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Neutrino phenomenology in NMSSM with $\mathbb{D}_4$ flavor symmetry

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We propose a next-to-minimal supersymmetric Standard Model (NMSSM) extended by a $\mathbb{D}_4$ discrete flavor symmetry to explain the current neutrino oscillation data within the type I seesaw mechanism. We found that the resulting neutrino mixing matrix is trimaximal. Then, we studied the phenomenology of neutrino parameters for the normal mass hierarchy. In particular, we numerically evaluated the three absolute neutrino mass observables, namely, the sum of absolute neutrino masses $\sum m_i$ which is restricted from cosmological observations, the electron neutrino mass m_{ν_e} estimated from single beta decay measurements, and the effective Majorana mass $|m_{ee}|$ derived from neutrinoless double beta decay experiments.

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1. Introduction

The discovery of neutrino oscillations by Super-Kamiokande in 1998 [1] was the undoubted evidence that neutrinos are massive. Since then, a high experimental effort has been performed to estimate and understand the properties of these mysterious particles. From a theoretical point of view, it describes the first evidence of physics beyond the Standard Model (SM). One of the widely studied extensions of the SM is the Next-to-Minimal Supersymmetric Standard Model (NMSSM), see for instance [2, 3]. However, further extensions are needed to explain the results of the neutrino oscillation experiments and, consequently, the observed mixing angles and tiny masses of neutrinos. On the other hand, the well-known trimaximal mixing (TM_2) is one of the most familiar neutrino mixing patterns, described as follows [4]

$$U_{TM_2} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{\sqrt{3}}{3} & \sqrt{\frac{2}{3}}\sin\theta e^{i\sigma} \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{\sin\theta}{\sqrt{2}}e^{-i\sigma} & \frac{\sqrt{3}}{3} & -\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}}e^{i\sigma} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{\sin\theta}{\sqrt{2}}e^{-i\sigma} & \frac{\sqrt{3}}{3} & \frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}}e^{i\sigma} \end{pmatrix} \quad (1)$$

Many strategies have been used to reveal the origin of neutrino masses and mixing patterns. One of the most appealing suggestions is that a flavor symmetry gets spontaneously broken by the non-zero vacuum expectation value (VEV) of some scalar fields, usually called flavons [5]. Indeed, for the lepton sector, where the mixing texture seems to be particularly well-specified, the use of flavor discrete symmetries has been expressly recommended [6].

In what follows, we build a model based on $\mathbb{D}_4$ discrete flavor symmetry within the framework of the NMSSM extended by three right-handed neutrinos. We then perform a brief phenomenological study of non-oscillation observables, such as the sum of neutrino masses $\sum m_i$, the electron neutrino mass m_{ν_e} and the effective Majorana mass $|m_{ee}|$.

2. Implementation of $\mathbb{D}_4$ symmetry in NMSSM

In this section, we begin by describing the particle content and the embedding of the $\mathbb{D}_4$ flavor symmetry in the NMSSM[1]. Before proceeding, let us first mention some properties of the $\mathbb{D}_4$ dihedral group. The tensor product of two doublets in the $\mathbb{D}_4$ group is given by

$$2_x \times 2_y = 1_{+,+} + 1_{+,-} + 1_{-,+} + 1_{-,-} \quad (2)$$

where

$$\begin{aligned} 1_{+,+} &= x_1y_2 + x_2y_1 \quad , \quad 1_{+,-} = x_1y_1 + x_2y_2 \\ 1_{-,+} &= x_1y_2 - x_2y_1 \quad , \quad 1_{-,-} = x_1y_1 - x_2y_2, \end{aligned} \quad (3)$$

while the singlets product are $1_{\alpha,\beta} \times 1_{\gamma,\delta} = 1_{\alpha\gamma,\beta\delta}$ with $\alpha, \gamma, \beta, \delta = \pm$.
In the present study, the usual NMSSM lepton and Higgs superfields are augmented by three right-handed neutrinos $N^c_{i=1,2,3}$ and four flavon superfields carrying quantum numbers under the $\mathbb{D}_4$, see Table 1. Moreover, as in the usual NMSSM, a global Z_3 symmetry is imposed under which all

[1]We perform the present study in the basis where the charged lepton mass matrix is assumed to be diagonal, which can be ensured by a specific choice of charges and representations of the right charged lepton fields; thus, the lepton mixing matrix is that of the neutrino. However, the very details are beyond the scope of this paper.

Superfields	(L_e, L_μ)	L_τ	(N_1^c, N_2^c)	N_3^c	H_u	H_d	S	F	Φ	Ω	Γ
$\mathbb{D}_4$	2	$1_{+,-}$	2	$1_{+,+}$	$1_{+,-}$	$1_{+,-}$	$1_{+,+}$	2	$1_{+,-}$	$1_{-,-}$	2

Table 1: Lepton and Scalar superfields and their quantum numbers under the flavor group.

fields transform non trivially and have the same Z_3 assignment, which is either ω or ω^2. Thus, the superpotential involved is strongly limited to cubic couplings[2].

3. Neutrino masses and mixing

We start this section by using the particle assignments shown in Table 1 to construct a $\mathbb{D}_4$-invariant neutrino superpotential, which is expressed in our case, as follows

$$\begin{aligned} \mathcal{W}_\nu &= \lambda_1 H_u L_{e,\mu} N_{1,2}^c + \lambda_2 H_u L_\tau N_3^c + \lambda_3 S N_3^c N_3^c + \lambda_4 S N_{1,2}^c N_{1,2}^c + \lambda_5 N_3^c N_{1,2}^c F \\ &\quad + \lambda_6 N_{1,2}^c N_{1,2}^c \Phi + \lambda_7 N_{1,2}^c N_{1,2}^c \Omega + \lambda_8 N_3^c N_{1,2}^c \Gamma \end{aligned} \tag{4}$$

where $\lambda_{i=1,\ldots,8}$ are the Yukawa coupling constants. The first two terms in $\mathcal{W}_\nu$ are Dirac mass terms while the remaining couplings are Majorana mass terms. Using the tensor product decomposition of $\mathbb{D}_4$ irreducible representations given above, and considering the typical VEVs alignment for the scalar fields as

$$\begin{array}{c} \langle H_u \rangle = \upsilon_u \quad , \quad \langle S \rangle = \upsilon_S \quad , \quad \langle \Phi \rangle = \upsilon_\Phi e^{i\phi} \quad , \quad \langle \Omega \rangle = \upsilon_\Omega \\ \langle \Gamma \rangle = (\upsilon_\Gamma, 0) \quad , \quad \langle F \rangle = (\upsilon_F, \upsilon_F), \end{array} \tag{5}$$

where the various VEVs υ_i are obtained solving the coupled differential equations $\frac{\partial V}{\partial \upsilon_i} = 0$, with V is the scalar potential. The mass matrices of the Dirac and Majorana neutrinos are expressed as follows, respectively

$$M_D = \lambda_1 \upsilon_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad M_R = \begin{pmatrix} x & y+z & y \\ y+z & 0 & y+x \\ y & y+x & z \end{pmatrix} \tag{6}$$

with $x = 2\lambda_6 \upsilon_\Phi e^{i\phi}$, $y = \lambda_5 \upsilon_F$, and $z = \lambda_3 \upsilon_S$. Note that here the VEV of Φ provides the unique source of CP violation as all other VEVs are assumed —without loss of generality— to be real. Obtaining magic neutrino mass matrix and trimaximal mixing required the following constraints[3], $\lambda_6 \upsilon_\Phi = \lambda_7 \upsilon_\Omega = \frac{\lambda_8}{2} \upsilon_\Gamma$, $\lambda_4 \upsilon_S = \lambda_3 \upsilon_S + \lambda_5 \upsilon_F$ and $\lambda_1 = \lambda_2$. Accordingly, M_R is diagonalized by $U_{TM_2}^*$, leading to the following eigenvalues[4]

$$\begin{aligned} M_1/|x| &= \sqrt{k^2 + k\epsilon - (2k+\epsilon)\cos\phi + 1}, \quad M_3/|x| = \sqrt{k^2 - k\epsilon + (2k-\epsilon)\cos\phi + 1} \\ M_2/|x| &= \sqrt{4k^2 + 4k\epsilon + (4k+2\epsilon)\cos\phi + 1}, \end{aligned} \tag{7}$$

[2] The well-known μ-term $\mu H_u H_d$ which undergoes a naturalness problem [3] is excluded by the Z_3 symmetry. The terms $\left(N_1^c, N_2^c\right)^2$ and $\left(N_3^c\right)^2$ are also forbidden.

[3] Assuming these constraints however has to be regarded as fine-tuning.

[4] The obtained eigenvalues are nonlinear in the deviation parameter ϵ; they may be treated perturbatively up to order $O(\epsilon^2)$.

respecting the subsequent conditions

$$\tan 2\theta = \frac{\sqrt{3}\epsilon\sqrt{k^2\cos^2\phi + \sin^2\phi}}{k\,(2-\epsilon\cos\phi)} \quad , \quad \tan\sigma = \frac{\tan\phi}{k}. \tag{8}$$

where $k = y/|x|$ and $\epsilon = z/|x|$. The light neutrino mass matrix is generated through type I seesaw formula $m_\nu = M_D^T M_R^{-1} M_D$, and therefore, we obtain

$$m_\nu = \frac{\lambda_1^2 \upsilon_u^2}{D}\begin{pmatrix} (x+y)^2 & -xy+yz-y^2+z^2 & -(y+z)(x+y) \\ -xy+yz-y^2+z^2 & -xz+y^2 & xy-yz+x^2-y^2 \\ -(y+z)(x+y) & xy-yz+x^2-y^2 & (y+z)^2 \end{pmatrix}, \tag{9}$$

with $D = (x+2y+z)\left(-xz+x^2-y^2+z^2\right)$. Thus, the three light neutrino masses are expressed as follows

$$m_1 = \frac{\lambda_0}{M_1}, \qquad m_2 = \frac{\lambda_0}{M_2}, \qquad m_3 = \frac{\lambda_0}{M_3}, \tag{10}$$

where $\lambda_0 = \frac{\lambda_1^2 \upsilon_u^2}{D}\,|x|$ and M_i are given in eq. (7). Furthermore, the mixing angles are given by

$$\sin^2\theta_{12} = \frac{1}{3-2\sin^2\theta} \quad , \quad \sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \quad , \quad \sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{3}\sin 2\theta}{2\left(3-\sin^2\theta\right)}\cos\sigma. \tag{11}$$

We now perform a numerical analysis of neutrino masses and mixing in the case of normal neutrino mass ordering[5]. For this purpose, we employ the current global analysis of the neutrino oscillation parameters given by NuFIT 5.0 at 3σ range [8]. Numericaly, using the second expression in eq. (11), we find that $0.1754 \lesssim \theta \lesssim 0.192$. Next, we plot in the left panel of Fig. 1 the correlation among the parameters k, ϵ and the arbitrary phase σ. Thus, the constrained ranges of the model

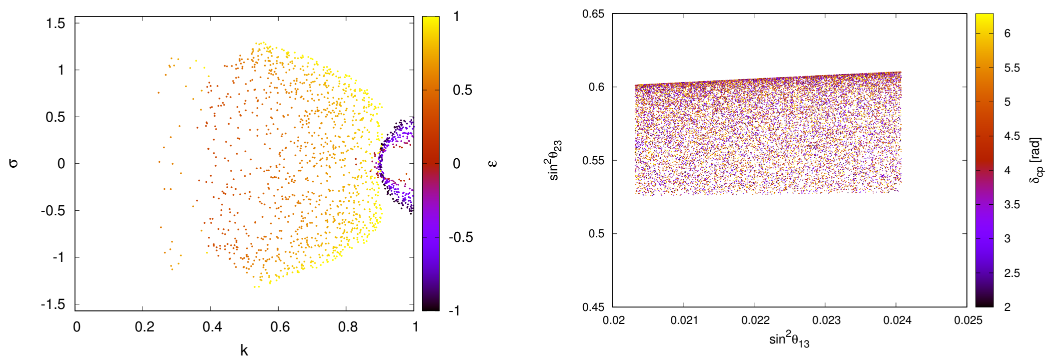

Figure 1: Left: Correlation plot among the parameters k, ϵ and σ. Right: the atmospheric angle as a function of the reactor angle with Dirac CP-violating phase in the palette.

parameters are given by

$$\begin{aligned} \sigma &\in [-0.418\pi, 411\pi] \quad , \quad k \in [0.249, 0.996]\,, \\ \epsilon &\in [-0.996, -0.828] \cup [-0.394, -0.246] \cup [0.36, 0.99]\,; \end{aligned} \tag{12}$$

[5]Recent experiments with observables more sensitive to normal mass ordering are more conceivable to give good results [7]

while the interval where ϕ varies can be easily derived using eq. (8), and thus we obtain $\phi \in [-0.245\pi, 0.411\pi]$. From the right panel of Fig. 1 we have

$$\sin^2\theta_{23} \in [0.526 - 0.612] \quad , \quad \sin^2\theta_{13} \in [0.02044 - 0.02437]. \tag{13}$$

As a result, the model allows only higher octant of θ_{23} (i.e. $\theta_{23} > 45°$). Notice that the recent global analysis has dismissed the maximal mixing value ($\theta_{23} = 45°$) [7], hence suggests towards the non-maximality of θ_{23}.

4. Neutrino Phenomenology

In this part of the analysis, we consider the neutrino phenomenology of the present model in light of the recent non-oscillation results from cosmology, and single and double beta decay. Among cosmological observations, cosmic microwave background (CMB) is the ultimate compelling probe for neutrino masses. The latest data from the Planck collaboration constrain the sum of absolute neutrino masses to be less than 0.12 eV at 95% confidence [9].

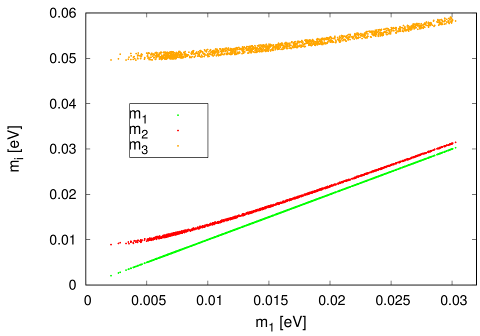

Figure 2: The absolute neutrino masses as a function of lightest neutrino mass.

In Fig. 2, we show the correlations of the three neutrino masses $m_{i=1,2,3}$ versus the lightest neutrino mass m_1 where we find (in meV)

$$2.07 \lesssim m_1 \lesssim 30.11 \quad , \quad 8.92 \lesssim m_2 \lesssim 30.86 \quad , \quad 49.6 \lesssim m_3 \lesssim 57.03. \tag{14}$$

Hence, the sum of the three neutrino masses is predicted to be restrained in the following range

$$0.06059 \text{ eV} \lesssim \sum m_i \lesssim 0.118 \text{ eV}. \tag{15}$$

The predicted lower bound on the sum of neutrino masses is about ~ 0.06 eV, and thus, it could be probed at upcoming experiments like CORE+BAO expecting bound on the sum of neutrino masses around 0.06 eV [10].

Apart from cosmology, the observation of the absolute values of neutrino masses is done through the measurements of the effective electron neutrino mass in the β-experiment and it is expressed in the light of this model as

$$m_{\nu_e} = \left[\frac{1}{3}\left(3m_1^2 + \Delta m_{21}^2 + 2\Delta m_{31}^2 \sin^2\theta\right)\right]^{1/2} \tag{16}$$

The KATRIN experiment has latterly upgraded the most stringent upper bound on the effective neutrino mass to $m_{\nu_e} < 1.1$ eV at 90% C.L. using tritium beta decay [11], and eventually aims at a sensitivity of 0.2 eV at 90% C.L. [12]. This bound would challenge the cosmological boundary on the sum of the neutrino masses of 0.12 eV. It is of interest to see in Fig. 3 that the allowed region

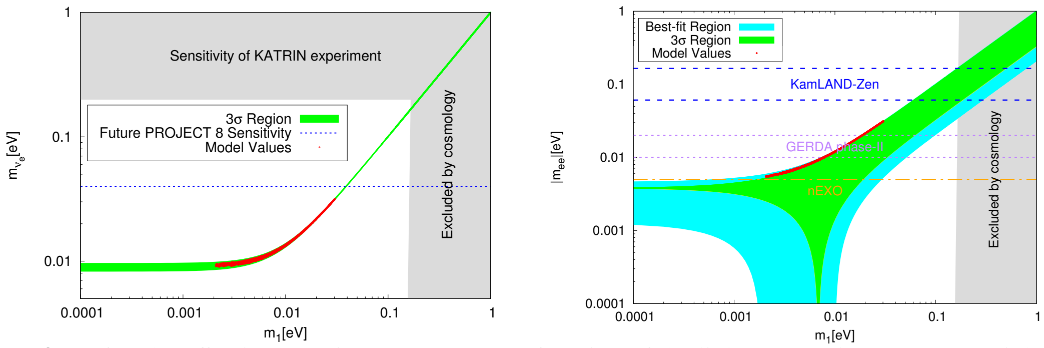

Figure 3: Left: The effective neutrino mass m_{ν_e} as a function of the lightest neutrino mass m_1. Right: The effective Majorana mass $|m_{ee}|$ as a function of the lightest neutrino mass m_1.

of m_{ν_e} $(9 - 32.5$ meV) is below the sensitivity of the KATRIN experiment and that it is therefore not possible to examine it in the near future. On the other hand, the Project 8 collaboration has developed a technique to measure the electron spectrum using atomic tritium with a sensitivity as low as 40 meV [13], and thus, it has the potential to probe our model predictions.

We recall that signals of neutrinoless double beta decay ($0\nu\beta\beta$) are able to clarify many fundamental aspects of neutrino physics. Nevertheless, the question of the order of the neutrino masses is not yet thoroughly resolved and experiments on a double beta decay can contribute to its solution. In our framework, the effective Majorana mass $|m_{ee}|$ is expressed as

$$|m_{ee}| = \frac{1}{3}\left|2m_1 \cos^2\theta + m_2 e^{i\alpha_{21}} + 2m_3 \sin^2\theta e^{i(2\sigma+\alpha_{31})}\right|. \tag{17}$$

where α_{21} and α_{31} are the Majorana CP-violating phases. The experiments on $0\nu\beta\beta$ attempt to derive $|m_{ee}|$, and especially to prove the non-conservation of the leptonic number. The most recent limits on $|m_{ee}|$ from the KamLAND-Zen and GERDA experiments are given respectively in the ranges $61 - 165$ meV [14] and $104 - 228$ meV [15]. From the right panel of Fig. 3, the predicted range on $|m_{ee}|$ is given in the range ($\sim 5 - 40$ meV). The obtained range of $|m_{ee}|$ is unattainable by the current sensitivities cited above; however, this range is already within the projected sensitivities of the next-generation experiments such as GERDA Phase-II ($\sim 10-20$ meV) [16], CUPID ($\sim 6-17$ meV) [17], and nEXO [18] which aims to reach a sensitivity of (~ 5 meV).

5. Conclusion

In the present framework, we have briefly investigated the neutrino phenomenology in the context of the NMSSM extended by three right-handed neutrinos, with the assistance of the non-abelian $\mathbb{D}_4$ discrete flavor symmetry. We have shown that this extension incorporates non-zero neutrino masses and a non-zero reactor mixing angle θ_{13}; while the atmospheric mixing angle θ_{23} predicted in the higher octant.

Moreover, we have reviewed and processed some observations about the absolute neutrino mass using non-oscillations data collected from cosmology, beta decay, and neutrinoless double beta decay constraints. In particular, the lightest neutrino mass found corresponds to 2.07 meV$\lesssim$ $m_1 \lesssim$ 30.11 meV, while the planned Project 8 experiment with a sensitivity of $m_{\nu_e} \sim 40$ meV has the potential to probe the model prediction. In addition, a large part with considerable enhancement in the predicted values for $|m_{ee}|$ can be probed by the upcoming $0\nu\beta\beta$ experiments like GERDA Phase-II, CUPID and nEXO.

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