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Chiral oscillations in three-flavor neutrino mixing

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Massive particles whose dynamics are described by the free Dirac equation can oscillate between left and right-handed chiral states, undergoing chiral oscillations. The phenomenon is prominent for non-relativistic particles, and can yield a depletion on the expected measured flux of cosmic neutrinos and a modification of quantum correlations encoded in lepton-antineutrino pairs. In this context, the interplay between chiral and flavor oscillations plays an important role in the description of neutrino flavor dynamics. In this paper, we extend previous results and obtain flavor oscillation formulas including chiral oscillations for N flavor mixing. We consider the general case of mixing that can distinguish between left and right-handed states and derive the flavor oscillation probabilities for Dirac and Majorana neutrinos within the bispinor formalism. We show that, for three flavors, oscillation probabilities between chiral left and right-handed flavor states allows for distinguishing between Dirac and Majorana neutrinos in the presence of additional CP-violation Majorana phases. To summarize, our work addresses to phenomenologically accessible chiral oscillation effects on non-relativistic neutrino dynamics and on quantum correlations encoded in flavor states.

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1. Introduction

Neutrino flavor oscillations play a prominent role in different areas of particle physics, from the explanation of the solar neutrino problem to implications for cosmological models [1, 2]. Flavor oscillations can be modelled by describing a state with a definite flavor as a superposition of states with different masses, called mass eigenstates [1]. While earlier frameworks for describing the dynamics of free neutrinos were based on plane-wave or wave-packet solutions of the Schrödinger equation [3–5], a fully relativistic treatment of the mass eigenstates is required for a correct description of the fermionic character of the particle [6–10].

The dynamics of single fermionic particle states is given by the Dirac equation [11]. In this framework, the state of a massive particle is described with a 4-component object, called a bispinor, and the underlying group structure to the Dirac equation, namely the complete Lorentz group [12], implies that such a particle state carries two discrete degrees of freedom: spin and chirality. Although the latter is a Lorentz invariant, it is not a conserved quantity under the free particle Dirac dynamics [11, 13]. In fact, the mass term of the Dirac equation couples the components of the bispinor with different chiralities [13], which we call right-handed (positive chirality) and left-handed (negative chirality), inducing *chiral oscillations* [10, 14–16]. Thus, a state describing a free particle initially with a definite chirality will evolve into a superposition of left and right-handed states. The amplitude of such oscillations depends on the ratio between mass and energy and, thus, is very small for relativistic particles.

Neutrinos are created via weak interaction processes, which violate parity symmetry and project the state into a definite chirality [17]. Since typically neutrinos are ultra-relativistic particles, chiral oscillations are negligible. Nevertheless, in recent proposals for measuring non-relativistic cosmic neutrinos via capture on Tritium [18–20], chiral oscillations could be prominent. Since the measurement process is only sensitive to left-handed neutrino states, chiral oscillations to the unmeasurable right-handed states would induce a depletion on the expected measured flux of electron neutrinos [21, 22]. Furthermore, chiral oscillations affect the pion decay rates into a lepton-antineutrino pair. If the antineutrino is non-relativistic, quantum correlations encoded in the antineutrino-electron pair are sensitive to chiral oscillations and, in principle, could be probed via a spin Bell inequality [23].

In this paper, we extend the formalism presented in [22] in order to describe chiral oscillation effects in flavor oscillations for non-relativistic neutrinos. We describe the mass eigenstates as bispinors whose temporal evolution is given by the Dirac equation, and consider a chirality-dependent flavor mixing [24]. The mixing matrix depends on whether the state is chiral left or right-handed [24] and oscillation probabilities for N flavor mixing subjected to chiral oscillation effects are depicted their corresponding time-averaged outputs [22]. In particular, we show that Majorana and Dirac nature are indistinguishable, unless additional CP-violation Majorana phases [25] differentiates left and right-handed chiral states. In this case, Dirac and Majorana neutrinos exhibit different oscillation patterns for right-handed states. In the three flavor mixing scenario, considering standard values for the mixing parameters, we show that the difference between the averaged oscillation probabilities for Dirac and Majorana neutrino oscillations could be $\sim 8\times 10^{-5}$ for non-relativistic neutrinos. Our results show how chiral oscillations can affect quantum correlations encoded in single-particle neutrino states [26, 27] considered in wave-packet treatments [28], and

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in lepton-antineutrino pair production [23].

The paper is organized as follows. In section 2 we review chiral oscillation in the context of the Dirac equation and briefly discuss their origin in terms of the group structure of the complete Lorentz group. In section 3, we obtain the dynamical evolution of a flavor state for a general mixing. We obtain generalized oscillation formulas for both left and right-handed states and evaluate the effects of chiral oscillations for three flavor mixing in terms of the averaged oscillation probabilities. In section 4, we discuss and evaluate the differences between Dirac and Majorana neutrinos driven by additional CP-violation Majorana phases. We draw our conclusions in section 5.

2. Dirac equation and chiral oscillations

The Dirac equation describing the temporal evolution of the state of a free massive fermion is given in terms of the free Dirac Hamiltonian, as

$$\hat{H}_D |\psi\rangle = \left(\hat{\mathbf{p}} \cdot \hat{\alpha} + m\hat{\beta}\right) |\psi\rangle = i\partial_t |\psi\rangle, \tag{1}$$

where natural units, $\hbar = c = 1$, have been considered, operators have been denoted with hats and vectors with boldface letters. The Dirac matrices, $\hat{\alpha}_x$, $\hat{\alpha}_y$, $\hat{\alpha}_z$, and $\hat{\beta}$, satisfy the anti-commutation relations

$$\begin{aligned} \hat{\alpha}_i\hat{\alpha}_j + \hat{\alpha}_j\hat{\alpha}_i &= 2\delta_{ij}\hat{I}_4, \\ \hat{\alpha}_i\hat{\beta} + \hat{\beta}\hat{\alpha}_i &= 0, \end{aligned} \tag{2}$$

which, with $\beta^2 = \hat{I}_4$, $\hat{I}_N$ the N-dim identity matrix, ensure the relativistic energy-momentum relation $E_{p,m} = \sqrt{p^2 + m^2}$ for free particles.

The solutions of the Dirac equation are 4-component wave-functions, the Dirac bispinors [13] which carry two intrinsic degrees of freedom: spin and chirality. Such internal structure of the bispinors can be better understood in terms of the group structure underlying the Dirac equation. In a group theory language, the Dirac equation is the dynamical equation for the irreducible representations (irreps) of the so-called complete Lorentz group [12], which consists on the proper Lorentz group plus parity. The latter connects the disjoint left and right-handed irreps of the proper Lorentz group (given in terms of Weyl spinors), each of which are isormorphic to $SU(2)$ and carry one intrinsic degree of freedom, the spin. To construct the irreps of the complete Lorentz group on thus requires a combination of left and right-handed Weyl spinors (which are the irreps of the Lorentz group), which carry two internal discrete degrees of freedom, spin and chirality (or intrinsic parity).

The internal structure of bispinors can be better appreciated when we consider a specific representation for the Dirac matrices. Here, we use the chiral representation of the Dirac matrices in which [29]

$$\hat{\alpha}_i = \begin{bmatrix} \hat{\sigma}_i & 0 \\ 0 & -\hat{\sigma}_i \end{bmatrix}, \quad \hat{\beta} = \begin{bmatrix} 0 & \hat{I}_2 \\ \hat{I}_2 & 0 \end{bmatrix}, \tag{3}$$

where $\hat{\sigma}_i$ ($i = x, y, z$) denotes the Pauli matrices. In this representation, the *chiral* matrix $\hat{\gamma}_5 = -i\hat{\alpha}_x\hat{\alpha}_y\hat{\alpha}_z$ is diagonal:

$$\hat{\gamma}_5 = \text{diag}\{\hat{I}_2, -\hat{I}_2\}, \tag{4}$$

accordingly, an arbitrary bispinor $|\xi\rangle$ can be decomposed as

$$|\xi\rangle = \begin{bmatrix} |\xi_R\rangle \\ |\xi_L\rangle \end{bmatrix}, \tag{5}$$

where $\left|\xi_{R,L}\right\rangle$ are two-component spinors, and R denotes right-handed (positive) chirality while L denotes left-handed (negative) chirality. Eq. (1) yields a set of coupled equations for the components

$$\begin{aligned} \hat{\mathbf{p}} \cdot \hat{\sigma} \, |\xi_R\rangle + m \, |\xi_L\rangle &= i\partial_t \, |\xi_R\rangle \,, \\ -\hat{\mathbf{p}} \cdot \hat{\sigma} \, |\xi_L\rangle + m \, |\xi_R\rangle &= i\partial_t \, |\xi_L\rangle \,, \end{aligned} \tag{6}$$

that is, the L and R components of the bispinor couple via the mass term of the Dirac equation which in yields chiral oscillations.

Turning our attention to the plane wave solutions of Eq. (1), we consider the simplified framework in which a free massive particle propagates along the $\boldsymbol{e}_z$ direction with momentum $\boldsymbol{p} = p\boldsymbol{e}_z$. The positive and negative plane-wave solutions of the Dirac equation are then given by $|\psi_+(p,t)\rangle = e^{ipz - iE_{p,m}t} \, |u_s(p,m)\rangle$ and $|\psi_-(p,t)\rangle = e^{ipz + iE_{p,m}t} \, |v_s(p,m)\rangle$. Here s indicates the polarization of the free-particle. Throughout the paper, we consider helicity bispinors: eigenstates of the Helicity operator $\frac{p \cdot \hat{\Sigma}_z}{p}$ [1]. Helicity is a conserved quantity under the free Dirac equation, and thus it is a convenient choice for describing free-particle states. The eigenstates of the Dirac equation with positive and negative helicities are therefore given in terms of [11, 30]

$$|u_\pm(p,m)\rangle = \sqrt{\frac{E_{p,m}+m}{4E_{p,m}}} \begin{bmatrix} \left(1 \pm \frac{p}{E_{p,m}+m}\right) |\pm\rangle \\ \left(1 \mp \frac{p}{E_{p,m}+m}\right) |\pm\rangle \end{bmatrix}, \quad |v_\pm(p,m)\rangle = \sqrt{\frac{E_{p,m}+m}{4E_{p,m}}} \begin{bmatrix} \left(1 \pm \frac{p}{E_{p,m}+m}\right) |\pm\rangle \\ -\left(1 \mp \frac{p}{E_{p,m}+m}\right) |\pm\rangle \end{bmatrix}, \tag{7}$$

where $\hat{\sigma}_z \, |\pm\rangle = \pm \, |\pm\rangle$. It is worth mentioning that for the ultra-relativistic limit, $m/p \to 0$, and the above described bispinors are either right-handed and positive-helicity or left-handed and negative helicity, i.e. massless particles have the same chirality and helicity [13]. On the other hand, in the non-relativistic limit, $p/m \to 0$, we have equal left and right-handed components irrespective of the helicity, i.e. chirality and helicity are different for massive particles. Helicity is related to the spin of particles, is a conserved quantity but is not Lorentz invariant. Chirality, on the other hand, is not a conserved quantity but is Lorentz invariant.

For a free massive particle under the Dirac equation dynamics, $[\hat{H}_D, \hat{\gamma}_5] = 2m\hat{\beta}\hat{\gamma}_5$. Therefore, a state with definite chirality at $t = 0$ will exhibit *chiral oscillations* as a consequence of the intrinsic structure of massive Dirac bispinor. To describe the dynamics of chiral oscillations, let us first define the left and right-handed bispinors

$$\left|\psi_{\pm,L}\right\rangle = \begin{bmatrix} 0 \\ |\pm\rangle \end{bmatrix}, \quad \left|\psi_{\pm,R}\right\rangle = \begin{bmatrix} |\pm\rangle \\ 0 \end{bmatrix}, \tag{8}$$

and consider the temporal evolution of a massive particle that at $t = 0$ is in a state with left-handed chirality and negative helicity $|\psi_m(0)\rangle = \left|\psi_{-,L}\right\rangle$. The time-evolved bispinor is given by

$$|\psi_m(t)\rangle = \left[\cos\left(E_{p,m}t\right) - i\frac{p}{E_{p,m}} \sin\left(E_{p,m}t\right)\right] \left|\psi_{-,L}\right\rangle - i\frac{m}{E_{p,m}} \sin\left(E_{p,m}t\right) \left|\psi_{-,R}\right\rangle \tag{9}$$

[1]Here $\hat{\Sigma}_i = \mathrm{diag}\{\hat{\sigma}_i, \hat{\sigma}_i\}$. Helicity is defined as the projection of the spin in the direction of momentum.

We then obtain the probability that the state is in its initial configuration

$$\mathcal{P}_{m,L}(t) = |\left\langle \psi_{-,L} \middle| \psi_m(t) \right\rangle|^2 = 1 - \frac{m^2}{E^2_{p,m}} \sin^2\left(E_{p,m}t\right), \tag{10}$$

and the probability that the state is right-handed

$$\mathcal{P}_{m,R}(t) = |\left\langle \psi_{-,R} \middle| \psi_m(t) \right\rangle|^2 = \frac{m^2}{E^2_{p,m}} \sin^2\left(E_{p,m}t\right). \tag{11}$$

The probability to a right-handed state (11) oscillates with amplitude $m^2/E^2_{p,m}$, and therefore is suppressed for ultra-relativistic particles. From (9), the average chirality is given by [14, 22]

$$\langle\hat{\gamma}_5\rangle(t) = \langle\psi_m(t)|\hat{\gamma}_5\,|\psi_m(t)\rangle = -1 + \frac{2m^2}{E^2_{p,m}} \sin^2\left(E_{p,m}t\right). \tag{12}$$

Although the above calculations consider Dirac particles, they can be readily modified to describe Majorana particles. The details are presented in [22], and the temporal evolution for a Majorana particle initially in a left-handed and negative helicity state is given by

$$\begin{aligned} \left|\psi^M_{-,L}(t)\right\rangle = &\left[\cos\left(E_{p,m}t\right) - i\frac{p}{E_{p,m}}\sin\left(E_{p,m}t\right)\right]\left|\psi_{-,L}\right\rangle \\ &- i\frac{m}{E_{p,m^{(M)}}}\sin\left(E_{p,m}t\right)\left|\psi^c_{-,R}\right\rangle, \end{aligned} \tag{13}$$

where $\left|\psi^c_{-,R}\right\rangle = C\left|\psi_{-,L}\right\rangle$ is a right-handed bispinor obtained via the charge conjugation of $\left|\psi_{-,L}\right\rangle$. The survival probability and average chirality of this state are the same as those of (9).

3. Chiral oscillations and flavor dynamics

We can now describe the interplay of chiral and flavor oscillations for massive neutrinos. Weak interactions that create neutrinos select specific chiral components of the state, creating particles with definite chirality [17]. We therefore, consider a neutrino whose state at $t = 0$ has a definite flavor α and left-handed chirality. In principle, the state could exhibit any spin polarization, including a superposition of positive and negative helicities. This is the case, for example, for anti-neutrinos produced in the pion decay [31–33]: for massive antineutrinos, the full state includes a superposition of positive and negative helicity states. To describe the intrinsic chirality of weak interactions, one considers the chiral projection of the antineutrino into the right-handed bispinor component [23]. As a consequence, it can be shown that the resulting superposition still involves positive and negative helicity components, but with the positive helicity amplitude suppressed for relativistic states. This is an entangled electron-antineutrino state, such that a measurement of the electron state can bring the polarization of the antineutrino state into a definite helicity state. In what follows, we consider the conceptually simpler framework in which the neutrino state at $t = 0$ has a definite negative helicity:

$$|\nu_\alpha(t=0)\rangle = \sum_i U_{\alpha,i}\left|\psi_{i,-,L}\right\rangle \otimes |\nu_i\rangle \equiv \left|\nu_{\alpha,L}\right\rangle, \tag{14}$$

where $U_{\alpha,i}$ are the elements of the mixing matrix, and $\left|\psi_{i,-,L}\right\rangle$ is given in (8). Here we have included the index i to specify that the bispinor describe the state of the mass eigenstate i. For now, we consider Dirac neutrinos, the extension to Majorana neutrinos shall be briefly discussed below. The temporal evolution of the mass eigenstates bispinors $\left|\psi_{i,-,L}\right\rangle$ are given in (9) and, assuming that all mass eigenstates have the same momentum, yields

$$\begin{aligned} |\nu_\alpha(t)\rangle = \sum_i U_{\alpha,i} \Bigg\{ & \left[\cos\left(E_{p,m_i}t\right) - i\frac{p}{E_{p,m_i}}\sin\left(E_{p,m_i}t\right)\right]\left|\psi_{i,-,L}\right\rangle \otimes |\nu_i\rangle \\ & - i\frac{m_i}{E_{p,m_i}}\sin\left(E_{p,m_i}t\right)\left|\psi_{i,-,R}\right\rangle \otimes |\nu_i\rangle \Bigg\}. \end{aligned} \tag{15}$$

We then consider the mixing transformation between mass and flavor states that takes into account possible differences between left and right chiral components [24]:

$$\left|\psi_{i,-,L}\right\rangle \otimes |\nu_i\rangle = \sum_\beta U^*_{\beta,i}\left|\psi_{i,-,L}\right\rangle \otimes \left|\nu_\beta\right\rangle, \quad \left|\psi_{i,-,R}\right\rangle \otimes |\nu_i\rangle = \sum_\beta V^*_{\beta,i}\left|\psi_{i,-,R}\right\rangle \otimes \left|\nu_\beta\right\rangle, \tag{16}$$

such that

$$\begin{aligned} |\nu_\alpha(t)\rangle = & \sum_{\beta,i} U_{\alpha,i}U^*_{\beta,i}\left[\cos\left(E_{p,m_i}t\right) - i\frac{p}{E_{p,m_i}}\sin\left(E_{p,m_i}t\right)\right]\left|\psi_{i,-,L}\right\rangle \otimes \left|\nu_\beta\right\rangle \\ & - i\sum_{\beta,i} U_{\alpha,i}V^*_{\beta,i}\frac{m_i}{E_{p,m_i}}\sin\left(E_{p,m_i}t\right)\left|\psi_{i,-,R}\right\rangle \otimes \left|\nu_\beta\right\rangle . \end{aligned} \tag{17}$$

The oscillation probability to a state of flavor β and left handed-chirality is therefore given by

$$\begin{aligned} P_{\nu_{\alpha,L}\to\nu_{\beta,L}}(t) = & \,|\left\langle \nu_{\beta,L}\middle|\nu_{\alpha,L}(t)\right\rangle|^2 \\ = & \sum_i |U_{\alpha,i}|^2|U_{\beta,i}|^2\mathcal{P}_{m_i,L}(t) \\ & + 2\sum_{\{i,j\}} \mathrm{Re}\left[U_{\alpha,i}U^*_{\alpha,j}U^*_{\beta,i}U_{\beta,j}\right]\left[\cos(E_{p,m_i}t)\cos(E_{p,m_j}t) + \frac{p^2}{E_{p,m_i}E_{p,m_j}}\sin(E_{p,m_i}t)\sin(E_{p,m_i}t)\right] \\ & - 2p\sum_{\{i,j\}} \mathrm{Im}\left[U_{\alpha,i}U^*_{\alpha,j}U^*_{\beta,i}U_{\beta,j}\right]\left[\frac{\sin(E_{p,m_j}t)\cos(E_{p,m_i}t)}{E_{p,m_j}} - \frac{\sin(E_{p,m_i}t)\cos(E_{p,m_j}t)}{E_{p,m_i}}\right], \end{aligned} \tag{18}$$

where in the last two lines $\sum_{\{i,j\}} \equiv \sum_{i=1}^{N-1}(\sum_{j=i+1}^{N})$, and $\mathcal{P}_{m_i,L}(t)$ was defined in (10).

The mass eigenstates can oscillate to right-handed states, which induce a finite probability that an initially left-handed neutrino of a flavor α is in a right-handed chirality of flavor β, which reads

$$\begin{aligned} P_{\nu_{\alpha,L}\to\nu_{\beta,R}}(t) = & \sum_i |U_{\alpha,i}|^2|V_{\beta,i}|^2\mathcal{P}_{m_i,R}(t) \\ & + \sum_{\{i,j\}} \mathrm{Re}\left[U_{\alpha,i}U^*_{\alpha,j}V^*_{\beta,i}V_{\beta,j}\right]\frac{m_i m_j}{E_{p,m_i}E_{p,m_j}}\sin(E_{p,m_i}t)\sin(E_{p,m_j}t). \end{aligned} \tag{19}$$

Notice that the third line of Eq. (18) contains a term that vanishes, unless the mixing matrix contains a CP-violation phase. A lengthy algebraic manipulation can explicitly show that both Eqs. (18) and (19) evolve with frequencies defined by both the energy difference $\Delta E_{ij} = E_{p,m_i} - E_{p,m_j}$ and by the energy sum $E_{p,m_i} + E_{p,m_j}$. The latter appears due to the relativistic treatment used here,

$\sin^2\theta_{12}$	0.36
$\sin^2\theta_{13}$	0.021
$\sin^2\theta_{23}$	0.42
m_1	0.8 eV
$\Delta m^2_{21} = m^2_2 - m^2_1$	7.5×10^{-5} eV^2
$\Delta m^2_{31} = m^2_3 - m^2_1$	2.8×10^{-3} eV^2

Table 1: Values for mixing angles and square mass difference.

which includes both positive and negative energy states in the time evolution for mass eigenstates [6, 7]. Such contributions are also obtained in a quantum field treatment of flavor oscillations [34–36], and the case in which $V_{\alpha,i} = U_{\alpha,i}$ describe the mixing that does not differentiate between left and right-handed neutrino states. Finally, all the terms composing (19) are multiplied by the mass-to-energy ratio, and therefore are suppressed in relativistic dynamical regimes. Oscillation probabilities, Eqs. (18) and (19), for the special case of two flavors, agree with those obtained in [24]. In the limit $m_i/p \to 0$ (for $i = 1, .., N$),

$$\begin{aligned} P_{\nu_{\alpha,L}\to\nu_{\beta,L}}(t) \to P^S_{\nu_\alpha\to\nu_\beta}(t) = {} & \sum_i |U_{\alpha,i}|^2|U_{\beta,i}|^2 \\ & + 2\sum_{\{i,j\}} \mathrm{Re}\left[U_{\alpha,i}U^*_{\alpha,j}U^*_{\beta,i}U_{\beta,j}\right]\cos(\Delta E_{ij}t) \\ & + 2\sum_{\{i,j\}} \mathrm{Im}\left[U_{\alpha,i}U^*_{\alpha,j}U^*_{\beta,i}U_{\beta,j}\right]\sin(\Delta E_{ij}t), \end{aligned} \tag{20}$$

where $P^S_{\nu_\alpha\to\nu_\beta}(t)$ is the standard oscillation probability obtained in the usual quantum mechanical treatment for neutrino oscillations.

Considering the mixing of three flavor parameterized by the mixing matrix

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{21}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ are given in terms of the mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$, and δ is the CP-violation phase, and Majorana CP-violation phases have been momently suppressed, and assuming the mass hierarchy $m_3 \gg m_2 > m_1$ [1], the square mass differences and the mixing angles used are summarized in Table 1.

Hence, the oscillation probabilities for an electron neutrino are depicted in Figure 1 for $p/m_1 = 1$ and in Figure 2 for $p/m_1 = 3$. Chiral oscillations manifest in two ways: as fast oscillations in $P_{\nu_{\alpha,L}\to\nu_{\beta,L}}(t)$ (shown in the upper row) and as prominent oscillation probabilities to right-handed neutrinos (displayed in the lower row). In the intermediate dynamical regime depicted in Fig. 2, chiral oscillations effects are highly suppressed. Even though there is still a clear effect of the fast chiral oscillations in $P_{\nu_{\alpha,L}\to\nu_{\beta,L}}(t)$, their amplitudes are smaller. Correspondingly, the oscillations $P_{\nu_{\alpha,L}\to\nu_{\beta,R}}(t)$ are less prominent when compared with the results depicted in Fig. 1. In fact the maximum oscillation probability to right-handed components for $p/m_1 = 3$ is around 20% of the value for $p/m_1 = 1$.

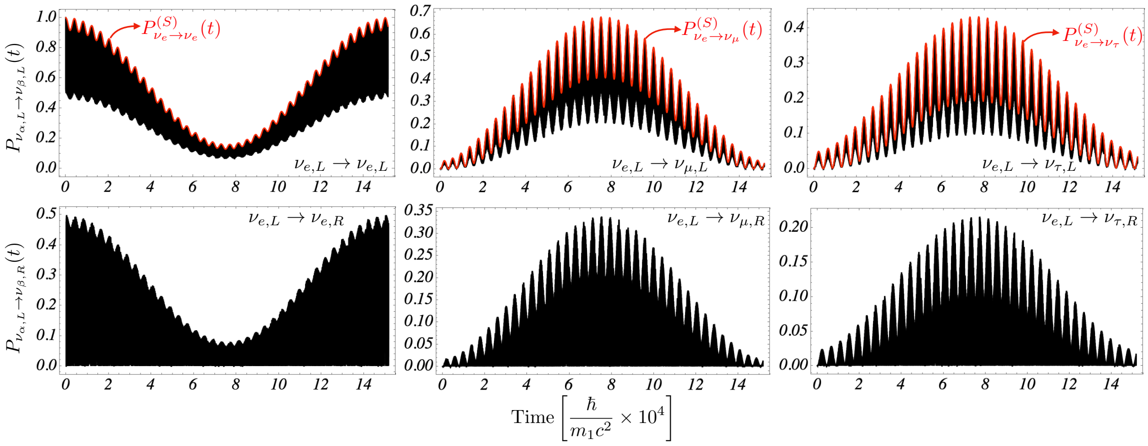

Figure 1: Oscillation probabilities (18) and (19) as a function of time for a non-relativistic electron neutrino. The flavor oscillations are accompanied by fast chiral oscillations, which have an amplitude $\propto m_i^2/E_{m_i,p}^2$, and are evident in the regime $m \sim p$ considered in this plots. The red curves in the first row depict the standard survival probability, which does not include chiral oscillations. Parameters as in Table 1 and $p/m_1 = 1$.

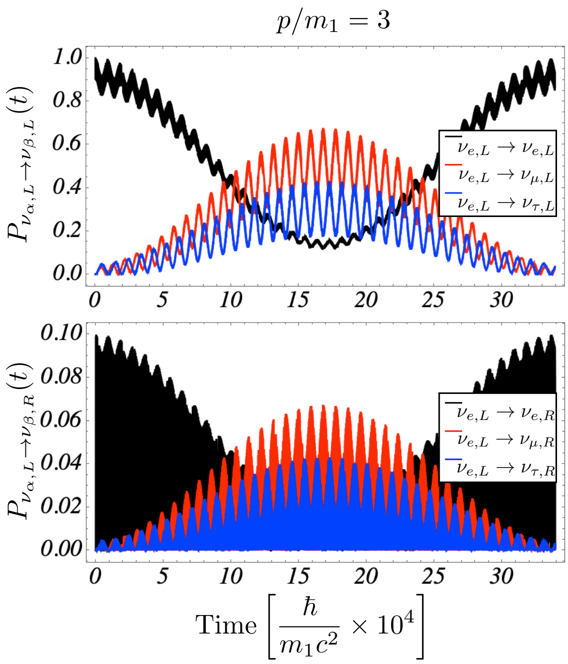

Figure 2: Oscillation probabilities (18) and (19) as a function of time at an intermediate dynamical regime between non-relativistic and relativistic. In comparison with Fig. 1, the fast chiral oscillations and the probabilities for chirality flip are highly suppressed. Parameters as in Table 1 and $p/m_1 = 3$.

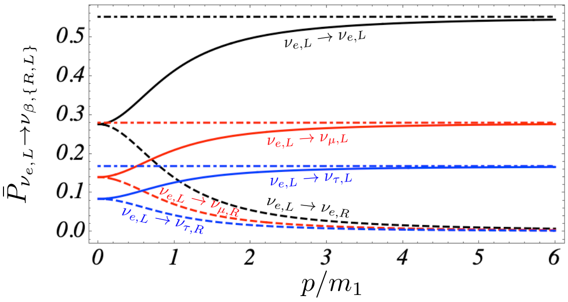

Figure 3: Time averaged oscillation probabilities as a function of the momentum following the framework set in Eq. (22). Oscillations to right-handed chiral states are suppressed as the momentum increase. Dashed lines represent the result for the standard formula. Parameters as in Table 1.

To further quantify the effects of chiral oscillations for three flavors, we follow the framework of Ref. [22] and consider the time averaged oscillation probabilities

$$\bar{P}_{\nu_{\alpha,L}\to\nu_{\beta,\{L,R\}}} = \frac{1}{\tau}\int_0^\tau dt P_{\nu_{\alpha,L}\to\nu_{\beta,\{L,R\}}}(t), \tag{22}$$

with a period of integration chosen as the longest one $\tau \equiv \tau_{12} = \frac{4\pi}{E_{p,m_2}-E_{p,m_1}}$. The results for all the possible oscillation channels of an initially left-handed electron neutrino are shown in Fig. 3. Oscillations to right-handed states are suppressed as p/m_1 increases, while the oscillation probabilities to left-handed states approach the standard result obtained by suppressing chiral oscillations – Eq. (20). For small ratios p/m_1, $\bar{P}_{\nu_{e,L}\to\nu_{\beta,L}} \sim \bar{P}_{\nu_{e,L}\to\nu_{\beta,R}}$, indicating the prominence of chiral oscillations in the non-relativistic regime.

Due to chiral oscillations, the averaged survival probability exhibits a depletion in the non-relativistic regime when compared with the standard result. Since measurement schemes for neutrinos involve weak interaction processes, they are sensitive only to left-handed neutrinos and therefore, the measurement of non-relativistic neutrinos would exhibit a depletion due to the prominent oscillations of the mass eigenstates from left to right-handed components. For two flavor mixing, as discussed in [22], such depletion can be 50% of the result obtained with the standard description of flavor oscillations not including chiral oscillations. This is relevant when probing cosmological neutrinos, which are non-relativistic and, as such, should exhibit a depletion on the expected measured flux due to oscillations to right-handed chiral components [21, 22].

4. Chiral and flavor oscillations of Dirac and Majorana neutrinos

In the previous section we have considered the effect of chiral oscillations on the flavor dynamics of Dirac neutrinos. Turning our attention to Majorana neutrinos, we conclude readily that the oscillation probabilities from (18) and (19) are also valid for Majorana neutrino, with the following correction: oscillations to right-handed components are $\nu_{\alpha,L} \to \nu^c_{\beta,R}$, as can be inferred from the time evolution for Majorana mass states given in (13). The impact of chiral oscillations on the measurement of cosmic neutrinos would be the same for Dirac and Majorana neutrinos, with the difference that capture by Tritium is sensitive to both left-handed and negative helicity neutrinos, and right-handed and positive helicity antineutrinos. The second is only present in the case of Majorana neutrinos, which yields a factor 2 difference between Majorana and Dirac neutrino capture rates in the non-relativistic regime even in the absence of chiral oscillations [20, 21].

Considering a more general framework, the mixing matrix for Majorana neutrinos can include more CP-violation phases than the one for the Dirac neutrinos. In fact, in the general case of N flavors, the number of CP-violation phases for Dirac neutrinos ($n_{\rm D}$) and for Majorana neutrinos ($n_{\rm M}$) are respectively [25]

$$n_{\rm D} = \frac{(N-1)(N-2)}{2}, \qquad n_{\rm M} = \frac{N(N-1)}{2}. \tag{23}$$

Therefore, the Majorana mixing matrix for three flavors can include two phases in addition to the Dirac phase δ in (21), which we indicate as ϕ_2 and ϕ_3:

$$U^M = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2/2} & 0 \\ 0 & 0 & e^{i\phi_3/2} \end{bmatrix} = U\Phi, \tag{24}$$

where we have defined $\Phi = {\rm diag}\{1, e^{i\phi_2/2}, e^{i\phi_3/2}\}$. We first notice that in the case $U = V$, $|U^M_{\alpha,i}|^2 = |U_{\alpha,i}|^2$ and $U^M_{\alpha,i} U^{M,*}_{\alpha,j} U^{M,*}_{\beta,i} U^M_{\beta,j} = U_{\alpha,i} U^*_{\alpha,j} U^*_{\beta,i} U_{\beta,j}$. Since Eqs.(18), (19) depend on those products, we conclude that the oscillation probabilities can not distinguish between Dirac and Majorana neutrinos, even in the presence of additional CP-violation phases. The distinction between Dirac and Majorana is possible only if $U \neq V$ and in the presence of additional CP-violation phases for Majorana neutrinos, as explicitly exemplified in the two flavor case in [24]. For three flavors, we consider the additional CP-violation phase matrices $\Phi_{L,R}$ with elements of the form $[\Phi_{L,R}]_{ij} = e^{i\phi_{i,(L,R)}/2}\delta_{ij}$, such that

$$U^M = U\Phi_L, \qquad V^M = V\Phi_R. \tag{25}$$

The oscillation probabilities to left-handed states (18) are the same for Dirac and Majorana, but the probability to right-handed states (19) for Majorana neutrinos reads

$$\begin{aligned} P^M_{\nu_{\alpha,L}\to\nu_{\beta,R}}(t) = {} & \sum_{i=1}^{3} |U_{\alpha,i}|^2 |V_{\beta,i}|^2 \mathcal{P}_{m_i,R}(t) \\ & + \sum_{\{i,j\}} \left\{ \cos\left(\frac{\Delta\phi_i - \Delta\phi_j}{2}\right) {\rm Re}\left[U_{\alpha,i} U^*_{\alpha,j} V^*_{\beta,i} V_{\beta,j}\right] \right. \\ & \left. - \sin\left(\frac{\Delta\phi_i - \Delta\phi_j}{2}\right) {\rm Im}\left[U_{\alpha,i} U^*_{\alpha,j} V^*_{\beta,i} V_{\beta,j}\right] \right\} \frac{m_i m_j}{E_{p,m_i} E_{p,m_j}} \sin(E_{p,m_i} t) \sin(E_{p,m_j} t), \end{aligned} \tag{26}$$

where $\Delta\phi_i = \phi_{i,L} - \phi_{i,R}$.

To illustrate the effects of the additional CP-violation phases combined with a mixing that differentiates between left and right-handed bispinors, we consider now the case in which the mixing angles are the same for left and right-handed states, but with one of the Majorana phases being different. In particular, we assume that $\Delta\phi_{1,3} = 0$ while $\Delta\phi_2 \neq 0$ in Eq. (26). In Figure 4, we show the difference between the time averaged oscillation probabilities to right-handed neutrinos for the Dirac and Majorana cases $\bar{P}_{\nu_{e,L}\to\beta_R} - \bar{P}^M_{\nu_{e,L}\to\beta_R}$ (see Eq. (22)), for different values of $\Delta\phi_2$.

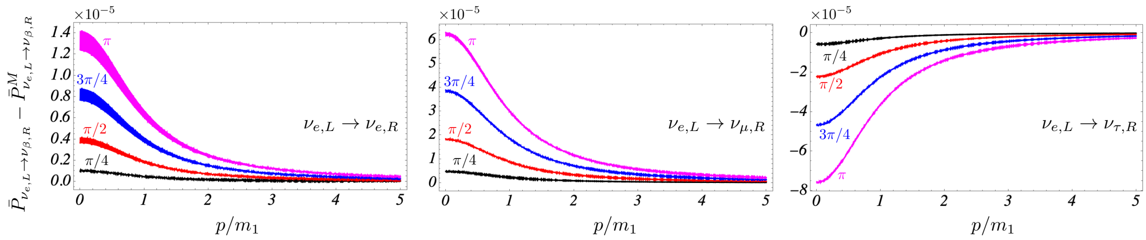

Figure 4: Difference between the time-averaged oscillation probability for Dirac and Majorana neutrinos to right-handed states, including extra CP-violation phases as a function of the momentum and for several values of $\Delta\phi_2$ (see Eq. (26)). Masses and mixing parameters given in Table 1.

We notice that the difference is more evident for the oscillations $\nu_{e,L} \to \nu_{\mu R}$ and $\nu_{e,L} \to \nu_{\tau R}$. For the latter, the averaged probability for Majorana neutrinos is bigger than for the Dirac case. For states with $p \gg m_1$, the difference between Dirac and Majorana vanishes, since chiral oscillations are suppressed, as such, the terms that depend on the difference between the additional Majorana phases also vanish.

5. Conclusions

In summary, we have extended and generalized the description of chiral oscillation effects in flavor oscillations for mixing involving N-flavors, including mixing that distinguish left from right-handed states, as well as additional CP-violation Majorana phases, and obtained oscillation formulas that agree with those recently derived in [24]. We have followed the framework presented in [22], and shown that the additional CP-violation Majorana phases induce a difference between Dirac and Majorana neutrinos for the time-averaged oscillation probabilities to right-handed states. Since those effects depend strictly on the chiral oscillations, they are prominent for non-relativistic neutrinos and are more evident on the oscillation probabilities to right-handed muon and tau neutrino states. The measurement of such difference requires the development of techniques sensitive to right-handed neutrinos, since the survival probability of a left-handed electron neutrino does not depend on the additional Majorana phases and therefore can not distinguish between Majorana and Dirac neutrinos. Alternatively, chiral oscillations can also affect quantum correlations encoded in single particle neutrino states [26–28] and in lepton-antineutrino pairs [23], which may yield novel procedures to measure the effects shown here.

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