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The Minimal $SU(5)$ Unification Model

Ilja Doršner[a,b,*]

[a] *University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture (FESB)*
Ruđera Boškovića 32, 21000 Split, Croatia
[b] *J. Stefan Institute, Jamova 39, P. O. Box 3000, 1001 Ljubljana, Slovenia*

E-mail: dorsner@fesb.hr

I present the most minimal realistic $SU(5)$ unification model to date and discuss its main predictions. The particle content of the model comprises 5_H, 24_H, 35_H, $\overline{5}_{Fi}$, 10_{Fi}, 15_F, $\overline{15}_F$, and 24_V, where subscripts H, F, and V denote whether a given representation contains scalars, fermions, or gauge bosons, respectively, while $i = 1, 2, 3$. The model employs all possible interaction terms, as allowed by the Lorentz group, the $SU(5)$ gauge symmetry, and the aforementioned particle content, to generate the Standard Model fermion masses through three different mechanisms. It also connects the neutrino mass generation mechanism to the experimentally observed mass disparity between the down-type quarks and charged leptons. The minimal structure of the model requires 24_H and 5_H not only to break $SU(5)$ and $SU(3) \times SU(2) \times U(1)$ gauge groups, respectively, but to accomplish one additional task each. The model furthermore predicts that neutrinos are strictly Majorana fields, that one neutrino is purely a massless particle, and that neutrino masses are of normal ordering. The current experimental bound on the $p \to \pi^0 e^+$ lifetime limit, in conjunction with the prediction of the model for the gauge mediated proton decays, implies that there are four new scalar multiplets at or below a 120 TeV mass scale if these multiplets are mass degenerate. If these multiplets are not mass degenerate, the quoted limit then applies, for all practical purposes, to the geometric mean of their masses. The scalar multiplets in question transform as $(1, 3, 0)$, $(8, 1, 0)$, $(\overline{3}, 3, -2/3)$, and $(\overline{6}, 2, 1/6)$ under the Standard Model gauge group $SU(3)\times SU(2)\times U(1)$ with calculable couplings to the Standard Model fields.

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1. A novel $SU(5)$ proposal

I present, in what follows, a minimal realistic $SU(5)$ model [1] and a phenomenological study [2] of the viability of its parameter space. The model under consideration addresses, in a very economical manner, the main shortcomings of the original Georgi-Glashow proposal [3]. These shortcomings are (*i*) a complete absence of the neutrino mass generation mechanism, (*ii*) a lack of viable gauge coupling unification, and (*iii*) an inability to accommodate either the down-type quark or charged lepton masses in a realistic manner.

The interactions of the model under consideration are built entirely out of the fields residing in the first five lowest lying non-trivial $SU(5)$ representations in terms of dimensionality. I will, for clarity of exposition, often use the Georgi-Glashow model as a starting point for the description of the most prominent features of this novel $SU(5)$ proposal [1].

The particle content of the Georgi-Glashow model comprises 5_H, 24_H, $\overline{5}_{Fi}$, 10_{Fi}, and 24_V, where subscripts H, F, and V denote whether a given representation contains scalars, fermions, or gauge bosons, respectively, while $i(=1,2,3)$ represents a generation index. (Note that the $SU(5)$ representations are simply identified through their dimensionality. I will use the same approach when denoting the Standard Model gauge group $SU(3)\times SU(2)\times U(1)$ multiplets.) The novel proposal in question [1] extends the Georgi-Glashow particle content with one 35-dimensional scalar representation, i.e., 35_H, and one set of vector-like fermions in 15-dimensional representation comprising 15_F and $\overline{15}_F$. The particle content of the model is presented in Table 1, where I also specify the decomposition of the $SU(5)$ representations into the Standard Model multiplets and introduce associated nomenclature.

$SU(5)$	$SU(3)\times SU(2)\times U(1)$	$SU(5)$	$SU(3)\times SU(2)\times U(1)$
$5_H \equiv \Lambda$	$\Lambda_1\left(1,2,\frac{1}{2}\right)$	$\overline{5}_{Fi} \equiv F_i$	$L_i\left(1,2,-\frac{1}{2}\right)$
	$\Lambda_3\left(3,1,-\frac{1}{3}\right)$		$d_i^c\left(\overline{3},1,\frac{1}{3}\right)$
$24_H \equiv \phi$	$\phi_0\,(1,1,0)$	$10_{Fi} \equiv T_i$	$Q_i\left(3,2,\frac{1}{6}\right)$
	$\phi_1\,(1,3,0)$		$u_i^c\left(\overline{3},1,-\frac{2}{3}\right)$
	$\phi_3\left(3,2,-\frac{5}{6}\right)$		$e_i^c\,(1,1,1)$
	$\phi_{\overline{3}}\left(\overline{3},2,\frac{5}{6}\right)$	$15_F \equiv \Sigma$	$\Sigma_1(1,3,1)$
	$\phi_8\,(8,1,0)$		$\Sigma_3\left(3,2,\frac{1}{6}\right)$
$35_H \equiv \Phi$	$\Phi_1\left(1,4,-\frac{3}{2}\right)$		$\Sigma_6\left(6,1,-\frac{2}{3}\right)$
	$\Phi_3\left(\overline{3},3,-\frac{2}{3}\right)$	$\overline{15}_F \equiv \overline{\Sigma}$	$\overline{\Sigma}_1\,(1,3,-1)$
	$\Phi_6\left(\overline{6},2,\frac{1}{6}\right)$		$\overline{\Sigma}_3\left(\overline{3},2,-\frac{1}{6}\right)$
	$\Phi_{10}\left(\overline{10},1,1\right)$		$\overline{\Sigma}_6\left(\overline{6},1,\frac{2}{3}\right)$

Table 1: The particle content of the novel proposal and the associated nomenclature at both the $SU(5)$ and the Standard Model levels. Subscripts H and F denote scalar and fermion representations, respectively, while $i(=1,2,3)$ is a generation index.

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The lagrangian of the model, except for the (gauge) kinetic terms, reads

$$
\begin{aligned}
\mathcal{L} \supset & \Big\{ +Y^u_{ij}\, T^{\alpha\beta}_i T^{\gamma\delta}_j \Lambda^\rho \epsilon_{\alpha\beta\gamma\delta\rho} + Y^d_{ij}\, T^{\alpha\beta}_i F_{\alpha j}\Lambda^*_\beta + Y^a_i\, \Sigma^{\alpha\beta} F_{\alpha i}\Lambda^*_\beta + Y^b_i\, \overline{\Sigma}_{\beta\gamma} F_{\alpha i}\Phi^{*\alpha\beta\gamma} \\
& +Y^c_i\, T^{\alpha\beta}_i \overline{\Sigma}_{\beta\gamma}\phi^\gamma_\alpha + \text{h.c.}\Big\} + M_\Sigma \overline{\Sigma}_{\alpha\beta}\Sigma^{\alpha\beta} + y\, \overline{\Sigma}_{\alpha\beta}\Sigma^{\beta\gamma}\phi^\alpha_\gamma \\
& - \mu^2_\Lambda\, (\Lambda^*_\alpha\Lambda^\alpha) + \lambda^\Lambda_0\, (\Lambda^*_\alpha\Lambda^\alpha)^2 + \mu_1\, \Lambda^*_\alpha\Lambda^\beta\phi^\alpha_\beta + \lambda^\Lambda_1\, (\Lambda^*_\alpha\Lambda^\alpha)\left(\phi^\beta_\gamma\phi^\gamma_\beta\right) + \lambda^\Lambda_2\, \Lambda^*_\alpha\Lambda^\beta\phi^\gamma_\beta\phi^\alpha_\gamma \\
& - \mu^2_\phi\, \left(\phi^\beta_\gamma\phi^\gamma_\beta\right) + \mu_2\, \phi^\alpha_\beta\phi^\beta_\gamma\phi^\gamma_\alpha + \lambda^\phi_0\, \left(\phi^\beta_\gamma\phi^\gamma_\beta\right)^2 + \lambda^\phi_1\, \phi^\alpha_\beta\phi^\beta_\gamma\phi^\gamma_\delta\phi^\delta_\alpha + \mu^2_\Phi\, \left(\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\gamma}\right) \\
& + \lambda^\Phi_0\, \left(\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\gamma}\right)^2 + \lambda^\Phi_1\, \Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\delta}\Phi^{*\delta\rho\sigma}\Phi_{\rho\sigma\gamma} + \lambda_0\, \left(\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\gamma}\right)\left(\phi^\sigma_\rho\phi^\rho_\sigma\right) \\
& + \lambda'_0\, \left(\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\gamma}\right)\left(\Lambda^*_\rho\Lambda^\rho\right) + \lambda''_0\, \Phi^{*\alpha\beta\gamma}\Phi_{\beta\gamma\delta}\Lambda^\delta\Lambda^*_\alpha + \mu_3\, \Phi^{*\alpha\beta\gamma}\Phi_{\beta\gamma\delta}\phi^\delta_\alpha \\
& + \lambda_1\, \Phi^{*\alpha\beta\gamma}\Phi_{\alpha\delta\epsilon}\phi^\delta_\beta\phi^\epsilon_\gamma + \lambda_2\, \Phi^{*\alpha\beta\epsilon}\Phi_{\alpha\beta\delta}\phi^\gamma_\epsilon\phi^\delta_\gamma + \left\{\lambda'\, \Lambda^\alpha\Lambda^\beta\Lambda^\gamma\Phi_{\alpha\beta\gamma} + \text{h.c.}\right\},
\end{aligned}
\tag{1}
$$

where the first two lines contain the fermion interactions. The Yukawa couplings are Y^u_{ij}, Y^d_{ij}, Y^a_i, Y^b_i, Y^c_i, and y, where $i, j = 1, 2, 3$. The fermion interactions of the model are thus completely governed by two 3×3 matrices, three 3×1 matrices, and a real number. The relevant terms are, again, featured in the first two lines of Eq. (1).

It is possible to freely rotate $SU(5)$ representations, prior to the $SU(5)$ symmetry breaking down to the Standard Model, in order to choose suitable basis to perform phenomenological analysis in. To that end, it is convenient to simultaneously redefine $\overline{5}_{F\,i}$ and $10_{F\,i}$ in such a way as to render Y^d real and diagonal whereas Y^u can be taken to be a symmetric matrix in the flavor space with complex entries. One can thus write that $Y^u_{ij} \equiv Y^u_{ji}$ and $Y^d_{ij} = Y^{d*}_{ij} \equiv \delta_{ij} Y^d_i$. One can also redefine 15_F and $\overline{15}_F$ to remove one of the phases in Y^c. These simplifications allow one to accurately determine that the total number of real Yukawa couplings of the model is nineteen while the total number of complex phases is fourteen. There is no other $SU(5)$ model in the literature that incorporates neutrino mass generation mechanism that has fewer parameters than the one under consideration. (For an extensive sample of the $SU(5)$ models to compare this proposal with, see Refs. [4–20].)

I will now briefly review the most prominent features of the model. The role of the field $\phi_0\,(1, 1, 0) \in 24_H$ in the Georgi-Glashow proposal is solely to break $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$ whereas the field $\Lambda_1(1, 2, 1/2) \in 5_H$ breaks $SU(3) \times SU(2) \times U(1)$ down to $SU(3) \times U(1)_{\text{em}}$ and, in the process, generates masses of the charged fermions. In this proposal, on the other hand, both $\phi_0(1, 1, 0)$ and $\Lambda_1(1, 2, 1/2)$ have additional tasks to accomplish beside the ones in the Georgi-Glashow model. Namely, $\phi_0(1, 1, 0)$ is instrumental in generating the experimentally observed mismatch between the down-type quark masses and the charged lepton masses whereas $\Lambda_1(1, 2, 1/2)$ helps to generate neutrino masses via a one-loop level mechanism. The fact that both $\phi_0(1, 1, 0)$ and $\Lambda_1(1, 2, 1/2)$ have additional roles when compared to the original Georgi-Glashow model attests to the economic nature of this novel proposal [1]. The relevant vacuum expectation values are $\langle 24_H\rangle \equiv \langle\phi_0(1, 1, 0)\rangle = v_{24}/\sqrt{15}\ \text{diag}(1, 1, 1, -3/2, -3/2)$ and $\langle 5_H\rangle \equiv \langle\Lambda_1(1, 2, 1/2)\rangle = (0 \quad 0 \quad 0 \quad 0 \quad v_5)^T$, where $v_5(= 174.104\,\text{GeV})$ is the Standard Model vacuum expectation value. (The effects associated with vacuum expectation values of the electrically neutral components of $\phi_1(1, 3, 0)$ and $\Phi_1(1, 4, -3/2)$ scalars are considered to be negligible.)

The model predicts that the neutrino masses are purely of the Majorana nature. The leading order contribution is generated at the one-loop level via the $d = 5$ operator. The relevant Feynman

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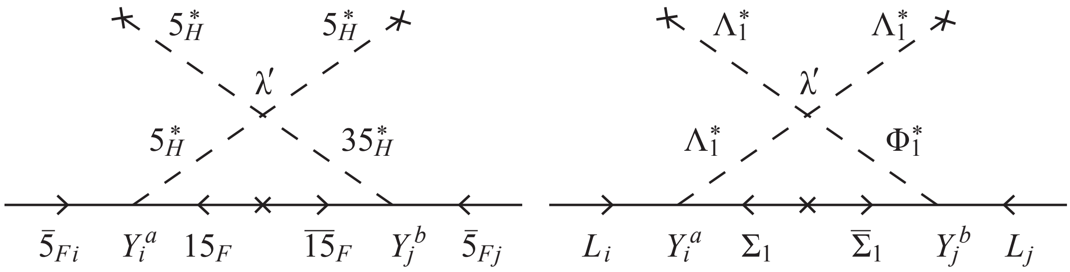

Figure 1: The Feynman diagrams of the leading order contribution towards the Majorana neutrino masses at the $SU(5)$ (left panel) and the Standard Model (right panel) levels.

diagrams, both at the $SU(5)$ and the Standard Model [21, 22] levels, are shown in Fig. 1. The neutrino mass matrix M_N reads

$$(M_N)_{ij} \approx \frac{\lambda' v_5^2}{8\pi^2}(Y_i^a Y_j^b + Y_i^b Y_j^a)\frac{M_{\Sigma_1}}{M_{\Sigma_1}^2 - M_{\Phi_1}^2}\ln\left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2}\right) = m_0(Y_i^a Y_j^b + Y_i^b Y_j^a)\,. \quad (2)$$

Clearly, M_N is constructed in the most minimal way imaginable out of two rank-one matrices with elements $Y_i^a Y_j^b$ and $Y_i^b Y_j^a$. This guarantees that one neutrino is massless which is one of the main predictions of the model.

Note that it is m_0 parameter of Eq. (2) that sets the neutrino mass scale through its dependence on *a priori* unknown parameters M_{Φ_1}, M_{Σ_1}, and λ'. There is thus a constrain on the available parameter space of the model in the M_{Φ_1}-M_{Σ_1} plane that originates solely from a need to generate sufficiently large m_0 parameter. I will, in order to enlarge m_0 as much as possible, use $|\lambda'| = 1$ in the numerical analysis. (Strictly speaking, Y^a and/or Y^b can always be redefined as to make λ' real and positive.)

The neutrino mass matrix elements, since $m_1 = 0$ in this model, can be written as

$$(M_N)_{ij} = m_0\left(Y_i^a Y_j^b + Y_i^b Y_j^a\right) = (N\,\mathrm{diag}(0, m_2, m_3)\,N^T)_{ij}\,, \quad (3)$$

where m_2 and m_3 are neutrino mass eigenstates and N is a unitary matrix. Since the charged leptons are already in the mass eigenstate basis because $Y_{ij}^d \equiv \delta_{ij} Y_i^d$, one can write N as

$$N = \begin{pmatrix} e^{i\gamma_1} & 0 & \\ 0 & e^{i\gamma_2} & 0 \\ 0 & 0 & e^{i\gamma_3} \end{pmatrix} V_{\mathrm{PMNS}}^*, \quad (4)$$

where V_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary mixing matrix with three mixing angles, one CP violating Dirac phase, and two Majorana phases. One can invert Eq. (3) using results of Refs. [23, 24] to obtain appropriate forms of Y^a and Y^b. Namely, the normal ordering yields

$$Y^a = \frac{1}{\sqrt{2}}\begin{pmatrix} i\,r_2\,N_{12} + r_3\,N_{13} \\ i\,r_2\,N_{22} + r_3\,N_{23} \\ i\,r_2\,N_{32} + r_3\,N_{33} \end{pmatrix}, \quad Y^b = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\,r_2\,N_{12} + r_3\,N_{13} \\ -i\,r_2\,N_{22} + r_3\,N_{23} \\ -i\,r_2\,N_{32} + r_3\,N_{33} \end{pmatrix}, \quad (5)$$

where $r_2 = \sqrt{m_2/m_0}$ and $r_3 = \sqrt{m_3/m_0}$. There are currently six phases in Eq. (5) that one can freely vary for the given M_{Φ_1}, M_{Σ_1}, and λ' to check the perturbativity of the Yukawa coupling elements in Y^a and Y^b. These phases are γ_1, γ_2, and γ_3 of Eq. (4), as well as one CP violating phase δ^{PMNS} and two Majorana phases in V_{PMNS}. Note that the fact that there are six arbitrary phases in Eq. (5) is expected since the six real parameters in Y^a and Y^b have been traded for three PMNS angles and three neutrino masses during the inversion procedure to obtain Eq. (5).

A presence of the vector-like fermions in 15_F and $\overline{15}_F$ induces experimentally observed mismatch between the masses of the charged leptons and the down-type quarks. The mismatch itself is due to a physical mixing between the vector-like fermions and fermions in 10_{Fi}. (The effect of this type of mixing on the charged fermion masses has already been studied within the context of a supersymmetric $SU(5)$ framework [25].) Namely, since the quark doublets $Q_i \in 10_{Fi}$ and $\Sigma_3 \in 15_F$ transform in the same way under the Standard Model gauge group, as can be seen from Table 1, these states interact at the $SU(5)$ symmetry breaking level, where the relevant mixing originates from the first term in the second line of Eq. (1) and explicitly reads

$$\mathcal{L} \supset \frac{1}{4}\sqrt{\frac{10}{3}} v_{24} Y^c_i \, Q_i \overline{\Sigma}_3 \,. \tag{6}$$

One can see, as argued before, that it is the vacuum expectation value of 24_H that plays an instrumental role in generating the observed mismatch between the masses of the charged leptons and the down-type quarks.

The mass matrices for the up-type quarks, down-type quarks, charged leptons, and neutrinos, in this model, are

$$(M_U)_{ij} = 4\, v_5 \,(Y^u_{ij} + Y^u_{ji}), \tag{7}$$

$$(M_D)_{ij} = v_5 \left(\delta_{ij} Y^d_i + \delta' \, Y^c_i Y^a_j \right), \tag{8}$$

$$(M_E)_{ij} = v_5 \, \delta_{ij} Y^d_i, \tag{9}$$

$$(M_N)_{ij} = m_0 \left(Y^a_i Y^b_j + Y^b_i Y^a_j \right), \tag{10}$$

where $\delta' \equiv (\sqrt{10/3} v_{24})/(4 M_{\Sigma_3})$. The mismatch between the down-type quark and charged lepton masses clearly originates from a single rank-one matrix with elements proportional to $Y^c_i Y^a_j$ product. This, again, attests to the simplicity of this $SU(5)$ proposal. It should also be noted that M_D and M_N share Y^a column matrix. The generation of the mismatch between the down-type quark masses and charged lepton masses is thus inextricably connected to the generation of the neutrino masses. Note also that the model simultaneously generates viable masses for the Standard Model fermions through the usual vacuum expectation value mechanism, the one-loop level mechanism, and the mixing between chiral fields and vector-like states [26].

This completes a brief introduction of the model.

2. Parameter space analysis

A numerical exploration of the entire parameter space of the model comprises three distinct steps that I outline in what follows.

The first step is to look at a viable gauge coupling unification at the one-loop level. To that end the masses of $\Phi_1, \Phi_3, \Phi_6, \Phi_{10} \in 35_H$, $\Sigma_1, \Sigma_3, \Sigma_6 \in 15_F$, $\phi_1, \phi_8 \in 24_H$, and $\Lambda_3 \in 5_H$ are varied to find the largest possible value of the gauge coupling unification scale $M_{\rm GUT}$. This approach gives the most conservative estimate of the available parameter space since the largest possible unification scale corresponds to the largest possible proton decay lifetimes one would need to probe to fully test the model. For the one-loop gauge coupling running the relevant Standard Model input parameters are $M_Z = 91.1876\,\mathrm{GeV}$, $\alpha_S(M_Z) = 0.1193 \pm 0.0016$, $\alpha^{-1}(M_Z) = 127.906 \pm 0.019$, and $\sin^2\theta_W = 0.23126 \pm 0.00005$ [27].

The lagrangian of Eq. (1) yields two mass relations that need to be taken into account for the gauge coupling unification analysis. These relations are

$$M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1}, \tag{11}$$

$$M^2_{\Phi_{10}} = M^2_{\Phi_1} - 3M^2_{\Phi_3} + 3M^2_{\Phi_6}, \tag{12}$$

where I denote the masses of the fields using the nomenclature of Table 1. It turns out that the viable unification of gauge couplings, in combination with Eqs. (11) and (12), prefers that M_{Σ_6}, M_{Σ_3}, and M_{Σ_1} are mass degenerate while the multiplets in 35_H split into two separate mass degenerate pairs $(M_{\Phi_{10}}, M_{\Phi_1})$ and (M_{Φ_3}, M_{Φ_6}). One also needs to insure that a proton does not decay too rapidly through the scalar leptoquark mediation. To that end one needs to have $M_{\Lambda_3} \geq 3 \times 10^{11}\,\mathrm{GeV}$ in order for the scalar induced proton decay to be under control [28].

A lower limit on the mass(es) of the new physics state(s) is introduced, before a numerical procedure for the generation of viable unification points is implemented, to explore the possible connection between the most accessible scale of new physics and $M_{\rm GUT}$. This limit is introduced through a mass parameter $M = \min(M_J)$, where $J = \Phi_1, \Phi_3, \Phi_6, \Phi_{10}, \Sigma_1, \Sigma_3, \Sigma_6, \phi_1, \phi_8, \Lambda_3$, that is set at 1 TeV, 10 TeV, and 100 TeV. It is already at this stage that a part of potentially viable parameter space can be discarded. Namely, since the neutrino mass scale explicitly depends on M_{Φ_1} and M_{Σ_1} via m_0 parameter of Eq. (2) it is easy to construct a two-dimensional parameter space spanned by M_{Φ_1} and M_{Σ_1} where one could, at least in principle, expect realistic explanation of neutrino masses, with perturbative couplings, within this model.

Once all unification points that allow for generation of viable neutrino mass scale are found, the second step of the numerical analysis is implemented. Namely, all the masses and mixing parameters of the Standard Model charged fermions are run to $M_{\rm GUT}$ using the factual new physics mass spectrum associated with a given unification point to account for all the threshold corrections between the low scale and $M_{\rm GUT}$ and then an accurate numerical fit of the Standard Model observables and neutrino mass parameters is performed. The charged fermion mass renormalization group running is performed at the one-loop level [29]. Note that one can separate the gauge coupling unification study from the running of the Standard Model charged fermion parameters, at this level of accuracy, since the latter provides feedback to the former only at the two-loop level whereas the former impacts the latter already at the one-loop level. The input for the numerical fit is presented in Table 2.

The combined numerical fit of the down-type quark and neutrino sectors demonstrates that this novel proposal cannot accommodate the inverted neutrino mass ordering. Note that Y^d is a hierarchical diagonal matrix, where its entries are completely determined by the charged lepton

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Yukawa couplings. Since the matrix elements $(M_D)_{ij}$ are proportional to the linear combination of $\delta_{ij}Y^d_j$ and $Y^c_i Y^a_j$ it is obvious that Y^a and Y^c should both be hierarchical column matrices to produce a good fit to data. This, however, is incompatible with the inverted ordering of the neutrino masses. Note also that the perturbativity of the couplings in Y^a and Y^b is accurately reassessed during the second step since both m_0 and M_N are fully known.

The third step of the analysis begins upon completion of the numerical fit of the fermion observables for all viable unification points. Namely, the constraints due to the proton decay signatures are tested against every single point that corresponds to a realistic gauge coupling unification and viable description of the Standard Model fermion observables. This produces an accurate constraint since all the relevant input parameters for such an analysis are known including $M_{\rm GUT}$, $\alpha_{\rm GUT}$, unitary transformations of the Standard Model fermions, Yukawa couplings, short-distance coefficients, et cetera. Here $\alpha_{\rm GUT}$ stands for the $SU(5)$ gauge coupling at $M_{\rm GUT}$.

It turns out that the most stringent experimental limit, i.e., the limit on the $p \to \pi^0 e^+$ partial lifetime, provides the best constraint on the available parameter space through the predictions for the gauge boson mediated proton decay, where the relevant gauge boson masses are identified with $M_{\rm GUT} = \sqrt{(5\pi)(6)\alpha_{\rm GUT}}v_{24}$. (For the full description of the proton decay analysis see Ref. [2].)

At the end of these three steps, one is left with a viable set of unification points that is in agreement with all currently accessible experimental results and that is what I present in left panels of Fig. 2 for three different values of $M(= 1\,\text{TeV}, 10\,\text{TeV}, 100\,\text{TeV})$. It is important to note that any further improvement in experimental values of the Standard Model parameters, such as the actual determination of the neutrino masses, measurement of the CP phase in the leptonic sector, or an input on Majorana phases, will only add to the precision of the model's predictions.

The viable parameter space of the model is given in the three left panels of Fig. 2 in the M_{Φ_1}-M_{Σ_1} plane, where I present the contours of constant $M_{\rm GUT}$, $\alpha_{\rm GUT}$, and m_0 for $|\lambda'| = 1$. The $M_{\rm GUT}$ contours are given in units of 10^{15} GeV and appear as vertical solid lines while the $\alpha_{\rm GUT}$ contours are given as dot-dashed lines that run horizontally. The contours of constant m_0, in units of $\sqrt{\Delta m^2_{31}}/2$ for $|\lambda'| = 1$, are shown as green solid curves. The unification scale, again, is maximized by varying masses M_J, where $J = \Phi_1, \Phi_3, \Phi_6, \Phi_{10}, \Sigma_1, \Sigma_3, \Sigma_6, \phi_1, \phi_8, \Lambda_3$, while taking into account additional constraints of Eqs. (11) and (12), requiring that $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV, and imposing a

$m(M_Z)$ (GeV)	Fit input	$\theta^{\rm CKM,PMNS}_{ij}$ & $\delta^{\rm CKM}$ & Δm^2_{ij} (eV^2)	Fit input
$m_u/10^{-3}$	1.158 ± 0.392	$\sin\theta^{\rm CKM}_{12}$	0.2254 ± 0.00072
m_c	0.627 ± 0.019	$\sin\theta^{\rm CKM}_{23}/10^{-2}$	4.207 ± 0.064
m_t	171.675 ± 1.506	$\sin\theta^{\rm CKM}_{13}/10^{-3}$	3.640 ± 0.130
$m_d/10^{-3}$	2.864 ± 0.286	$\delta^{\rm CKM}$	1.208 ± 0.054
$m_s/10^{-3}$	54.407 ± 2.873	$\Delta m^2_{21}/10^{-5}$	7.425±0.205
m_b	2.854 ± 0.026	$\Delta m^2_{31}/10^{-3}$	2.515±0.028
$m_e/10^{-3}$	0.486576	$\sin^2\theta^{\rm PMNS}_{12}/10^{-1}$	3.045±0.125
m_μ	0.102719	$\sin^2\theta^{\rm PMNS}_{23}$	0.554±0.021
m_τ	1.74618	$\sin^2\theta^{\rm PMNS}_{13}/10^{-2}$	2.224 ± 0.065

Table 2: Experimental observables associated with charged fermions [30] and neutrinos for normal ordering [31] with $1\,\sigma$ uncertainties (except for charged leptons).

condition that $M = \min(M_J)$ is equal to 1 TeV, 10 TeV, and 100 TeV in the panels of the first, second, and third row of the left column of Fig. 2, respectively.

In the right panels of Fig. 2 I present the running of the gauge couplings for one particular unification point, i.e., when $M_{\Phi_1} = M_{\Sigma_1} = 10^{13.19}$ GeV, for the $M = 1$ TeV, $M = 10$ TeV, and $M = 100$ TeV scenarios in the first, second, and third row, respectively. The unification points that correspond to these new physics mass spectra are denoted with A, A′, and A″ in the left panels of Fig. 2. It is clear that scalar multiplets $\phi_1(1,3,0)$, $\phi_8(8,1,0)$, $\Phi_3(\overline{3},3,-2/3)$, and $\Phi_6(\overline{6},2,1/6)$ need to be light to maximize M_{GUT} whereas the multiplets $\Sigma_1, \Sigma_3, \Sigma_6 \in 15_F$ tend to be mass degenerate.

The parameter space that is viable with respect to the experimental input when $M = 1$ TeV, $M = 10$ TeV, and $M = 100$ TeV can be read off from the left panels of Fig. 2. Namely, it is bounded from the left by the proton decay curve and from the right by the outermost dashed curve. The outermost dashed curve delineates the region after which it is not possible to address phenomenologically viable neutrino mass scale with perturbative couplings. It is obtained by setting $|\lambda'|$ to one and freely varying M_{Φ_1}, M_{Σ_1}, and six phases in Eq. (5) to find the region where the product $\max(|Y_i^a|)\max(|Y_j^b|)$, where $i, j = 1,2,3$, does not exceed one. Contours of constant m_0 show that the novel proposal predicts neutrino mass scale to be within three orders of magnitude, in the most conservative scenario when $M = 1$ TeV. This is another nice feature of the model.

The proton decay bound in Fig. 2 is generated by the experimental limit on the partial lifetime for the $p \to \pi^0 e^+$ process that is currently at $\tau^{\text{exp}}_{p\to\pi^0 e^+} > 2.4 \times 10^{34}$ years, as given by the Super-Kamiokande Collaboration [32]. An improvement of the current $p \to \pi^0 e^+$ lifetime limit by a factor of 2, 15, and 96 would completely rule out the $M = 100$ TeV, $M = 10$ TeV, and $M = 1$ TeV scenarios, respectively. The last viable point to be eliminated by the aforementioned improvement, in all three left panels of Fig. 2, is $(M_{\Phi_1}, M_{\Sigma_1}) = (10^{13.2}\,\text{GeV}, 10^{13.6}\,\text{GeV})$. This is to be expected since α_{GUT} grows with a decrease in the Σ_1 mass for a fixed value of M_{Φ_1}, whereas M_{GUT} remains constant.

Note that the numerical fit of fermion masses explicitly yields all unitary transformations and Yukawa couplings except for the phases associated with the up-type quark sector. The couplings and, accordingly, interactions of scalar multiplets $\phi_1(1,3,0)$, $\phi_8(8,1,0)$, $\Phi_3(\overline{3},3,-2/3)$, and $\Phi_6(\overline{6},2,1/6)$ are thus fully calculable. An analysis of the decay modes and associated lifetimes of these scalars as well as a full-fledged study of the proton decay signatures via the gauge boson and scalar leptoquark mediations is left for future publications.

3. Conclusion

I present a phenomenological study of the viable parameter space of the most minimal realistic $SU(5)$ model to date. The structure of the model is built entirely out of the fields residing in the first five lowest lying representations in terms of dimensionality that transform non-trivially under the $SU(5)$ gauge group. These representations are 5_H, 24_H, 35_H, $\overline{5}_{Fi}$, 10_{Fi}, 15_F, $\overline{15}_F$, and 24_V, where subscripts H, F, and V denote whether a given representation contains scalars, fermions, or gauge bosons, respectively, while $i = 1,2,3$. The Yukawa couplings are $Y^u_{ij} = Y^u_{ji}$, $Y^d_{ij} = Y^{d*}_{ij} = \delta_{ij} Y^d_i$, Y^a_i, Y^b_i, Y^c_i, and y, where $i, j = 1,2,3$. The model has nineteen real parameters and fourteen phases, all in all, to address experimental observables of the Standard Model fermions and accomplishes that

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via simultaneous use of three different mass generation mechanisms. It inextricably links the origin of the neutrino mass to the experimentally observed difference between the down-type quark and charged lepton masses. The main predictions of the model are that (a) the neutrinos are Majorana particles, (b) one neutrino is massless, (c) the neutrinos have normal mass ordering, and (d) there are four new scalar multiplets at or below a 120 TeV mass scale if they are mass degenerate. An improvement of the current $p \to \pi^0 e^+$ lifetime limit by a factor of 2, 15, and 96 would require these four scalar multiplets to reside at or below the 100 TeV, 10 TeV, and 1 TeV mass scales, respectively, under the assumption of the multiplet mass degeneracy. If these multiplets are not mass degenerate, the quoted limits then apply, for all practical purposes, to the geometric mean of their masses. The scalar multiplets in question transform as $(1, 3, 0)$, $(8, 1, 0)$, $(\overline{3}, 3, -2/3)$, and $(\overline{6}, 2, 1/6)$ with calculable couplings to the Standard Model fields.

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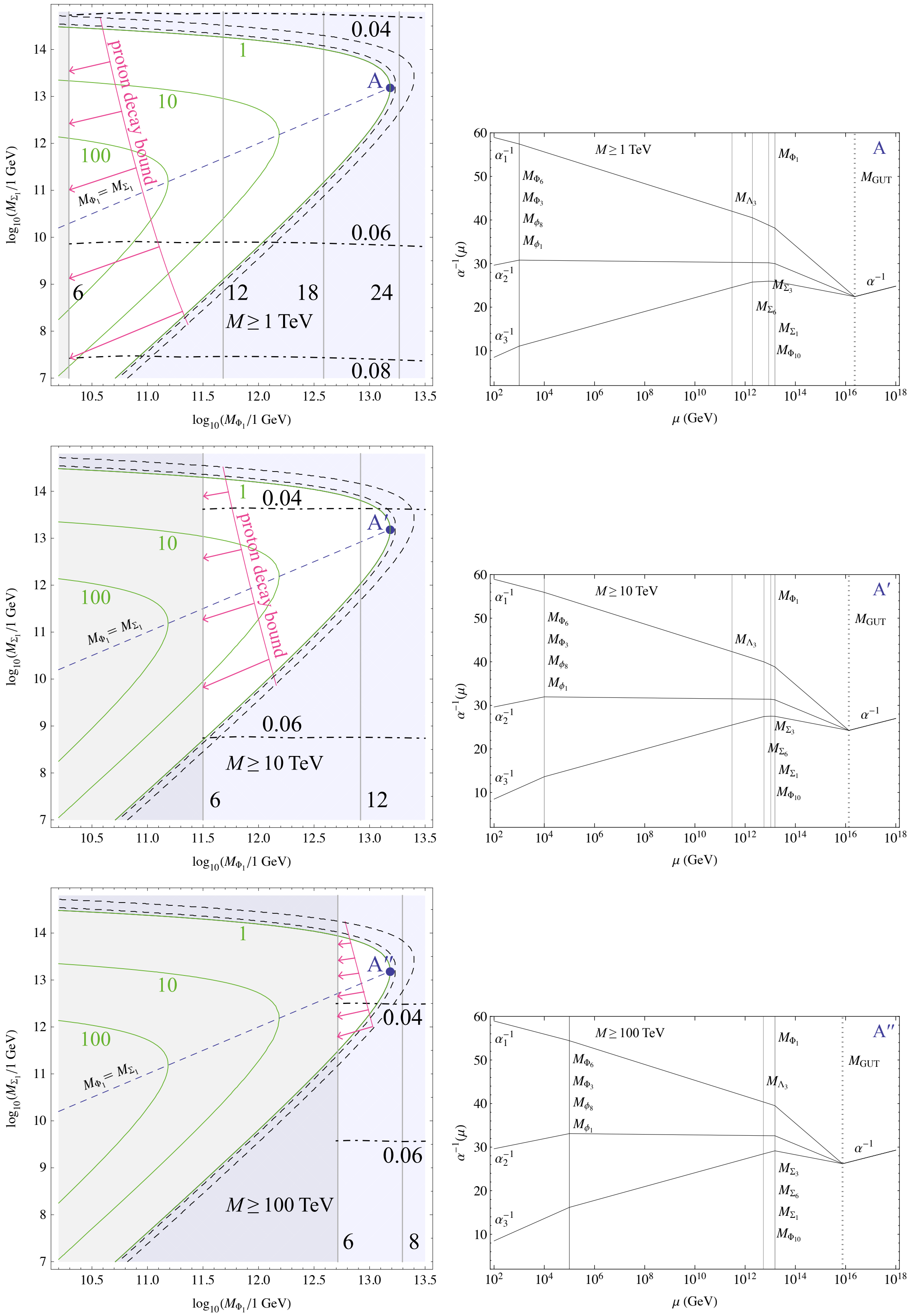

Figure 2: Experimentally viable parameter space of the model (left panels) and the gauge coupling unification for the unification points A, A′, and A″ (right panels) when $M \geq 1, 10, 100$ TeV, as indicated. For additional details see the text.