PROCEEDINGS OF SCIENCE

Symmetries of spin amplitudes: applications for factorization and Monte Carlo solutions

Zbigniew Was[a,*]

[a],*Institute of Nuclear Physics, Polish Academy of Sciences*
Radzikowskiego 152, Krakow, 31-342 Poland

E-mail: z.was@cern.ch

For phenomenology of High Energy Physics experiments, Known Physics and New Physics phenomena, predictions need to be available. One has to take into account results of diverse research activities. On experiments side this includes control of detector acceptance and its response details, also observable definitions and different types of background subtractions. On theoretical side one needs to keep in mind results on: higher order matrix elements calculations, structure functions, evolution kernels, Monte Carlo and/or semi-analytic integration algorithms, algebraic manipulation programs.
I will not address foundations of Yennie-Frautchi-Suura exclusive exponentiation, or of factorizations used in QCD. That would be too broad a subject, obscuring the main purpose of my talk. I will concentrate on the investigation of spin amplitude gauge invariant parts and how they were useful in case of my work.

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1. introduction

Symmetry is one of the principles that define the foundation of physics. We all recall, already from early physics courses, lectures on the Noether theorem. Later, we could realize that every subgroup of interaction symmetry may open the gate for simplification or for systematization of results. For example, at LEP-time, and later at Tevatron and even in LHC phenomenology, one separately performed calculations for electroweak and strong interactions. Even work for QED could have been performed alone. This simplified the burden for calculations a lot. Expansions, where mass terms are introduced into masless QCD calculations as additional (external) interactions could be mentioned in this context too. Needless to say, QED as a building block for the Electroweak sector, enabled the introduction of large higher order terms without a need of multi-loop full electroweak calculations. Conformal symmetry helps to identify parts of QED Matrix Elements, which can be obtained by iteration to all orders. This became a starting point for analytic and Monte Carlo methods. These results can be corrected perturbatively order by order, thus improving convergence significantly. If solution is implemented in Monte Carlo, detector acceptance effects for multiparticle final states, can be studied already at the lowest perturbative order.

In every practical case one needs to decide what should be taken into account for the required precision, and then, how these distinct components can be combined. In my talk, I will rely on projects I participated in. I will concentrate on matrix elements originating from Field Theory calculations. What was needed for their actual implementation in phenomenology programs (Monte Carlos) and semi analytical and how properties originating from symmetries were used. These aspects were never at the center of my activities, but may be they are, nonetheless, worth to review.

In particular, I will address how to inject, the effects of New Physics into complicated precision simulation programs. Note that precision of New Physics implementation does not need to be high, but precision of Standard Model (interfering) background cannot be compromised. Some properties of spin amplitudes are useful in that effort, in particular their separation into parts; often gauge invariant and when separation is valid all over the phase space.

Of course, most of these issues, reviewed well e.g. in [1], go far beyond the scope of my talk. Here I concentrate on practical aspects of spin amplitudes calculation and their implementation into numerical programs. My talk is organized as follows. Section 1 is devoted to simple features which enable one to identify eikonal factors in QED amplitudes, and later consecutive perturbative corrections for eikonal level amplitudes. These perturbative corrections break conformal symmetry of eikonal terms, but otherwise remain individually gauge invariant too. In Section 2 an indication is given that division of spin amplitudes into gauge invariant parts, can be extended to QCD and even to Scalar QED. Some of these gauge invariant parts can be associated with terms, present in QCD factorization schemes, such as DGLAP or BFKL. Section 3 is on how to implement extra interactions into predictions of Standard Model (or strong interactions only). In particular, how to implement electroweak interactions into picture based on strong interaction only, or of New Physics into predictions (Monte Carlo programs) of Standard Model. Section 4, Summary, closes the presentation.

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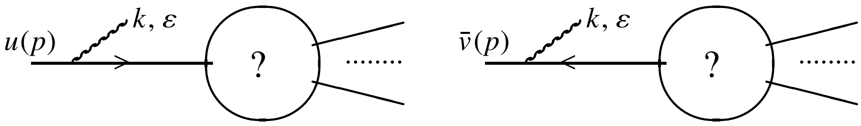

a) Incoming electron b) Incoming positron

Figure 1: *Feynman diagrams for photon emission in initial state from electron and positron respectively. Dots represent all other fields entering amplitude (initial or final). Note that in case of positron arrow points in the opposite direction, even though it is also an initial state particle.*

2. Spin amplitudes for exclusive exponentiation.

Exponentiation very useful, in particular it improves convergence of QED perturbative expansion. It has been long known [2] and the concept was instrumental in the design of a broad spectrum of Monte Carlo programs [3–10]. My experience with spin amplitudes originates from work and discussions on these projects. Let me quote, from my old radiative correction lectures [11], where details and notation can be found too. The Feynman diagram for a photon attached to external fermion line, see Fig. 1, left-hand side, can be expressed as

$$\mathcal{M} = \; ... \frac{\not p - \not k + m}{-2pk} e \not\varepsilon u(p,s). \tag{1}$$

The $\sim \not k$ parts of contributions from left- and right-hand side plot are gauge invariant. They lead to real photon contribution to β_1 of Yennie-Frautchi-Suura exponentiation. Longitudinal contribution from $\not\varepsilon$ trivially cancels out, because it stands next to $\sim \not k$, thus gauge invariance is assured. The remaining gauge dependent part of Eq. (1) read

$$\mathcal{M} = -e\frac{\varepsilon p}{pk} ..., \tag{2}$$

but once combined with analogous contribution from right-hand side diagram gauge invariant eikonal factor is obtained. Note factorization of $... = \mathcal{M}_B$, the Born-like (or lower perturbative level) amplitude. Energy-momentum conservation constraint is essential if formula is to be used outside soft photon limit. For the $\mathcal{M}_B$ definition, the extrapolation from lower order perturbation expression is needed to compensate for energy momentum non-conservation due to the photon of momentum k. This can be cumbersome if strong kinematic dependence is present, in case of non-negligible k. The extrapolation of $\mathcal{M}_B$ definition can be specially complicated, if in diagrams contributing to $\mathcal{M}_B$ exchange of bosons both in s- and t-channel is present.

Nonetheless in many cases Formula (2) iterates nicely. If partial integration is performed, Formula (2) contributes to double logarithms, thus to numerically largest radiative corrections for integrated cross sections. After extension to all orders, if care of energy momentum conservation is taken, Formula (2) encapsulate to all orders, terms of real emission contributions, simultaneously singular in collinear and infrared limits. When all details are taken into account, this provides a non trivial, powerful scheme, developed for Monte Carlo simulations. It is explained in [6]. There, complete second order, double emission amplitudes are used.

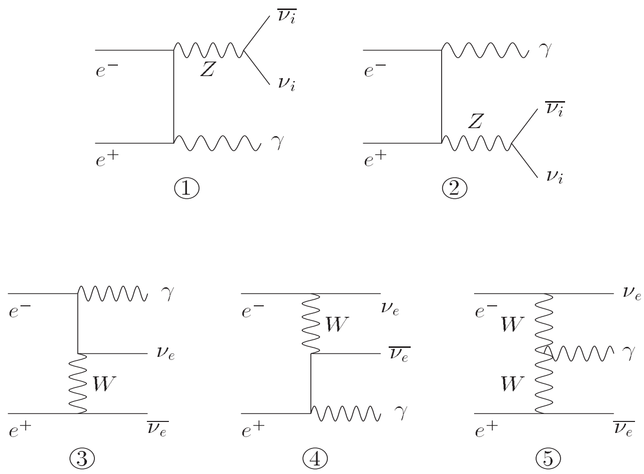

Figure 2: Feynman diagrams for $e^+e^- \to \nu_e\bar{\nu}_e\gamma$ process.

Things complicate if t-channel exchanges appear, like in case of $e^+e^- \to \nu_e\bar{\nu}_e\gamma$ processes, see Fig 2. Then, t-channel W exchange necessitates diagrams (like plot 5) of emission from t-channel W to ensure gauge invariance. Fortunately, because W is massive, this does not bring new singular terms. Expansion with respect to contact interaction is possible, corrections are not excessive numerically. The structure of singularities remains the same as for s-channel interactions.

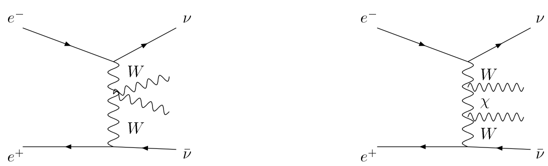

Figure 3: Feynman diagrams of four boson coupling and coupling for non-physical χ field $e^+e^- \to \nu_e\bar{\nu}_e\gamma$ amplitudes.

For double photon emission amplitudes, in order to preserve gauge invariance seemingly non-QED diagrams for $e^+e^- \to \nu_e\bar{\nu}_e\gamma$ process need to be taken into account, including four-boson interaction and non-physical χ field on internal lines. Study of spin amplitude separation into gauge invariant parts was documented in [12]. It was beneficial for Monte Carlo.

Surprisingly, gauge invariance alone was enough to localize parts that were responsible for the most singular parts of the amplitudes, which are also the origin of leading (next to leading etc.) logarithms for integrated quantities (cross sections or asymmetries).

As in the single emission case, contact interaction was useful to identify terms essential for exponentiation of s-channel exchange processes. New, t-channel dependent corrections turned out not large numerically. That is why, `KKMC` Monte Carlo [13] of s-channel exchange processes could be easily extended to the case when t-channel exchange is present too.

From the perspective of principles, one should keep in mind that to preserve gauge invariance of these essentially QED amplitudes, four boson interaction and nonphysical χ field contributions had to be taken into account. In this case we went slightly, outside pure QED framework where exponentiation is granted. By explicit calculation we were able to check extension of the approach beyond pure QED at least for order α and α^2 terms, necessary for required precision.

3. Non QED bremsstrahlung processes.

Gauge invariance can be useful for the preparation of spin amplitudes for radiative corrections and for construction of Monte Carlo programs, also for the decays of scalars. Then amplitudes are calculated from scalar QED [14, 15] and form-factors may be needed. For QCD [16], amplitudes can be divided into gauge invariant parts as well. Not all of these parts are of course as those of QED exponentiation. That should be expected. Amplitude parts can be useful for other purposes, for example for identification of terms corresponding to BFKL, DGLAP or CCFM of QCD, see Section 5 of Ref. [16].

One may thus expect that gauge invariance and Lorentz invariance of spin amplitude parts, are generally useful, in majority of renormalized field theories calculations, or even for effective theories such as scalar QED. In all quoted references, terms of similar analytic form appeared. This seems to be the consequence of Lorentz group invariance. Amplitudes can be understood as elements of its reducible representations: products of the ones for spin 0, 1/2 or 1 fields.

Such considerations are helpful to understand foundations of factorization in QCD and QED. Even though it is not the purpose of the talk, lest us point to some references which were helpful in the design of Monte Carlo programs. In Ref. [17] it was shown, that cross-section for $e^+e^- \to l^+l^-$ can be written in a way that Born-like factor is explicit, and no approximation is needed. In [18] it was argued that similar property holds for QCD as well. The first terms outside such scheme are $\sim \alpha_s^2 \simeq 0.01$: no logarithmic enhancements are present. It was argued already in [19], that electroweak interaction parts can be factorized in a form of spherical harmonics. These results were inspiring for works [20, 21] on the validation of `TauSpinner` the tool introducing, with event weights, electroweak corrections into high energy pp collisions event samples.

This broad spectrum of applications resulting implicitly from properties of spin amplitudes of QCD is somewhat out of main purpose of our presentation. Instead, let us turn our attention back to interaction of leptons. Investigation of spin amplitudes can be performed for the processes where instead of QED bremsstrahlung, emission of addition lepton pairs was introduced. That was behind work for pair emissions and `Photos` Monte Carlo [22]. There, dominant part of spin amplitude for $\gamma^* \to 4l$ process was identified again as gauge invariant part of the whole amplitude. It was then installed into the program, insuring that implementation was valid all over the four-lepton phase-space. Approximations were introduced, but at the same time framework for tests and extensions beyond soft pair emissions (if needed), was prepared and exploited later in Refs. [23, 24].

Let us comment now on phase-space parametrisations. In all mentioned above solutions prepared for various Monte Carlo programs, parametrisations were exact and explicit. The full phase-space coverage was assured. In case of `photos` Monte Carlo, this was the case since Ref. [22].

4. Separation into parts and approximate amplitudes.

Let us briefly recall a scheme of the work, in case when spin amplitudes are too complicated for easy identification of dominant gauge invariant parts and separation of remain analytically clearly defined rest. The strategy for the implementation of lepton pair emission into Monte Carlo simulation algorithm, can be explained in steps:

- Analyze tree level amplitudes for the four fermion final states of the Z boson (Z/γ^* mixed state) decay to identify its numerical most important parts.
- Prepare and run parallel simulation where tree-level four-fermion final-states are simulated without any approximation.
- With this parallel simulation, we can check how numerically significant is the approximation when these most important parts are used (all over the phase-space).
- The advantage of developed approximation should be, that it can define kernel for iterative solution. For that purpose and for optimization, several attempts may need to be performed.
- Once kernel for pairs emissions is ready (and found to work well in case when it is iterated), it can be checked again, first against simulations with exact matrix elements, later against simulations where dominant parts of the exact amplitudes were used only.
- The simplified kernel can then be fed into Monte Carlo. It can be combined (iterations can be mixed) with the ones for bremsstrahlung photons, as it was done for `Photos` Monte Carlo for generation of bremsstrahlung in decay of particles or resonances.
- Finally, it can be verified if kernel and the corresponding algorithm, work with e.g. `KKMC` generated events. Then, pair emission algorithm, does not need to form an integral part of the whole program, but is used to modify events of previous simulation steps. That means, there is no need to modify the code for the programs like `KKMC`.

Such approach can in general be justified for implementation of relatively small effect which contribute to simulated sample at the permille level or so. Accuracy for implementation of such relatively small effect is usually not demanding. On the other hand, its introduction should not compromise precision of the previously installed effects in large projects, e.g. in `KKMC`. These requirements are met for additional lepton pair emission originating from QED interactions.

Recently, the approach for introduction into `KKMC` simulated events, lepton pairs was extended to work for intermediate states, such as dark photons or exotic light scalars. Previously, pairs were to originate from QED processes only. Again, first dominant part of the amplitude for four fermion final states had to be localized and checked with other Monte Carlo. In our case [25] `MadGraph` [26] was used. Fortunately, even if it was now not granted by any general principle, these dominant parts could have factorized (or separable) form. Previously developed algorithms could be thus used with minor adjustments only. The final form of the kernel was optimized with the help of educated guesses exploiting the form of Altarelli-Parisi kernels for scalars. The differences with respect to `MadGraph` generated sample were evaluated with the help of all possible invariant masses constructed from final state momenta. For the purpose of comparison and numerical evaluation of differences, `MC-tester` [27] was used.

Such simulation solutions should be sufficient, because for still to be observed New Physics signatures like $e^+e^- \to \tau^+\tau^- dark\ scalar$, with τ decays taken into account, precision requirements are not very demanding. Results of implementation tests are encouraging, see Fig. 4. The two plots were selected from Ref. [25].

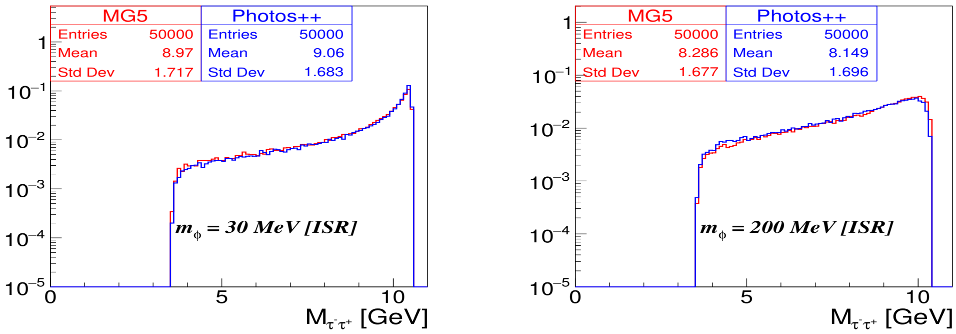

Figure 4: Belle 2 center of mass energies $e^-e^+ \to \tau^-\tau^+\phi_{\text{Dark Scalar}}(\to e^-e^+)$. Case of dark scalar of 30 and 200 GeV. Simulation of `KKMC+Photos` is compared with the one based on `MadGraph`.

5. Summary

Strategies for the adaptation of spin amplitudes and their gauge invariant parts for phenomenology programs were reviewed. Presentation started from a rigorous case of QED, where formal framework is available. Attention was later moved to cases of QCD, scalar QED applications and finally to New Physics signatures, where precision of introduced additional effects is less important than preservation of previously developed basic Standard Model calculation schemes.

We have followed with the discussion of results for final states with additional lepton pairs were first implemented for QED bremsstrahlung-like production. Later this work was extended towards New Physics simulations as well. The first practical examples of dark photon or light scalar production in association with τ-lepton pairs production at Belle 2 energies were presented. Also in this case separation of amplitudes into parts was useful.

My presentation was brief, many details and proofs were omitted. As a consequence, without checking the references, formal aspects of the approaches may remain blurred (at the best). Nonetheless, I hope, that main message of my presentation can be deciphered and it can serve as an invitation for further reading.

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