

## Model-independent test of T violation in neutrino oscillations

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We propose a method to establish time reversal symmetry violation at future neutrino oscillation experiments in a largely model-independent way. We introduce a general parametrization of flavour transition probabilities which holds under weak assumptions and covers a large class of new-physics scenarios. This can be used to search for the presence of T-odd components in the transition probabilities by comparing data at different baselines but at the same neutrino energies. We show that this test can be performed already with experiments at three different baselines and might be feasible with experiments under preparation/consideration.

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## 1. Introduction

The violation of time reversal ( $T$ ) and charge-parity ( $CP$ ) symmetries are central topics in particle physics.  $CP$  violation ( $CPV$ ) is one of the necessary conditions to generate a matter-antimatter asymmetry in the early Universe [1], and under the well founded assumption of  $CPT$  conservation,  $CPV$  is equivalent to  $T$  violation ( $TV$ ). A particularly active field is the search for  $CPV$  in neutrino oscillations [2–4]. Unfortunately, the experimental signature is rather indirect, and it is not possible to construct model-independent  $CP$ -asymmetric observables in neutrino oscillation experiments. This is related to the fundamental obstacle that experiments and detectors are made out of matter (and not antimatter). Moreover, the passage of the neutrino beam through Earth matter introduces environmental  $CPV$  due to matter effects [5].

The standard approach to this problem is to perform a model-dependent fit to data. This involves the assumptions that neutrino production, detection and propagation is fully understood in terms of Standard Model ( $SM$ ) interactions, that neutrino mixing is unitary, and only the three  $SM$  neutrino flavours exist. In this case oscillation physics can be parametrized in terms of a unitary  $3 \times 3$  lepton-mixing matrix [6, 7] and two neutrino mass-squared differences.  $CPV$  is then described by a complex phase  $\delta$  in the mixing matrix [2, 8] which can be fitted against data. “Observation of  $CPV$ ” is considered equivalent to establishing that  $\delta$  is different from 0 and  $\pi$  at a certain confidence level. Within this restricted framework, current data start to provide first indications of preferred regions for the parameter  $\delta$ .

Large activity is devoted to study the impact of non-standard scenarios on the search for  $CPV$  in neutrino oscillations. Examples are non-unitary mixing, non-standard neutrino interactions, or the presence of sterile neutrinos. Typically one adopts a specific parameterization of new-physics and again performs a parametric fit in the extended model. Our aim in this letter is to go a step beyond such approaches and develop a largely model-independent test, covering a wide class of non-standard scenarios.

## 2. The $TV$ test

We focus on the experimentally relevant  $\nu_\mu \rightarrow \nu_\mu$  disappearance and  $\nu_\mu \rightarrow \nu_e$  appearance channels, adopting the following assumptions:

(i) Propagation of the three  $SM$  neutrino states is described by a hermitian Hamiltonian  $H(E, x)$ , which depends on neutrino energy  $E$  and in general on the matter density at the position  $x$  along the neutrino path.

(ii) We assume that for the experiments of interest, medium effects can be described to sufficient accuracy by a constant matter density which is approximately the same for all considered experiments. This is a good approximation for experiments with baselines less than several 1000 km [9, 10].

(iii) We allow for arbitrary (non-unitary) mixing of the energy eigenstates  $\nu_i$  with the flavour states  $\nu_\alpha$  relevant for detection and production,

$$|\nu_\alpha\rangle = \sum_{i=1}^3 N_{\alpha i}^{\text{prod,det}} |\nu_i\rangle. \quad (1)$$

We make no specific assumption on the complex coefficients  $N_{\alpha i}$ : we allow them to be arbitrary (sufficiently smooth) functions of energy, and they can be different for neutrino production and detection. But we do assume that they are the same for different experiments (at the same energy).

(iv) We impose that the oscillation frequencies  $\omega_{ij}$  deviate only weakly from the ones corresponding to the standard three-flavour oscillation case.

Under the (rather general) assumptions (i), (ii), (iii), which cover a large class of new physics scenarios, the corresponding probabilities are obtained as [11, 12]

$$P_{\mu\alpha} = \sum_i |c_i^\alpha|^2 + 2 \sum_{j<i} \text{Re}(c_i^\alpha c_j^{\alpha*}) \cos(\omega_{ij}L) - 2 \sum_{j<i} \text{Im}(c_i^\alpha c_j^{\alpha*}) \sin(\omega_{ij}L), \quad (2)$$

with  $c_i^\alpha \equiv (N_{\alpha i}^{\text{det}})^* N_{\mu i}^{\text{prod}}$  and  $\omega_{ij} \equiv \lambda_j - \lambda_i$ . The first line of Eq. (2) is invariant under  $T$ , whereas the second line is  $T$ -odd. Fundamental  $TV$  can be established by proving the presence of the  $L$ -odd term in the probability.

The strategy we propose to probe the  $L$ -odd terms is to measure the oscillation probability as a function of  $L$  at a fixed energy and check whether  $L$ -even terms are enough to describe the data or if  $TV$  is required. Under these conditions, the effective frequencies and mixings in the Hamiltonian are the same, and so the data at different baselines (but at the same energy) can be consistently combined. Notice that antineutrino data cannot be analyzed together with neutrino data, as their effective frequencies and mixings are in general different from the neutrino's.

In the absence of  $TV$ , all  $c_i^\alpha$  are real and the data points could be described by the  $L$ -even part of the oscillation probability. We define ( $c_i^\alpha$  real)

$$P_{\mu\alpha}^{\text{even}}(L, E; \theta) = \sum_i (c_i^\alpha)^2 + 2 \sum_{j<i} c_i^\alpha c_j^\alpha \cos(\omega_{ij}L). \quad (3)$$

For the two relevant channels, these probabilities depend on 8 parameters, which we collectively denote by  $\theta$ : 6 real coefficients  $c_i^\mu, c_i^e$  ( $i = 1, 2, 3$ ) and two independent  $\omega_{ij}$ , e.g.,  $\omega_{21}$  and  $\omega_{31}$ . We assume now that the probabilities  $P_{\mu\mu}$  and  $P_{\mu e}$  are measured at a fixed energy at several baselines  $L_b$ . We denote the corresponding measured values by  $p_b^{\text{dis}}$  and  $p_b^{\text{app}}$  with the uncertainties  $\sigma_b^{\text{dis}}$  and  $\sigma_b^{\text{app}}$ , respectively. Below we are going to assume that  $p_b^{\text{dis}}$  and  $p_b^{\text{app}}$  correspond to the values predicted by standard three-flavour neutrino ( $3\nu$ ) oscillations in matter.

We now ask the question if we can exclude the hypothesis of  $T$  conservation parametrised by Eq. (3), if the data correspond to  $3\nu$  oscillations *with*  $TV$ , i.e., for a  $CP$  phase  $\delta$  different from 0 or  $\pi$ . To this aim we construct the  $\chi^2$  function

$$\chi_{\text{even}}^2(E; \theta) = \sum_{b=1}^{N_L} \left[ \frac{P_{\mu\mu}^{\text{even}}(L_b, E; \theta) - p_b^{\text{dis}}}{\sigma_b^{\text{dis}}} \right]^2 + \sum_{b=1}^{N_L} \left[ \frac{P_{\mu e}^{\text{even}}(L_b, E; \theta) - p_b^{\text{app}}}{\sigma_b^{\text{app}}} \right]^2 + \left[ \frac{\Delta\tilde{m}_{21}^2(E) - 2E\omega_{21}}{\sigma_{21}} \right]^2. \quad (4)$$

The best-fit  $T$ -conserving model is obtained by considering  $\chi_{\text{min}}^2(E) = \min_{\theta} [\chi_{\text{even}}^2(E; \theta)]$ . We will take the value of  $\chi_{\text{min}}^2(E)$  as a rough indication of how strongly  $T$  conservation can be excluded by

data, and leave a more detailed statistical analysis for future work. The last term, associated to our assumption ( $i\nu$ ) of BSM effects being sub-leading, is a prior to ensure that the oscillation frequency  $\omega_{21}$  deviates only weakly from its standard  $3\nu$  value in matter—we estimate  $\sigma_{21} = 0.1\Delta\tilde{m}_{21}^2$ , and find that no similar term is required for  $\omega_{31}$ .

Considering that each baseline provides 2 data points (appearance and disappearance) and that the T-even model has 8 parameters, together with the prior term, it is clear that one would need more than 3 experiments at different baselines. Let us note, however, that our parameterization includes so-called zero-distance effects, so the near-detector(s) of long-baseline experiments provide already two data points at  $L \approx 0$  and effectively only 3 experiments are needed.

The crucial requirement, however, is sufficient overlap in neutrino energy. If experiments have overlapping energy ranges, we can combine information from different energies. However, to be completely model-independent, the minimization has to be done individually for each energy, since we do not want to make any assumptions about the energy dependence of the unknown new physics. This is an important difference to usual model-dependent analyses.

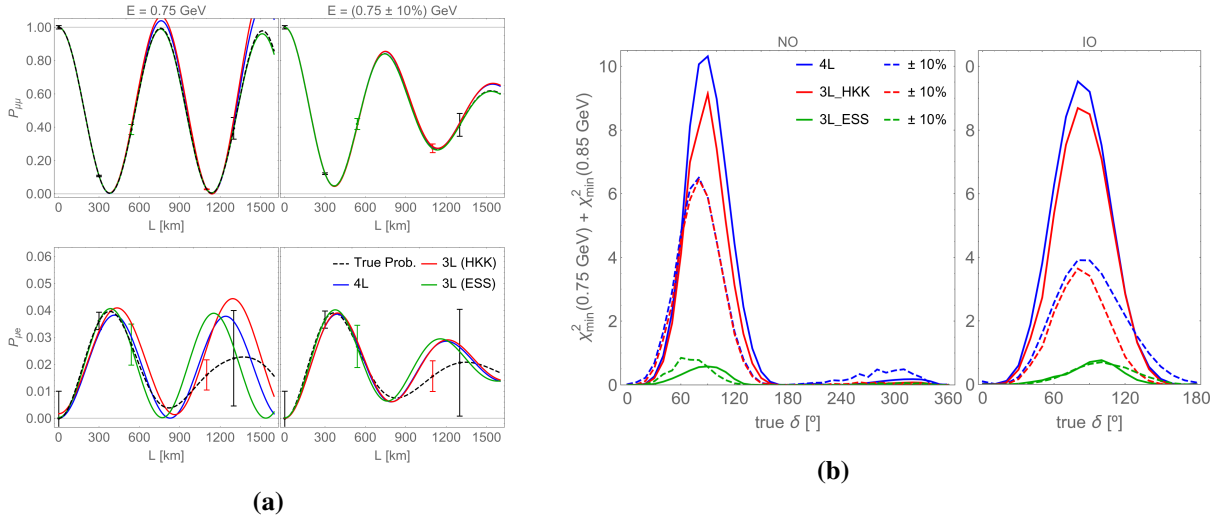
### 3. Realistic baselines and energies

We consider planned long-baseline accelerator experiments in order to see if such a test realistically can be carried out in the future: the DUNE project in USA ( $L = 1300$  km), T2HK in Japan ( $L = 295$  km), with the option of a second detector in Korea, T2HKK ( $L = 1100$  km), and a long-baseline experiment at the European Spallation Source in Sweden, ESS $\nu$ SB ( $L = 540$  km).

There is only limited overlap in energy with sufficient events, in particular between DUNE and HKK. In practice, we will see that only the two energy bins between 0.7 and 0.9 GeV provide relevant sensitivity. We note that the energy spectrum from the NO $\nu$ A experiment has no overlap with the T2K beam and therefore it cannot be used for this analysis. We use the expected number of events at these experiments to estimate the statistical uncertainties in Eq. (4) as  $\sigma_{br}/P^{\text{even}}(L_b, E_r) = \sqrt{S_{br} + B_{br}}/S_{br}$  at baseline  $b$  and energy bin  $r$ . We take the background events  $B_{br}$  directly from the experimental studies and estimate the number of signal events from the  $N_{br}$  assuming  $S_{br} = N_{br} \times P^{\text{even}}(L_b, E_r; \theta)/P^{3\nu}(L_b, E_r)$ . For the near detector data points, we assume the standard  $P_{\alpha\beta}(L \rightarrow 0) = \delta_{\alpha\beta}$  with  $\sigma = 0.01$ .

In Fig. 1a we show the data points for the appearance and disappearance probabilities as a function of the baseline for the 0.7–0.8 GeV energy bin. We can see that the disappearance data points essentially fix the oscillation frequency, whereas the appearance data are crucial for the TV test. We find that no satisfactory  $L$ -even fit is possible for the 4L and 3L (HKK) combinations at this energy. The essential information is obtained from the relative heights of the first and second appearance oscillation peaks.

In Fig. 1b we show the summed  $\chi_{\text{min}}^2$  contributions from the 0.75 and 0.85 GeV bins as a function of the value of the  $3\nu$  CP phase  $\delta$  assumed to calculate the “data” to which the T-even model is fitted. In addition to the features mentioned above, we see from Fig. 1b that the test is sensitive only to  $\delta \simeq 90^\circ$ , whereas no sensitivity appears around  $270^\circ$ . This behaviour stems from the enhancement of the second oscillation maximum in the latter case (contrary to its suppression around  $90^\circ$ ), which produces a much more oscillatory-like  $L$ -dependence that can be effectively fitted with an  $L$ -even function.



**Figure 1:** (a) Data points for the disappearance (top) and appearance (bottom) channels at the baselines of DUNE, T2HK, T2HKK, ESS $\nu$ SB and a near detector location for  $E = 0.75$  GeV and true  $\delta = 90^\circ$ , and the corresponding oscillation probability is shown as black-dashed. The solid curves show the best-fit model-independent  $L$ -even probabilities using all baselines (4L, blue), DUNE + T2HK + T2HKK (3L (HKK), red), or DUNE + T2HK + ESS $\nu$ SB (3L (ESS), green). Left (right) panels are without (with) the smearing due to a 10% energy resolution. (b)  $\chi^2_{\min}$  summed for the energy bins around 0.75 and 0.85 GeV, with perfect (solid) or 10% (dashed) energy resolution. Neutrino data is assumed, with normal (inverted) mass ordering for the left (right) panel.

The results for inverted mass ordering (IO) are qualitatively similar to the one from normal ordering (for IO we show only the relevant range of  $\delta$  in Fig. 1b). If antineutrino data are assumed (instead of neutrino data) the result is roughly obtained for  $\delta \rightarrow 2\pi - \delta$  in Fig. 1b, with highest sensitivity around  $\delta \simeq 270^\circ$ . This is to be expected, since antineutrino oscillation probabilities are obtained from the neutrino ones by replacing  $\delta \rightarrow -\delta$  (in addition to the sign-flip of the matter potential). Hence, in order to cover all  $T$ -violating values of  $\delta$ , data for neutrinos and antineutrinos are necessary.

#### 4. Summary

We propose a largely model-independent test to search for  $T$  violation in neutrino oscillations by comparing transition probabilities at the same energy and different baselines. The test can be done under rather general assumptions covering a wide range of new physics scenarios. Within some modest assumptions, the test can be performed already with experiments at three different baselines plus near detectors. The crucial requirements are sufficient event numbers in the neutrino energy overlap region between the experiments and good neutrino energy reconstruction [13, 14]. Our estimates show that with the planned long-baseline experiments DUNE, T2HK, and T2HKK, this test can be potentially carried out. We stress that a detector at the Tokai-Korea baseline is required in addition to DUNE and T2HK. Some optimization studies, especially in the low-energy region of the DUNE and high-energy region of the T2HKK beams, may be required. The results

presented here warrant more detailed sensitivity studies based on realistic experiment simulations and statistical analyses, which we leave for future work.

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