-I seesaw mechanism. A single flavon that gets a complex vacuum expectation value (vev) and is thereby responsible for spontaneous CP violation at a high scale is the sole source of CP in the theory. In addition to $\theta_{23} = 45°$ and $\delta = -90°$, the model makes specific predictions for absolute neutrino mass and absolute mass parameter appearing in the neutrinoless double beta decay for both hierarchies of light neutrino masses with trivial values for the Majorana phases.

In the next section 2 we discuss the detailed flavor structure of the model and in section 3 we mention the prediction involving the neutrino parameters of the model. Finally in section 4 we conclude.

2. Structure of the model

In this framework, we start with a canonical type-I seesaw framework [7, 8] to explain the smallness of light neutrino masses in conjugation with A_4 discrete symmetry to obtain the lepton

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mass matrices. Here A_4 symmetry is assisted by additional $Z_3 \times Z_2$ symmetry which forbids unwanted contribution in the lepton mass matrices. The particle content and the symmetries of the model are provided in Table 2.

	ℓ	α_R	H	N_R	ϕ_T	ξ	ϕ_S
A_4	3	3	1	3	1	1	3
Z_3	ω^2	ω^2	1	ω^2	1	ω^2	ω^2
Z_2	1	- 1	1	1	-1	1	1

Table 1: Symmetry and Particle content of the model

Form Table 2 we find that along with the right-handed neutrinos (N_R) the Standard Model is extended by three gauge singlet scalar fields known as flavons. We further assume that only one flavon here gets a complex vacuum expectation value and is thereby responsible for spontaneous CP violation at a high scale [9, 10]. All other flavons have real vevs and all the couplings involved are considered to be real. The VEV alignment for these flavons are considered as : $\langle\phi_T\rangle = v_T$, $\langle\xi\rangle = v_\xi$ and $\langle\phi_S\rangle = (v'_S, v_S e^{i\gamma}, v_S e^{-i\gamma})$. Thus the CP symmetry is spontaneously broken by the complex vacuum expectation value of a singlet field ϕ_S and this is the only source of CP violation of the theory. Here the construction is such that the lepton doublets ℓ transform as an A_4 triplet and e_R, μ_R, τ_R together also forms A_4 triplet α_R. Due the imposed Z_2 symmetry, the tree contribution to the charged $\bar{\ell} H \alpha_R$ sector is forbidden. However, with the involvement of the A_4 singlet ϕ_T, charged masses can be generated from a dim-5 operator $\bar{\ell} H \alpha_R \phi_T/\Lambda$ which yields a diagonal mass matrix for the charged leptons. Now the Lagrangian for neutrino sector can be written as

$$-\mathcal{L}_\nu = y\left(\bar{\ell} N_R\right)_{\mathbf{1}} \tilde{H} + (y_x \xi + y_\phi \phi_S)\overline{N^c_R} N_R + h.c. \tag{3}$$

Following the multiplication rules given Appendix A.1, the Dirac (m_D) and Majorana (M) neutrino mass matrices therefore can be written as

$$m_D = vy\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = vY_\nu; \quad M = \begin{pmatrix} a & be^{-i\gamma} & be^{i\gamma} \\ be^{-i\gamma} & a & c \\ be^{i\gamma} & c & a \end{pmatrix} \tag{4}$$

where $a = 2y_x v_\xi$, $b = 2y_\phi v'_S$, $c = 2y_\phi v_S$. Through type-I seesaw effective light neutrino mass therefore takes the form

$$\begin{aligned} m_\nu =& - m_D M^{-1} m_D^T \\ =& \lambda \begin{pmatrix} a^2 - b^2 & c(be^{-i\gamma} - ae^{i\gamma}) & c(be^{i\gamma} - ae^{-i\gamma}) \\ c(be^{-i\gamma} - ae^{i\gamma}) & a^2 - c^2 e^{-2i\gamma} & c^2 - ab \\ c(be^{i\gamma} - ae^{-i\gamma}) & c^2 - ab & a^2 - c^2 e^{2i\gamma} \end{pmatrix}, \end{aligned} \tag{5}$$

which has the similar form of the neutrino mass matrix compatible with the cobimaximal neutrino mixing given in Eq. (2).

3. Neutrino masses and mixing

Now, substituting $\theta_{23} = 45^\circ$ and $\delta = -90^\circ$ in the standard form of the lepton mixing matrix [11] and diagonalizing m_ν one can find the neutrino mixing parameters as a function of the model parameters a, b, c and γ and can be written as

$$\sin 2\theta_{13} = \frac{4c(a+b)}{A\sin\delta}; \quad \sin 2\theta_{12} = \frac{c^2 \sin 2\gamma}{B\sin\delta\sin\theta_{13}}. \tag{6}$$

where

$$A = (2c^2(1+\cos 2\gamma) - (a+b)^2); \quad B = (c^2(1+\cos 2\gamma) - a(a+b)). \tag{7}$$

The real positive mass eigenvalues for the light neutrinos therefore can be written as

$$\begin{aligned} m_1^2 =& \frac{\lambda^2}{4}\left[4B^2 c_{12}^4 s_{13}^4 s_{2\delta}^2 + \{2C\sin^2\theta_{12} + 2c(a-b)\cos\gamma\cos\theta_{13}\sin 2\theta_{12}\right. \\ &\left. +((a^2-b^2)\cos^2\theta_{13} - 2B\cos 2\delta\sin^2\theta_{13})\cos^2\theta_{12}\}^2\right], \\ m_2^2 =& \frac{\lambda^2}{4}\left[4B^2\sin^4\theta_{12}\sin^4\theta_{13}\sin^2 2\delta + \{2C\cos^2\theta_{12} - 2c(a-b)\cos\gamma\cos\theta_{13}\sin 2\theta_{12}\right. \\ &\left. +((a^2-b^2)\cos^2\theta_{13} + 2B\cos 2\delta\sin^2\theta_{13})\sin^2\theta_{12}\}^2\right], \\ m_3^2 =& \frac{\lambda^2}{4}\left[(a^2-b^2)^2 s_{13}^4 s_{2\delta}^2 + \{2Bc_{13}^2 + (a^2-b^2)c_{2\delta}s_{13}^2\}\right], \end{aligned} \tag{8}$$

where

$$\lambda = v^2 y^2/(ab^2 - a^3 - 2bc^2 + 2ac^2\cos 2\gamma); \quad C = a(a-b) + c^2(1-\cos 2\gamma). \tag{9}$$

Using the 3σ allowed range for the neutrino oscillation data taken from [3], the model parameters can be constrained using the method used in [12]. Here we find the a to be in the range $3.52 - 3.83$, whereas the upper-limit for b is found to be $b \leq 0.02$. The phase γ is found to be concentrated around 3.2 and 6.16 with $12 \leq c \leq 13$ for normal hierarchy of light neutrino mass. Using these allowed ranges for the parameters a, b, c and γ we present correlation between neutrino oscillation parameters given in Fig. 1. In the left panel, we plot the correlation between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$

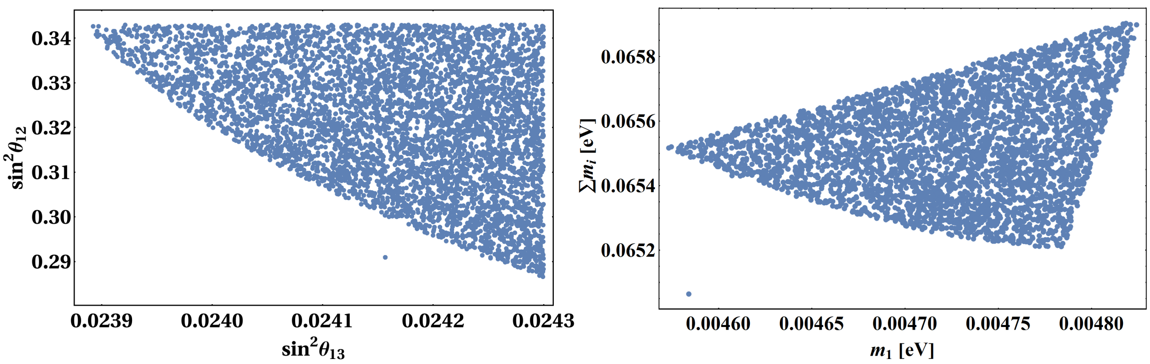

Figure 1: [left panel] Correlation between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ for the allowed values of the model parameters satisfying 3σ range of neutrino oscillation parameters. [Right panel] Correlation between the lightest neutrino mass m_1 and sum of all three light neutrinos Σm_i for normal hierarchy.

using 3σ range of neutrino oscillation parameters whereas in the right panel we plot the correlation between the lightest neutrino mass m_1 and sum of all three light neutrinos Σm_i. In the left panel of Fig. 1 the clear boundary indicates how the 3σ allowed range of the solar and reactor mixing angles restrict the parameter space. On the other hand in the right panel the prediction and pattern for the absolute neutrino masses is mainly dictated the by the tho mass-squared difference. Clearly, this model for cobimaximal neutrino mixing predicts the lightest neutrino mass m_1 (for the normal hierarchy of light neutrino mass) is found to be in the range 0.0043 eV to 0.0048 eV while the sum of the absolute masses of the three light neutrinos is found to be in the range 0.064-0.066 eV. Similarly, the effective mass parameter appearing in the neutrinoless double beta decay is found to be in the range 0.004-0.006 eV. Here we have carried out the analysis only for the normal hierarchy of the light neutrino masses. This analysis can also be extended to the inverted hierarchy of the light neutrino mass.

4. Conclusion

Here in this work, we have formulated a framework for cobimaximal neutrino mixing which predicts $\theta_{23} = 45°$ and $\delta = -90°$ within well-motivated A_4 discrete flavor symmetry. In this framework, the both charged leptons and Dirac neutrino mass matrix is diagonal to start with and the RH mass matrix plays an instrumental role in reconstructing an effective mass matrix for the light neutrino through a type-I seesaw. In this model, the CP is spontaneously violated and fixed around 3.2 and 6.16 respectively. Our model predicts a definite correlation between the neutrino oscillation parameters and the sum of the absolute masses of the three light neutrinos is found to be in the range 64-66 meV. In the present model, owing to the considered flavor symmetry, the generation of lepton asymmetry gets constrained as the Dirac Yukawa coupling is diagonal at leading order. Interestingly, by considering the higher-dimensional operators in the Dirac Yukawa coupling adequate lepton asymmetry can be successfully generated as exercised in [13].

Appendix

A. A_4 Multiplication Rules:

It has four irreducible representations: three one-dimensional and one three dimensional which are denoted by $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ and $\mathbf{3}$ respectively. The multiplication rules of the irreducible representations are given by [14]

$$\mathbf{1}\otimes\mathbf{1} = \mathbf{1}, \mathbf{1}'\otimes\mathbf{1}' = \mathbf{1}'', \mathbf{1}'\otimes\mathbf{1}'' = \mathbf{1}, \mathbf{1}''\otimes\mathbf{1}'' = \mathbf{1}', \mathbf{3}\otimes\mathbf{3} = \mathbf{1}+\mathbf{1}'+\mathbf{1}''+\mathbf{3_a}+\mathbf{3_s} \tag{A.1}$$

where **a** and **s** in the subscript corresponds to anti-symmetric and symmetric parts respectively. Now, if we have two triplets as $A = (a_1, a_2, a_3)^T$ and $B = (b_1, b_2, b_3)^T$ respectively, their direct product can be decomposed into the direct sum mentioned above. The product rule for this two triplets in the S diagonal basis[1] can be written as

$$(A \times B)_{\mathbf{1}} \backsim a_1 b_1 + a_2 b_2 + a_3 b_3, \tag{A.2}$$
$$(A \times B)_{\mathbf{1}'} \backsim a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \tag{A.3}$$
$$(A \times B)_{\mathbf{1}''} \backsim a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \tag{A.4}$$
$$(A \times B)_{\mathbf{3}_{\mathrm{s}}} \backsim (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1), \tag{A.5}$$
$$(A \times B)_{\mathbf{3}_{\mathrm{a}}} \backsim (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1), \tag{A.6}$$

here ω $(= e^{2i\pi/3})$ is the cube root of unity.

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References

[1] S.F. King and C. Luhn, *Neutrino Mass and Mixing with Discrete Symmetry*, *Rept. Prog. Phys.* **76** (2013) 056201 [1301.1340].

[2] G. Chauhan, P.S.B. Dev, B. Dziewit, W. Flieger, J. Gluza, K. Grzanka et al., *Discrete Flavor Symmetries and Lepton Masses and Mixings*, in *2022 Snowmass Summer Study*, 3, 2022 [2203.08105].

[3] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, *The fate of hints: updated global analysis of three-flavor neutrino oscillations*, *JHEP* **09** (2020) 178 [2007.14792].

[4] P.F. Harrison and W.G. Scott, *mu - tau reflection symmetry in lepton mixing and neutrino oscillations*, *Phys. Lett. B* **547** (2002) 219 [hep-ph/0210197].

[5] W. Grimus and L. Lavoura, *A Nonstandard CP transformation leading to maximal atmospheric neutrino mixing*, *Phys. Lett. B* **579** (2004) 113 [hep-ph/0305309].

[6] W. Grimus and L. Lavoura, *Cobimaximal lepton mixing from soft symmetry breaking*, *Phys. Lett. B* **774** (2017) 325 [1708.09809].

[7] P. Minkowski, *$\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?*, *Phys. Lett. B* **67** (1977) 421.

[8] R.N. Mohapatra and G. Senjanovic, *Neutrino Mass and Spontaneous Parity Violation*, *Phys. Rev. Lett.* **44** (1980) 912.

[1] Here S is a 3×3 diagonal generator of A_4.

[9] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and H. Serodio, *Spontaneous leptonic CP violation and nonzero θ_{13}*, *Phys. Rev. D* **86** (2012) 076008 [1203.2646].

[10] B. Karmakar and A. Sil, *Spontaneous CP violation in lepton-sector: A common origin for θ_{13}, the Dirac CP phase, and leptogenesis*, *Phys. Rev. D* **93** (2016) 013006 [1509.07090].

[11] Particle Data Group collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01.

[12] A. Datta, B. Karmakar and A. Sil, *Flavored leptogenesis and neutrino mass with A_4 symmetry*, *JHEP* **12** (2021) 051 [2106.06773].

[13] B. Karmakar and A. Sil, *Nonzero θ_{13} and leptogenesis in a type-I seesaw model with A_4 symmetry*, *Phys. Rev.* **D91** (2015) 013004 [1407.5826].

[14] G. Altarelli and F. Feruglio, *Discrete Flavor Symmetries and Models of Neutrino Mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [1002.0211].