

Nanometer Size Dirty Dark Matter Pearls, e^- -signal, IMP or SIDM, not WIMP *

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Through several articles we have developed a model for dark matter as consisting of bubbles of a new (speculated) type of vacuum, starting from cm-sized pearls or balls down to atomic size ones and now we believe they have nanometer sizes. In the latest development of our model we have the bubbles of the new vacuum imbedded in dust grains very similar to the grains present in interstellar and intergalactic space anyway, although the presence of the bubble with a very large homolumo gap in its single electron spectrum influences the dust grain material so as to become denser and harder.

We have earlier explained how our dark matter particles get stopped in the shielding, so that normally expected nucleonic collisions are not observable. The signal of the dark matter in the underground experiments rather becomes decays of excited particles actually with the energy of the homolumo gap, which is also equal to the photon energy of the X-ray line presumably observed astronomically from galaxy clusters etc.

A new calculation here is a fitting of the velocity dependence of the dark matter self-interaction as estimated by Correa [15], using deviations from the only gravitationally interacting dark matter in dwarf galaxies.

Let us stress that apart from the speculated new vacuum we have no new physics, and if the couplings in the Standard Model were adjusted to make degenerate vacua as speculated according to our Multiple Point Principle (MPP) we would only need the Standard Model, so dark matter would *not* require new physics.

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*IMP=" Interacting Massive Particles ".

SIDM = "Self interacting Dark Matter".

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1. Introduction

One of the most mysterious facts about the dark matter is that the DAMA-LIBRA experiment, which only recognizes dark matter from its annually varying impact rate, sees the dark matter, while seemingly very similar experiments using xenon and able to distinguish, if the dark matter particle has hit a nucleus or say an electron, do not see dark matter. We take this mystery to point towards models of the type of Khlopov [1–4], wherein the dark matter particle is larger and more composite, as e.g. Khlopov’s model with an a priori doubly negatively charged object having been neutralized by a He-nucleus.

In fact we have proposed for some time dark matter model(s) based on the idea of having several vacua with the same energy density [5–14]. Then the dark matter particle is a bubble or pearl of one of the new vacua filled with some ordinary matter - e.g. diamond - under a very high pressure caused by the surface tension of the surface - the domain wall - separating the two phases of vacuum. But recently we got to expecting these bubbles to be surrounded by an essentially normal dust grain, of which there are so many anyway in the interstellar or intergalactic medium. This dust material may though be made harder and more dense by effects of the homolumo gap in the pearl influencing the material of the dust.

The crux of the matter for our model is now that the dark matter pearls are so large that they interact significantly with each other and with ordinary matter. In fact especially Correa [15] found phenomenological evidence for an even velocity dependent cross section in as far as the ratio $\frac{\sigma}{M}$ of the cross section σ to the mass M , which is the only quantity you can extract when the observation is indirect in the sense that you only can measure the dark matter via its influence on the ordinary matter, stars say, turned out to fall with velocity. It is shown in Figure 8 how this ratio falls off with collision velocity v .

A rather recent calculation consists in seeking to understand the velocity dependence of the ratio $\frac{\sigma}{M}$ in terms of our model of a highly dense pearl surrounded by a rather usual dust grain only made somewhat harder and more condensed by the homolumo gap effect on the dust grain.

When the dark matter particles reach the Earth we imagine that the dust grain around the genuine new vacuum bubble gets boiled off, presumably already in the atmosphere. At the same time the pearl gets excited and some electrons in the interior get excited across the homolumo gap. This leads some electrons first losing energy to about the lumo-state energy (just at the bottom of the unfilled states) and, if some excitations can survive so long that the excited pearl can reach down to the experiments looking for dark matter underground, the emission of photons or electrons with energies given by the homolumo gap may be detected. So our model predicts that the underground observations should mainly be electron events just with energy equal to homolumo gap size 3.5 keV - taken to be so as to identify such decays also with the source of the 3.5 keV X-ray radiation observed by satellites [16–21]. It is indeed remarkable that the average of the energy per event, both in the DAMA-LIBRA experiment [22, 23] and in the recently observed mysterious excess of events seemingly coming from electron collisions observed with low statistics by Xenon1T [24], is close to the value 3.5 keV!

This difference of our model from the most popular type of model, the WIMP type of models, can be claimed by saying: Our model could be called IMP or SIDM but not WIMP, where IMP = “Interacting Massive Particles” and SIDM = “Self-Interacting Dark Matter”.

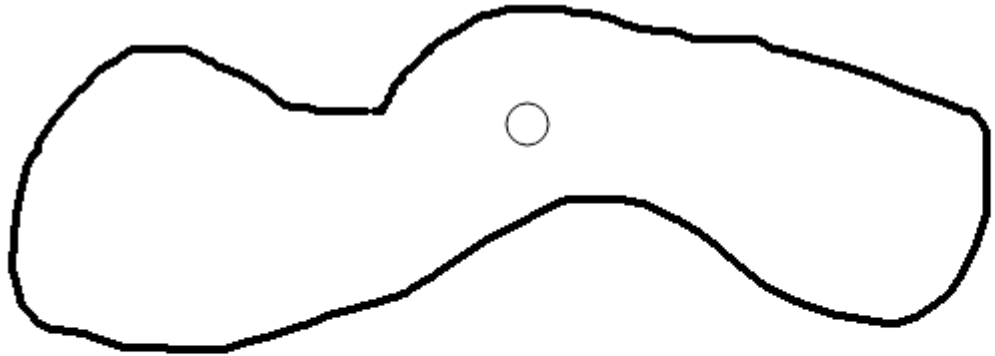


Figure 1: Simplified crude picture of our dark matter particle as a small accurately spherical bubble of new vacuum with some ordinary matter inside, say carbon, under high pressure. Around it is a more irregular grain of dust having been collected by the little bubble, much like an ordinary grain of dust in the intergalactic or interstellar space.

A bit similar to Khlopov’s model of double negatively charged particles leading to zero charged He that can be stopped, our model assumes:

- *The dark matter particles interact so strongly as to get stopped in the earth shielding before reaching the direct dark matter searches like DAMA, LUX, XenonIT,...*
- *They can get excited and emit energy as electrons or photons, say hours or even years after hitting the Earth..*

Thus DAMA-LIBRA may see them as dark matter, but experiments needing recoil nuclei will not accept them as dark matter.

2. Pearl

2.1 Dark Nanometer Size Pearls, Electronic 3.5 keV Signal, IMP or SIDM, not WIMP

Figure 1 illustrates a fraction of a micrometer size small dust grain with an about a nanometer big new vacuum bubble inside. This complicated/macroscopic object corresponds to a dark matter particle in our model. There are about $10^{12}/12 \approx 10^{11}$ carbon “atoms” inside the bubble.

- In the middle is a spherical bubble of radius

$$R \approx r_{cloud\ 3.3MeV} \approx 10^{-9}m \quad (1)$$

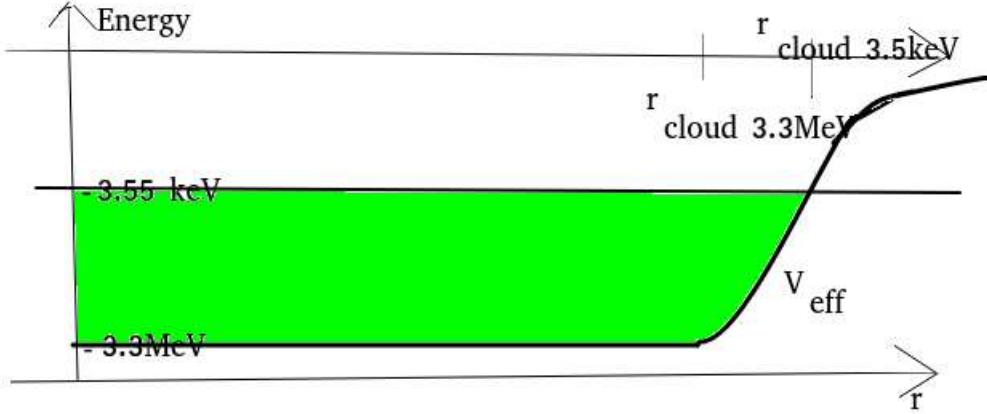


Figure 2: The green filled out region symbolizes the Fermi-sea of electrons. They have energies above the effective potential, which is the lower end of this ‘green’ region. On the left axis is the electron energy. The numbers $r_{\text{cloud } 3.5 \text{ keV}}$ and $r_{\text{cloud } 3.3 \text{ MeV}}$ give the distances from the center to the points where the potential numerically has the values in the index.

Here $r_{\text{cloud } 3.3 \text{ MeV}}$ denotes the radius where the electron potential is 3.3 MeV , which is identified with the Fermi energy E_f of the electrons in the bulk of the pearl - i.e. inside the radius R . We estimated the value $E_f = 3.3 \text{ MeV}$ in previous papers [11–13] by fitting the overall rate of the intensity of the 3.5 keV line emitted by galactic clusters and the very frequency 3.5 keV of the radiation in our model.

- The radius $r_{\text{cloud } 3.5 \text{ keV}}$ is where the electron potential is 3.5 keV . By our story of the ‘‘homolumo gap’’: the electron density crudely goes to zero at this radius. (It falls a lot in the range between $r_{\text{cloud } 3.3 \text{ MeV}}$ and $r_{\text{cloud } 3.5 \text{ keV}}$). The distance between the two different radii $R = r_{\text{cloud } 3.3 \text{ MeV}}$ and $r_{\text{cloud } 3.5 \text{ keV}}$ is only about 10^{-12} m , so with nanometer size pearls the essentially purely electron region between the two is only a rather thin layer.

2.2 The electron density and potential in the pearls

- Due to an effect we call the *homolumo-gap effect* [10, 25], the nuclei in the bubble region and the electrons themselves become arranged in such a way as to prevent there from being any levels in an interval of width 3.5 keV . So outside the distance $r_{3.5 \text{ keV}} = r_{\text{cloud } 3.5 \text{ keV}}$ from the center of the pearl at which the Coulomb potential is $\sim 3.5 \text{ keV}$ deep there are essentially (\sim in the Thomas-Fermi approximation) no more electrons in the pearl-object, as illustrated in Figure 2 .
- The radius $r_{3.3 \text{ MeV}} = r_{\text{cloud } 3.3 \text{ MeV}}$ at which the potential felt by an electron is 3.3 MeV deep, is supposed to be just the radius to which the many nuclei replacing the only one nucleus in ordinary atoms reach out. So inside it the potential is much more flat.

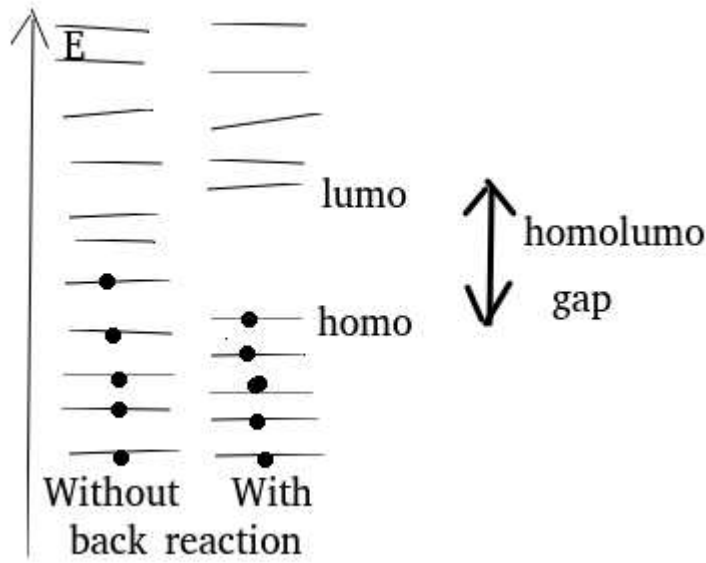


Figure 3: The electron energy spectrum in the left column is supposed to be what one would get without *back reaction*, meaning ignoring that the electrons act back on the nuclei and other electrons around, so as to put them in a way influenced by the electrons in the electron eigenstates. The spectrum to the right is supposed to be modified relative to the one to the left by such a back reaction. The energy of the total system can be lowered, if the nuclei and other electrons adjust so as to lower the filled levels.

- The energy difference between the zero energy line and the effective Fermi surface above which there are no more electrons is of order of 3.5 keV, the value so crucial to our work.
- Since in the Thomas Fermi approximation there are no electrons outside roughly the radius $r_{3.5keV} = r_{cloud\ 3.5keV}$, this radius will give the maximal self-interaction cross section, even for very low velocity $\sigma_{v \rightarrow 0}$, were it not for the dust grain supposedly having built up around the pearl. We shall discuss the dust extension more below.

2.3 The homolumo-gap effect.

Think first of the spectrum of energy levels for the electrons by assuming at the start the positions or distributions of the charged particles in the piece of material studied, e.g. one of our pearls, as fixed.

Then the ground state is just a state built e.g. as a Slater determinant for the electrons being in the lowest single electron states, so many as are needed to have the right number of electrons.

But now if the charged particles can be moved due to their interactions, the ground state energy could be lowered by moving them so that the energy of the filled electron state levels get lowered.

So we expect introducing such a “back reaction” will lower the filled states.

When the filled levels get moved downwards, then the homo= “highest occupied molecular orbit” level will be lowered and its distance to the lumo= “lowest unoccupied molecular orbit” increased by the effect of the back reaction, and so an exceptionally large region, the “homolumo-

gap” with no single particle electron levels, will appear on the energy axis, as illustrated in Figure 3.

We believe we can estimate the homolumo-gap E_H .

Using the Thomas Fermi approximation - or crudely just some dimensional argument, where the fine structure constant has dimension of velocity - we calculated the homolumo gap in highly compressed ordinary matter, for relativistic electrons [12]:

$$E_H \sim \left(\frac{\alpha}{c}\right)^{3/2} \sqrt{2} p_f \quad (2)$$

$$\text{where } p_f = \text{Fermi momentum} \quad (3)$$

$$\frac{\alpha}{c} = \frac{1}{137.03\dots} \quad (4)$$

(the $\sqrt{2}$ comes from our Thomas Fermi calculation).

It is by requiring this homolumo-gap to be the 3.5 keV energy per photon of the X-ray radiation mysteriously observed by satellites astronomically from clusters of galaxies, Andromeda and the Milky Way Center [16–21] that we estimate the Fermi-energy to be $E_f \approx p_f = 3.3$ MeV in the interior bulk of the pearl.

3. The Dirty Pearls

The most important and heaviest parts of the dark matter particles are the small regions of a new vacuum just described with its surrounding range of electrons which, because of the high pressure in the degenerate collection of electrons, tend to be somewhat pressed out compared to the nuclei which, because they are not degenerate, are easier to be kept inside. However, these pearls survive in the intergalactic or interstellar space for more than 13 milliard years, and so we expect them to become dirty. In fact there exist dust grains in space, typically of the order of $0.1\mu\text{m}$ in size and consisting of silicates or chemical compositions with carbon. A typical model picture of such a dust grain in the outer space is given in Figure 4.

It is indeed very likely that such a dust grain would collect in many cases on top of one of our dark matter particles, which is in itself very much like a seed atom. We may illustrate that by drawing our little pearl with its new vacuum as a ball inside the usual dust grain model picture 5.

3.1 Influence of Homolumo gap on the Dust Matter

In this subsection we want to speculate that the adjustment of the degrees of freedom mainly in the pearl or bubble of new vacuum will influence the properties of the dust grain built up around this little heavy pearl - we shall return in section 4.1.3 to this subject. The argument suggesting that a rather strong effect of the homolumo gap is expected is the following: we might consider the whole object - dust grain plus the bubble in the middle - as one could say a molecule, so that the single electron spectrum *for the whole dark matter particle with both dust and bubble* should have the homolumo gap 3.5 keV. Now this gap size 3.5 keV is for usual chemistry a very high number, only reached as a binding energy for the very tightest bound electrons. Thus if the wave functions filled in the whole dark matter object should have the binding energy of the order of 3.5 keV, it would either be needed that an electron wave function would have so much overlap with the interior

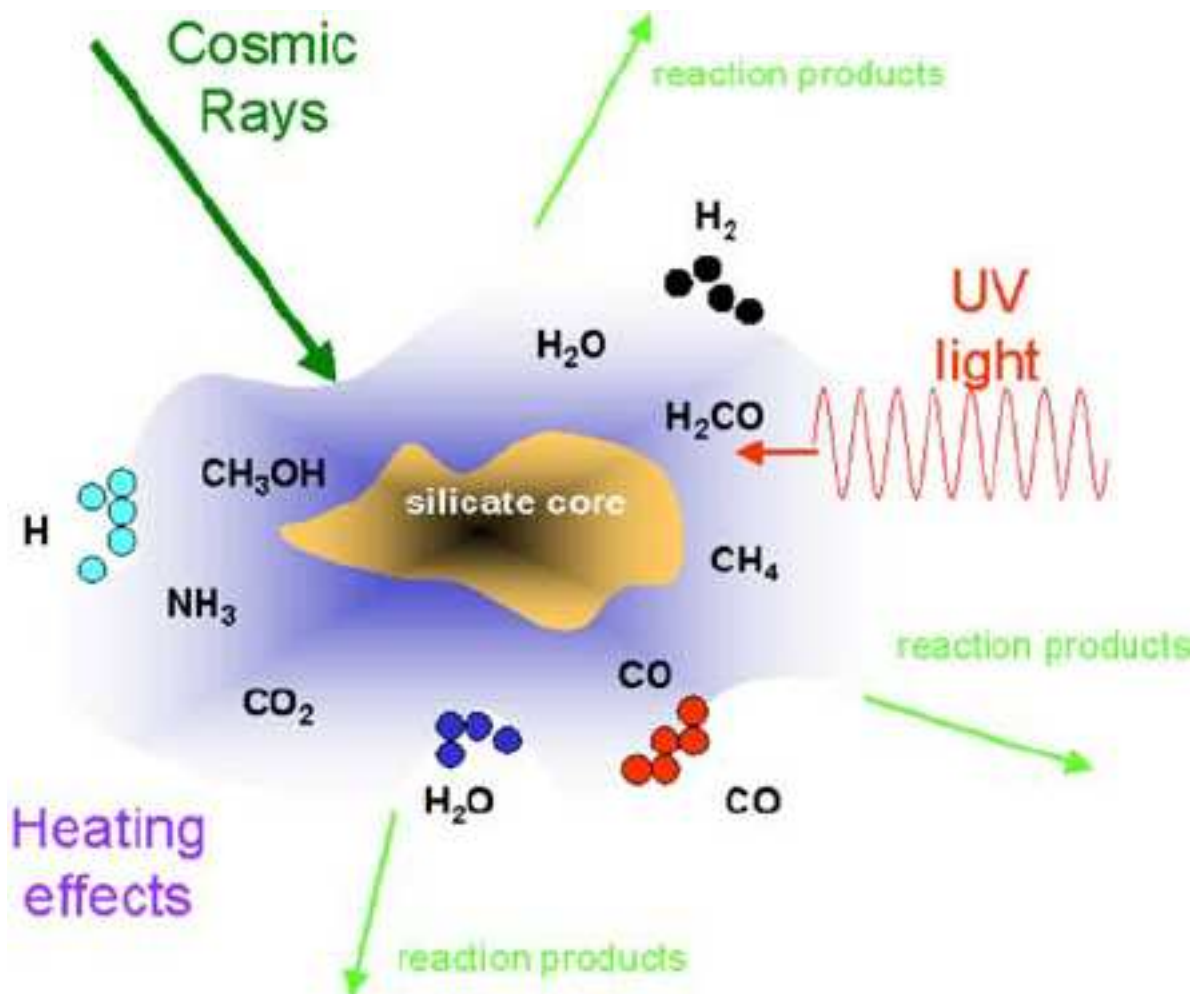


Figure 4: Model of Dust Particle

pearl region that this overlap could quite dominate the binding energy of the whole wave function, or the part of the wave function in the region outside in the dust grain should concentrate around nuclei very strongly. Essentially in this latter possibility only the most strongly bound electronic orbits in the dust region should be filled.

To get an idea about, what we should expect the filled wave functions to look like in the outside region in the dust grain, let us think of constructing these wave functions in this dust grain region by expanding them as macroscopically slowly varying superpositions of one orbit around the atoms in the outside. This is very close to what one does by using Bloch wave functions in the tight binding approximation in solid state physics.

In fact, if we take a wave function ansatz to be composed from only one specific atomic level with binding energy E_l as a from atom to atom smoothly varying function $\psi(\vec{x})$ of the atom positions, we can in the macroscopic approximation, that there are many atoms inside the length scale we are interested in, describe the electron as behaving freely except that it feels a constant negative potential of E_l in the whole dust grain region. Only in the very small sphere, where the bubble sits, there is a very strong and quite different effective potential. But now we must - assuming

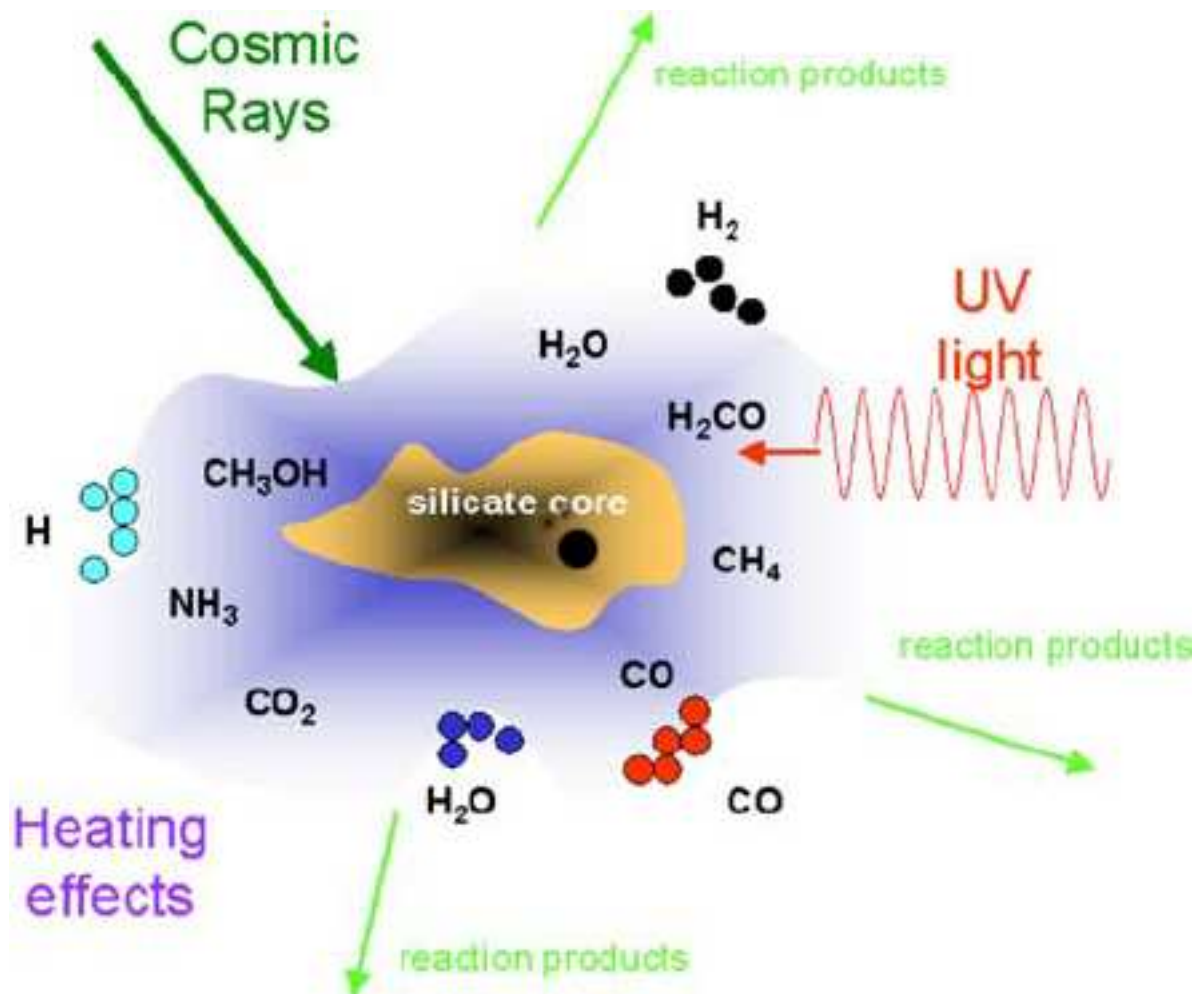


Figure 5: The little dot inserted in the foregoing figure here symbolizes the bubble of new vacuum. It is supposedly much heavier than the rest of the dust grain.

that the homolumo gap for the whole system shall be of the large value 3.5 keV - believe that the system has adjusted, so that all the filled states have negative energy of the order of 3.5 keV. This means that considering a wave function ansatz constructed from the upper levels in the atoms in the dust grain region - thus having only relatively small potential E_l - it will be strongly / exponentially concentrated around the bubble (with its high binding). Only if the atomic level from which we construct the wave function ansatz has a numerically large binding E_l , preferably -3.5 keV, will the ansatz wave function be able to extend far out in the dust grain.

Thus it appears that looking at the outskirts of the dust grain the electrons in the filled states reaching out there must be essentially all along in the most strongly bound atomic orbits for the atoms in the dirt. This means that this dirt-material should be imagined to be significantly different in internal structure from the dirt in a dust grain without our bubble pearl inside it.

Actually we expect the dust grain with a bubble inside to have the electrons only in the lowest atomic orbits bound by orders of keV rather than as usual eV. This might even mean that these atoms become positive ions in as far as there will be too few deeply bound atomic orbits. However,

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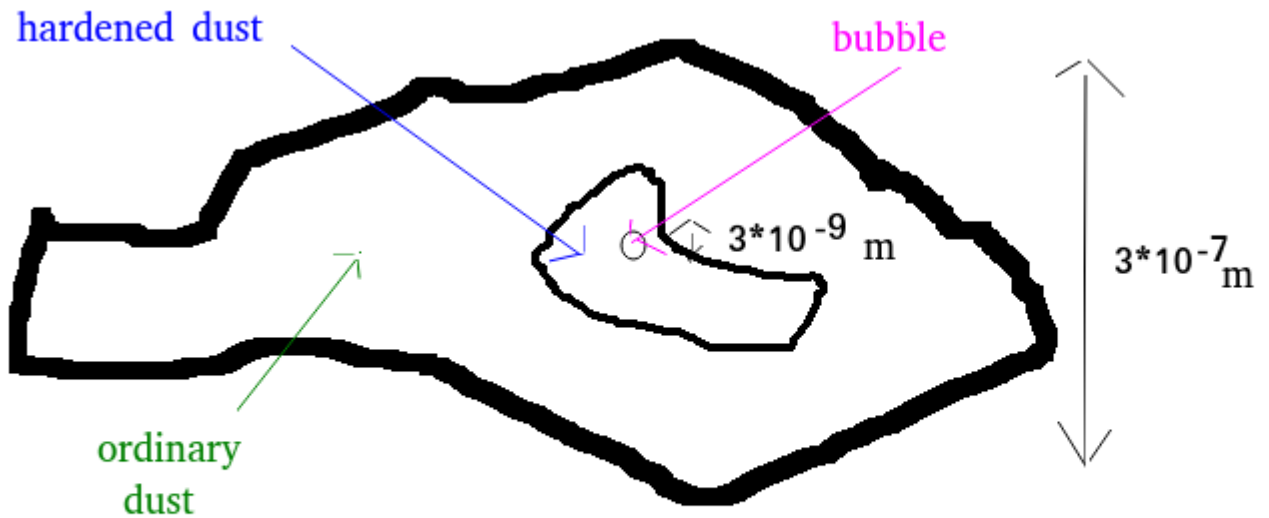


Figure 6: The crude dimensions of our dark matter model particle including its dust grain, which is typically irregular. The much smaller genuine bubble of new vacuum is, however, very accurately spherical, because it is so as to minimize the skin or surface area for the essentially given volume of highly compressed ordinary material on the background of the second vacuum. In the dust material but near to the pearl we expect the stuff to be hardened due to an effect from the homolumo gap, which inside the bubble has for ordinary matter an unusually high value 3.5 keV.

presumably the density of the nuclei can adjust so that the total charge density can be zero anyway.

Next one may then wonder how this by the homolumo gap modified structure of the atoms would influence the properties of the material. Presumably the geometrically smaller size of the deeper bound orbits would make the density of the dust material higher. The atoms can come closer. Also the scale of the binding energy of the atoms to each other would likely be scaled up and the material would be harder.

3.2 Our Picture of Dark Matter Pearls:

- **Principle** Nothing but Standard Model! (*seriously would mean not in a BSM-workshop.*)
- **New Assumption** Several Phases of Vacuum with Same Energy Density [26–31].
- **Central Part** Bubble of New Phase of Vacuum with E.g. carbon under very High Pressure, surrounded by a surface with tension S (=domain wall) providing the pressure.
- **Outer part** Cloud of Electrons much like an ordinary Atom with a nucleus having a charge of order of the number of electrons in the thin layer of electrons outside the genuine bubble region.

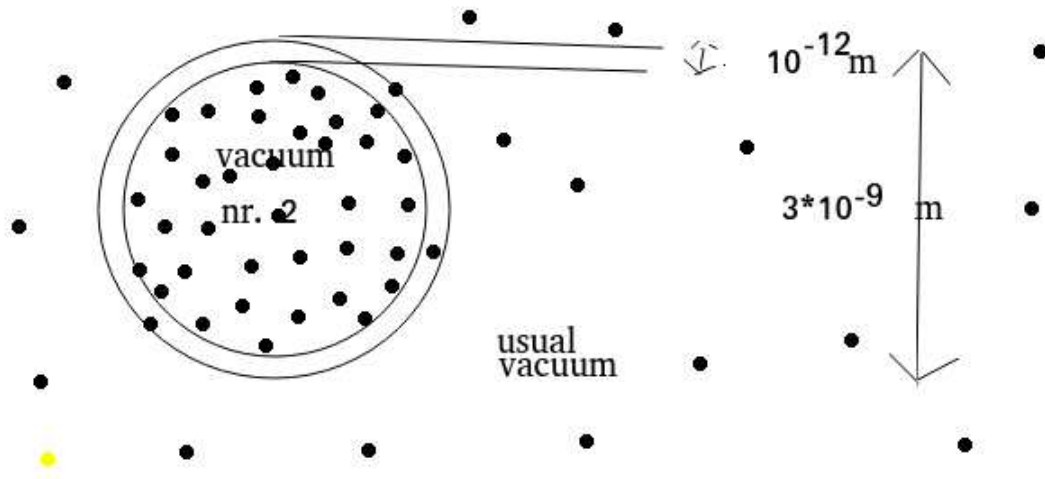


Figure 7: Illustration of the bubble of new vacuum as imbedded into a dust grain, which is not fully drawn. The size of the bubble is of the order of a few nanometers surrounded by a much thinner layer of electrons having been pushed a bit, about $10^{-12}m$, outside the domain wall separating the two vacuum phases. Because of the high pressure from this domain wall (=skin), the density of atomic nuclei inside the skin is expected to be larger than outside, even though the dust material just outside should have been hardened by an effect of the homolumo gap (see subsection 3.1). The density of nuclei shown on the figure here, both inside and outside, is lower compared to reality in our model in as far as the distance between the nuclei is of the order of $10^{-12}m$ in our model, so that a chain of atoms across the pearl has a few thousand atoms in it.

- **Even further out** Further out one finds the dust collected by the pearl (through milliards of years) with the hardening and densification due to the effect of the unusually high homolumo gap.

The structure of our dark matter particles is illustrated in Figures 6 and 7.

4. Achievements

4.1 Fitting the Velocity Dependence of the Ratio $\frac{\sigma}{M}$

One somewhat supporting property of our model is that we can get it to match crudely with the actual interaction of dark matter particles as seemingly needed to fit the star velocities especially in dwarf galaxies, as estimated by Correa [15].

The a priori story, that dark matter has only gravitational interactions, seems not to work perfectly [32–34]: Especially in dwarf galaxies (around our Milky Way), where dark matter moves slowly, an appreciable cross section to mass ratio $\frac{\sigma}{M}$ seems to be needed going in the limit of low velocity to $150cm^2/g$, according to the fits in Figures 8 and 9.

We believe that our bubbles or pearls have become dirty in the sense that they are typically surrounded by a dust grain much like the dust grains that exist in outer space. Then, if as we

7 dark matter and Milky Way dwarf galaxies 9

Name	Final profile			Preferred cross section
	M_{200} [$10^9 M_\odot$]	c_{200}	r_{core} [pc]	σ/m_χ [cm^2g^{-1}]
UM	0.13	34.2	180.8	40 – 50
Draco	1.17	26.8	472.9	20 – 30
Carina	1.09	19.1	648.4	40 – 50
Sextans	0.32	20.8	395.5	70 – 120
CVnI	0.46	25.7	356.8	50 – 80
Sculptor	1.65	25.8	553.2	30 – 40
Fornax	2.29	15.3	1036.7	30 – 50
LeoII	0.05	30.6	148.8	90 – 150
LeoI	1.17	31.1	410.8	50 – 70

Table 2. From left to right: name of the dSph galaxy, present-time virial mass, concentration parameter and core size of the subhalo hosting the dSph and range of preferred cross section values that reproduce the observed DM central densities.

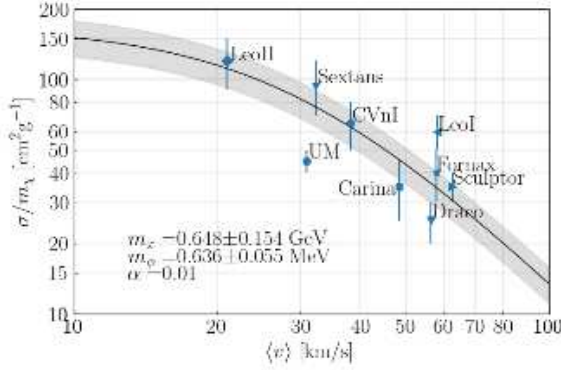


Figure 6. Cross section per unit mass, σ/m_χ , as a function of the average collision velocity, $\langle v \rangle$, of DM particles within each subhalo’s core. Symbols show the range of σ/m_χ needed for the SIDM model to reproduce the central DM densities reported by Kaplinghat et al. (2019). The solid line corresponds to the best-fit relation given by eq. (15) to the MW dSph data.

Figure 8: Extract from Correa’s paper illustrating the fits of the cross section to mass ratio obtained for different dwarf galaxies, as function of the estimated velocity for the relevant galaxy.

shall guess (and get confirmed) the dust grain is appreciably larger than the bubble, it will be the size of this dust grain that determines the cross section σ for the dark matter particle. The dark matter particles do not just end up being ordinary dust grains because according to big bang nucleosynthesis there is not enough ordinary matter to make up all the dark matter. Only if the ordinary matter is packed so compactly inside pearls so as to remain hidden already in the big bang nucleosynthesis era can we maintain a model in which the dark matter is basically ordinary matter. But we need to have the significantly bigger part “packed away” so that it is not seen as ordinary matter. So we must have the majority of the mass M of our dark matter dirty particle sit inside the

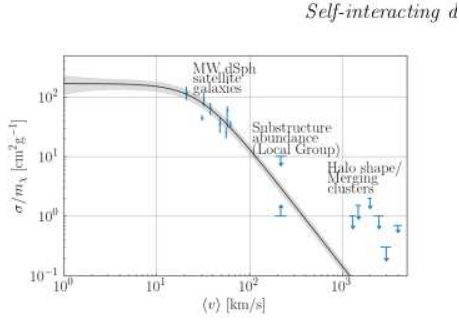


Figure 7. Same as Fig. 6, but extended to cover the range of MW- (~ 200 km/s) and cluster-size ($\sim 1000 - 5000$ km/s) haloes' velocities. The figure shows upper and lower limits for σ/m_χ taken for substructure abundance studies (e.g. Volgelberger et al. 2012 and Zavala et al. 2013), as well as based on halo shape/ellipticity studies and cluster lensing surveys (see text).

Figure 9: Same as Figure 8 but extended to cover the range of Milky Way and cluster-size haloes' velocities.

new vacuum bubble under high pressure, and not be free to show up as ordinary matter. So the full mass M of the dark matter particles in our model must be dominated by the mass of the little bubble, which thus essentially also has the mass M .

Now one shall have in mind that since dark matter is only safely observed via its gravitational force on stars and galaxies and in the case of gravitational lensing on light, and since one looks over very large distances, it is impossible to distinguish by these main observations of dark matter whether there are many small dark matter particles or fewer but heavier ones. In fact it is thus only the cross section over the mass $\frac{\sigma}{M}$ which is observable.

Correa [15] fitted this ratio for a series of dwarf galaxies and there were also other estimates of this ratio from galaxies and galaxy clusters giving some estimates for higher velocities.

4.1.1 Using the Dust grain to get a Mass

If we took it as a crude estimate that the dust grain around our bubbles would have essentially the usual size $0.1\mu m - 0.2\mu m$ as for the dust without any dark matter inside, then we could use the low velocity value of the ratio

$$\frac{\sigma}{M}|_{v \rightarrow 0} = 150 \text{ cm}^2/\text{g} = 15 \text{ m}^2/\text{kg} \quad (5)$$

to estimate the mass and the cross section separately

$$\sigma = (0.2\mu m)^2 = 4 * 10^{-14} \text{ m}^2 \quad (6)$$

$$M = \frac{\sigma}{\frac{\sigma}{M}} \quad (7)$$

$$= \frac{4 * 10^{-14} \text{ m}^2}{15 \text{ m}^2/\text{kg}} = 3 * 10^{-15} \text{ kg} \quad (8)$$

$$= 1.6 * 10^{12} \text{ GeV}. \quad (9)$$

4.1.2 The Size of the Bubble with New Vacuum

In section 4.4 we shall mention, that we already [10–14] some time ago have fitted both the value 3.5 keV for the photon energy of an X-ray emission line found by satellite X-ray observations and not easily attached to any known line, and the intensity with which it was observed, by a single parameter combination $\frac{\xi_{fS}^{1/4}}{\Delta V} = 0.6 \text{ MeV}^{-1}$. In our model this quantity is rather trivially connected with the fermi momentum p_f of the degenerate electrons in the new vacuum, when filled with some ordinary matter so as to balance the pressure from the skin around the bubble of new vacuum. In fact this fermi momentum becomes with our fitted parameter combination

$$p_f = 2 / \frac{\xi_{fS}^{1/4}}{\Delta V} = \frac{2}{0.6 \text{ MeV}^{-1}} = 3.3 \text{ MeV}. \quad (10)$$

Such a relativistic fermi momentum $p_f = 3.3 \text{ MeV}$ leads to a number density of electrons and thus mass density:

$$\rho_{\text{number } e} = \frac{2 * 4\pi}{3 * (2\pi)^3} * p_f^3 \quad (11)$$

$$= \frac{1}{3\pi^2} * p_f^3, \quad (12)$$

which for two nucleons per electron leads to a mass density

$$\rho_B = 2m_N * \frac{1}{3\pi^2} * p_f^3 \quad (13)$$

$$= 5.2 * 10^{11} \text{ kg/m}^3. \quad (14)$$

If the bubble shall have a mass $M = 3 * 10^{-15} \text{ kg} = 1.6 * 10^{12} \text{ GeV}$ it shall have a radius R given by

$$\frac{4\pi}{3} R^3 = M / \rho_B = 6 * 10^{-27} \text{ m}^3 \quad (15)$$

$$\text{or } R = 1.2 * 10^{-9} \text{ m} = 1.2 \text{ nm} \quad (16)$$

Indeed with the dust grain radius being $0.2 \mu\text{m}$, the bubble inside would be about 200 times smaller in extension than the dust part. But if we speculate that the dust grain around the bubble is more hard to break into pieces than the normal dust grain, it could have grown somewhat bigger and then the dust grain size relative to the bubble would be even bigger than the here first mentioned factor 200.

If we e.g. took it that the grain around the bubble would be say 3.5 keV/ 3eV times bigger than the normal dust grain, then we would get a factor 200000 rather than only the 200 for the size of the dust grain in the dark matter pearl relative to its bubble of new vacuum.

4.1.3 The Effect of the Homolumo gap on the Dust Region

Now we want to estimate the effect of the homolumo gap - as already alluded to above in section 3.1 - on the dust region into which the bubble has come to be a seed.

For this purpose we would like to get an idea about how the part of the wave functions for the filled states in the dust region looks. Now in this region in which nuclei for the dust are present we

shall use, what one could call a tight binding approximation, in which we build up a wave function ansatz being a superposition of single atom eigenfunctions such as 1s, 2s, 2p, ... That is to say we take ansatzes of the form

$$\psi_{ans}(\vec{x}) = \sum_{nucl.} \psi(\vec{x}(nucl.)) * \psi_n(\vec{x} - \vec{x}(nucl.)), \quad (17)$$

where ψ_n is the eigenwave function for a certain level n in the atom corresponding to the nucleus type making up the dust, and the sum runs over the nuclei in the dust region, which are enumerated by the name *nucl.*. The adjustable part of the ansatz is the smoothly varying (weight) function ψ .

In the approximation that this ψ is smooth the eigenvalue equation for such an ansatz ψ_{ans} will be essentially the usual Schrödinger equation, just with an extra constant potential V equal to the energy eigenvalue of the level n . If the energy eigenvalue for the level n is E_n , then the effective Schrödinger equation formulated for the adjustable function ψ in the ansatz becomes in the dust region

$$-\frac{1}{2m_{eff}} \Delta \psi - |E_n| \psi = E \psi. \quad (18)$$

Now because of the homolumo gap we expect that the eigenvalue for the filled states shall be negative and of the order -3.5 keV. Since this value -3.5 keV is a large number numerically compared to many of the $|E_n|$ values, the solution in the dust region for such a filled wave function will fall off exponentially as one goes further and further away from the central bubble.

This then means that the wave functions for filled states only survive far out in the dust region, when $|E_n|$ is of the order of magnitude 3.5 keV. That is to say only the levels with very strong binding will be filled for the atoms in the dust region.

The other levels in the dust atoms will only have a chance to be filled very close to the bubble - see Figure 10.

Having argued that only the lowest levels in the dust atoms are filled, we expect that the electron clouds around the dust region nuclei become much smaller than the usual size of such atoms. If the distance between the atoms in the dust region is determined by the size of these clouds, then the dust material would be expected to contract due to the here studied effect of the homolumo gap for this material. Taking this scaling to make the dust atoms smaller seriously, then we estimate the scaling ratio to be of the order of the energy ratio of the 3.5 keV relative to a typical binding energy in the usual valence electron level, say a few electron volts, say for simplicity 3.5 eV. Since the Coulomb potential around a nucleus in the dust is proportional to the inverse radius

$$V \propto 1/r, \quad (19)$$

we expect the radius of the electron cloud around the nuclei in the dust to be $\frac{3.5keV}{3.5eV} = 1000$ times smaller in size.

This should then mean, that the density should be increased by a factor 1000 in each of the three dimensions. We must, however, unfortunately admit that so strong a contraction of the dust material is absurd in as far as it would make the hardened dust more compact than the very bubble itself, which is supposed to cause this contraction. The material in the bubble has namely, according to our fit to the 3.5 keV X-ray line and its intensity, a density $5.2 * 10^{11} kg/m^3$, which is only about

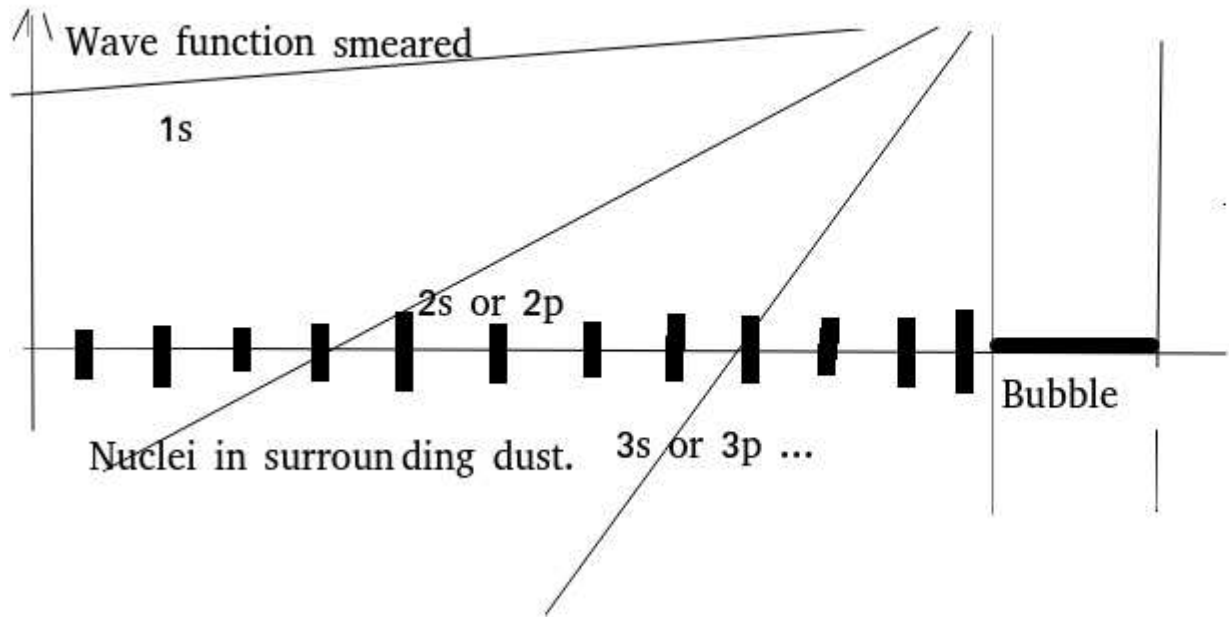


Figure 10: Here we imagine to make a wave function ansatz as a superposition of certain wave functions around the nuclei in the dust - symbolized by the short vertical thick lines in the left-hand region - but smeared out in the high nuclear density approximation. As the (local) wave function around the nuclei one can choose the 1s tightest bound state or the 2s or 2p states in the next level or 3s Since the eigenvalue for the electron energy is of the order of -3.5 keV (a numerically large number from the usual atomic physics point of view) the smoothed wave function describing the relative amplitudes for various atoms in the dust range will fall drastically as we go away from the bubble with its special and strong effects for the majority of the atom levels. Only if the binding to the state used for the ansatz is of order (-)3.5 keV will the smeared weighting wave be more flat. We imagine that this plot is logarithmic, so that the exponential fall off (towards the left in this figure) is represented by straight lines.

$10^8 = 464^3$ times bigger than a “normal” density, say $5 * 10^3 kg/m^3$. So a contraction of more than a factor of 464 in each direction is at least unreasonable, and may even be unbelievably too much.

From similar dimensional arguments the energy of moving an atom closer to a neighbour by a distance of the order of the distance to this neighbour should go up by a factor 1000. Thus the force arising from compressing a cubic meter of the material by a strain of order unity would be $1000^2 = 10^6$ times strengthened by the homolumo gap effect. But if we take the little more reasonable value of at most a factor 464 for each direction, we would rather only get a $464^4 = 5 * 10^{10}$ times bigger elastic modulus for the hardened material than it had before.

We could by such dimensional arguments, using a unit system in which the fine structure constant is conceived of as a velocity and put to unity also taking the Planck constant $\hbar = 1$, claim that the length has dimension $[1/energy]$ and e.g calculate the dimensionality of the elastic modulus E of a material to be $[energy^4]$. Then using the dimensional argument and screwing up the energy by a factor 1000, the modulus would go up by $1000^4 = 10^{12}$. But this is too much and we should not believe more than say $400^4 = 2.5 * 10^{10}$.

Above we assumed that the modification of the dust material was so as to let the homolumo gap of 3.5 keV get extended from the bubble throughout the dust grain. But that is presumably not true, because the tails of the wave functions reaching out in the dust region get thinner and thinner, so that the connection to the interior of the bubble gets very weak. Indeed in Figure 10 we sketch how the wave functions fall off exponentially with the distance to the bubble. As soon as the wave function truly feeling the bubble, meaning its homolumo gap effect, drops down exponentially and becomes small presumably its effect in modifying the atomic structures also gets small. So when even the wave functions dominating furthest out become small, we must expect that the appearance of relatively undisturbed standard atomic wave functions for atoms in the dust will be allowed by the Pauli principle. Thus the further out the dust atoms, the more and more normal they become and the effective homolumo gap is expected to diminish gradually with the distance from the bubble. In other words only the most nearby part of the dust gets fully hardened. So to expect the correction factor 1000^4 for the modulus E should be considered rather an upper limit for the correction.

Even the more moderate number $400^4 = 2.5 * 10^{10}$ is only an upper limit.

Very far out we may expect quite normal dust, and we might take this as an argument to allow us to take the quite normal size $0.1\mu m$ even for the dust grain around the dark matter bubble.

But we should have in mind that if the hardening has an influence on the size, it is presumably by making the hardened dust grain less easy to break into pieces. Thus the hardened grain will survive without breaking up more easily than the normal dust grain. That of course will tend to make the dust grain around the dark matter bubble larger than the normal dust grain.

4.1.4 $\frac{\sigma}{M}$ Velocity Dependence, Characteristic Velocity

When the collision velocity v is very low of course the geometrical size of the dust grain around the bubble determines the cross section, because there is not enough energy in the collision to cause any appreciable penetration of the dust grains into each other.

However, when the velocity gets higher the grains get deformed, but the bubbles are extremely heavy and in first approximations they are very hard to turn into a motion in another direction. Now it is exactly turning them into an appreciably different direction which is needed, in order that the collision will be significant for the motion of the dark matter pearls and give a cross section that can be detected in say the dwarf galaxy simulations and fits.

Let us imagine the collision at higher velocity like this:

- In first approximation - of high velocity - the bubbles which come in with a relative impact parameter b just move undisturbed through with undisturbed velocity until they have compressed the dust grains so much that the energy of the deformation has become of the order of the incoming kinetic energy of the bubbles. (Since the bubbles are in mass supposed to dominate, essentially all the kinetic energy sits in the bubbles.)
- We describe the deformation of the dust grains as the displacement of the bubble inside the grain as far as is needed, assuming the elastic deformation of the grain going on mainly around the bubble requires the least amount of elastic energy.
- The dust grains themselves are in our crudest approximation considered so light compared to the bubbles, that they can be accelerated and decelerated without any significant energy use.

So the grains must be imagined to pass through each other when the impact parameter b is smaller than the sum of their radii by the dust particles going a bit aside so as to be able to pass. Then they immediately turn back to essentially the original orbits because the small but heavy bubbles force them to return to their original orbits, unless of course the bubbles have themselves been stopped and brought out of their straight trajectories during the detour.

- The estimation of the elastic energy stored during the passage of the two dark matter particles is thus mainly due to - the square, one could say, of - the displacement during the passage of the bubbles relative to their associated grains of dust.

At the most stressed point during this passage the displacement distance is

$$\text{“ displacement”} = r_{dust} - b/2, \quad (20)$$

where r_{dust} is the radius of the dust grains. In reality the dust grains are not at all spherical, so that the radius is only a very crude concept and also they will typically not all be of the same radius even to the approximation that they are spherical.

- The crudest way to estimate the elastic energy from this $r_{dust} - b/2$ displacement is to say that a volume of dust grain material at least pressed through is

$$\text{“ volume”} = \pi * R^2 * (r_{dust} - b/2), \quad (21)$$

where $\pi * R^2$ is the area for the heavy bubble.

The corresponding elastic energy may crudely be estimated to be

$$\text{“elastic energy”} = E * R^2 * (r_{dust} - b/2)^2/b. \quad (22)$$

The b in the denominator is the “original” or rather “final” length of the column being compressed by the amount $r_{dust} - b/2$ (if you wish you can consider it that the length of the column of dust matter compressed has the length $b/2$ and that this factor of a half is cancelled by the $1/2$ in the potential energy; or just take it as an order of magnitude estimate).

Really this expression for the elastic energy does not quite correspond to a correct picture of the deformation of the dust grain, in order to allow the two dust grains to pass each other without any deviations from the unchanged velocity motion of the very heavy bubbles. This means of course that the bubbles must be displaced relative to its dust grain so as to come to only a distance $b/2$ from the surface of the grain. This not so correct deformation pattern is thought to be that of the dust material in the very neighbourhood of the column of dust with length of the order $b/2$ - the most compressed part being of this length - and cross section $\pi * R^2$, the cross section of the bubble. But it is very likely that one can find a more smoothed out deformation with a somewhat lower energy. The true deformation should be the one with the minimal elastic energy, but still with the correct displacement of the bubble relative to the dust grain. Thus really the above order of magnitude estimate for the elastic energy is rather an upper bound. So rather we have

$$\text{“elastic energy”} \leq E * R^2 * (r_{dust} - b/2)^2/b. \quad (23)$$

But we may still believe that order of magnitudewise it is rather close to equality.

The condition on the kinetic energy $\frac{1}{2}M * v^2$ of a dark matter particle with mass dominated by that of the bubble M and velocity v , which is just barely being stopped in such a collision, is provided order of magnitudewise by

$$\frac{1}{2}M * v^2 \approx E * R^2 * (r_{dust} - b/2)^2 / b \quad (24)$$

$$\text{or} \quad \sqrt{\frac{M}{2E * R^2}} * v \approx \frac{r_{dust} - b/2}{\sqrt{b}} \quad (25)$$

$$= r_{dust} / \sqrt{b} - \sqrt{b} / 2. \quad (26)$$

$$\text{or} \quad (\sqrt{b})^2 + \sqrt{\frac{2M}{E * R^2}} * v * \sqrt{b} - 2r_{dust} \approx 0 \quad (27)$$

It should be borne in mind that these equations are only relevant for the impact parameter $b \leq 2r_{dust}$, since for larger b of course the dust grains do not even touch. For the variable in the second order equation, the square root of the impact parameter \sqrt{b} , this relation for the interesting cases mean $\sqrt{b} \leq \sqrt{2r_{dust}}$. The solution to this second order equation is

$$\sqrt{b} = -\sqrt{\frac{M}{2E * R^2}} * v \pm \sqrt{\frac{M}{2E * R^2}} * v^2 + 2r_{dust} \quad (28)$$

We can check that for very low velocity $v \approx 0$ we of course obtain the cross section corresponding to the impact parameter of just stopping

$$\sigma = \pi b^2 = \pi (\sqrt{2r_{dust}})^4 \quad (29)$$

$$= \pi * 4r_{dust}^2. \quad (30)$$

For the purpose of estimating the ratio $\frac{\sigma}{M}$ it is more suitable to write the equation for the combination $\frac{\sqrt{b}}{M^{1/4}}$, i.e.

$$\frac{\sqrt{b}}{M^{1/4}} = -\sqrt{\frac{\sqrt{M}}{2E * R^2}} * v \pm \sqrt{\frac{\sqrt{M}}{2E * R^2}} * v^2 + \frac{2r_{dust}}{\sqrt{M}}. \quad (31)$$

In the case of low velocity we can make the approximation of dropping the v^2 term under the square root and obtain

$$\frac{\sqrt{b}}{M^{1/4}} = -\sqrt{\frac{\sqrt{M}}{2E * R^2}} * v \pm \sqrt{\frac{2r_{dust}}{\sqrt{M}}} \quad (\text{for small } v) \quad (32)$$

$$= \sqrt{\frac{2r_{dust}}{\sqrt{M}}} * \left(1 - v * \sqrt{\frac{M}{2r_{dust} 2E * R^2}}\right) \quad (\text{using of course } + \text{ for } \pm) \quad (33)$$

$$= \sqrt[4]{\frac{\sigma}{M\pi}} \Big|_{v \rightarrow 0} \left(1 - \frac{v}{v_0}\right), \quad (34)$$

where

$$v_0 = \sqrt{\frac{4r_{dust} E * R^2}{M}}. \quad (35)$$

Taking the fourth power we then obtain

$$\frac{\sigma}{M} = \pi \left(\frac{\sqrt{b}}{M^{1/4}} \right)^4 \quad (36)$$

$$= \frac{\sigma}{M} |_{v \rightarrow 0} * (1 - v/v_0)^4. \quad (37)$$

Using the data of Figure 8 from Correa [15] and fitting to the formula (37) above for $v = 100$ km/s we obtain $v_0 = 230$ km/s. Taking the smaller velocity $v = 35$ km/s to fit we get 220km/s.

The value of the fermi momentum $p_f = 3.3$ MeV determines the mass density of the material in the bubble of the second type of vacuum:

$$\rho_B = \frac{2m_N}{3\pi^2} p_f^3 = 5.2 * 10^{11} \text{ kg/m}^3. \quad (38)$$

So we have

$$\frac{4\pi}{3} * R^3 * 5.2 * 10^{11} \text{ kg/m}^3 = M. \quad (39)$$

and therefore

$$\frac{R^2}{M} = \frac{1}{\sqrt[3]{M}} * \left(\frac{3}{4\pi * 5.2 * 10^{11} \text{ kg/m}^3} \right)^{2/3} \quad (40)$$

$$= \frac{1}{\sqrt[3]{M} * 1.68 * 10^8 \text{ kg}^{2/3}/\text{m}^2} \quad (41)$$

Taking $v_0 = 220 \text{ km/s} = 2.2 * 10^5 \text{ m/s}$ we thus obtain from equation (35):

$$2.2 * 10^5 \text{ m/s} = \sqrt{\frac{4r_{dust}E}{\sqrt[3]{M} * 1.68 * 10^8 \text{ kg}^{2/3}/\text{m}^2}} \quad (42)$$

$$\text{or } 4r_{dust}E = (2.2 * 10^5 \text{ m/s})^2 * 1.68 * 10^8 \text{ kg}^{2/3}/\text{m}^2 * \sqrt[3]{M} \quad (43)$$

$$= 8.1 * 10^{18} \text{ kg}^{2/3}/\text{s}^2 * \sqrt[3]{M}. \quad (44)$$

For say $M = 10^{-15} \text{ kg}$, we then obtain:

$$4r_{dust}E = 8.1 * 10^{13} \text{ kg/s}^2. \quad (45)$$

Without a story about the influence of the homolumo gap effect this number for $4r_{dust}E$ is far too high. In fact without the homolumo gap modification we expect

$$E \approx 10^9 \text{ N/m}^2 \quad (\text{usual materials}) \quad (46)$$

$$r_{dust} \approx 10^{-7} \text{ m} \quad (\text{typical cosmic dust grain}) \quad (47)$$

$$\text{giving } 4Er_{dust} \approx 400 \text{ N/m}. \quad (48)$$

(Have in mind $N/m = \text{kg/s}^2$).

However in section 4.1.3 we suggested a possible homolumo gap correction factor to the elastic modulus E of $1000^4 = 10^{12}$ or perhaps a little more reasonable factor of $400^4 = 2.5 * 10^{10}$.

Assuming r_{dust} is not changed these two possible homolumo gap corrections increase $4Er_{dust}$ up to

$$4Er_{dust}|_{\text{homolumo corrected}} = 400N/m * 10^{12} = 4 * 10^{14}N/m \quad (49)$$

and

$$4Er_{dust}|_{\text{homolumo corrected}} = 400N/m * 2.5 * 10^{10} = 1 * 10^{13}N/m \quad (50)$$

respectively. These values are to be compared with the value $4Er_{dust} = 8.1 * 10^{13}$ N/m obtained in equation (48) using a characteristic velocity $v_0 = 220$ km/s and pearl mass $M = 10^{-15}$ kg.

Clearly we really need such a homolumo gap correction to even get just crude agreement.

Formally we could adjust the mass M to make the estimates agree completely and declare we had fitted the mass to be the third power of the deviation ratio 5 bigger than the 10^{-15} kg, thus getting $M \approx 10^{-13}$ kg, but of course it makes little sense, since the dependence on the mass M is so weak for this fitting of the velocity dependence, coming essentially via its sixth root. The uncertainty gets huge.

4.2 Mass Discussion

In the philosophy that the dark matter particles are provided with dust grains around them, it is of course clear that the density in the universe of such dark matter particles should better be smaller than the number of dust grains estimated from observations, since otherwise our model would predict more dust grains than observed.

If we at first ignored the suspicious corrections of the density of the dust material and just assumed that each dark matter bubble were associated with a dust grain of usual density ~ 2000 kg/m³ and size $r_{dust} = 0.1\mu m = 10^{-7}m$, we could estimate the grain number density - very crudely - by taking it that 10-logarithm of the ratio DGR of dust mass to gas mass is [35]

$$\log DGR = 2.45 \log \frac{Z}{Z_{\odot}} - 2.03. \quad (51)$$

Here Z is the metallicity in the region in question (i.e. the amount of heavier than helium elements relative to all elements) while Z_{\odot} is this metallicity in the surface of the sun (\odot).

Let us crudely estimate:

$$\text{“mass of typical grain”} = 2000 \text{ kg/m}^3 * (10^{-7} \text{ m})^3 \quad (52)$$

$$= 2 * 10^{-18} \text{ kg} \quad (53)$$

$$\text{“critical density”} = \frac{3H^2}{8\pi G} = 10^{-26} \text{ kg/m}^3 \quad (54)$$

$$\text{“ordinary matter density”} = 5\% \text{ of } 10^{-26} \text{ kg/m}^3 \quad (55)$$

$$= 5 * 10^{-28} \text{ kg/m}^3 \quad (56)$$

$$\text{hereof say “gas”} = 3 * 10^{-28} \text{ kg/m}^3. \quad (57)$$

$$\text{If } Z = Z_{\odot} \text{ then } DGR = 0.009, \quad (58)$$

$$\text{and “dust density” } \rho_{dust} = 3 * 10^{-28} \text{ kg/m}^3 * 0.009 \quad (59)$$

$$= 2.7 * 10^{-30} \text{ kg/m}^3 \quad (60)$$

$$\text{“Typical grain number density”} = \frac{2.7 * 10^{-30} \text{ kg/m}^3}{2 * 10^{-18} \text{ kg}} \quad (61)$$

$$= 1.4 * 10^{-12} \text{ m}^{-3}. \quad (62)$$

This would correspond to a typical distance between the grains of 11 km.

The dark matter average density is similarly

$$\text{“DM density”} = 24\% \text{ of } 10^{-26} \text{ kg/m}^3 \quad (63)$$

$$= 2.4 * 10^{-27} \text{ kg/m}^3. \quad (64)$$

So with a dark matter pearl mass M the number density of the dark matter pearls is

$$\rho_{DM \text{ number}} = \frac{2.4 * 10^{-27} \text{ kg/m}^3}{M}. \quad (65)$$

If this number density should be equal to that of the dust grains, we should have

$$\frac{2.4 * 10^{-27} \text{ kg/m}^3}{M} = 1.4 * 10^{-12} \text{ m}^{-3} \quad (66)$$

$$\text{leading to } M = 2 * 10^{-15} \text{ kg}. \quad (67)$$

Of course there would be a lot of dust grains without any dark matter bubble in it, and thus we should consider the number $2 * 10^{-15} \text{ kg}$ as only a *lower* limit for the mass M of the dark matter particle:

$$M \geq 2 * 10^{-15} \text{ kg} = 1.2 * 10^{12} \text{ GeV}. \quad (68)$$

Corresponding to such a mass we have a radius

$$R \geq \sqrt[3]{\frac{2 * 10^{-15} \text{ kg}}{\frac{4\pi}{3} * 5.2 * 10^{11} \text{ kg/m}^3}} \quad (69)$$

$$= 1.0 * 10^{-9} \text{ m} \quad (70)$$

Below in section 4.3.1 we shall present yet another lower bound for the mass M of the dark matter pearls: $3.9 * 10^{13} \text{ GeV}$, see equation (88).

That order of magnitudewise, including the speculation of the hardened dust material, we can achieve a consistent dark matter picture as these dirtified bubbles should be considered an achievement of our model.

4.3 Resolving Seeming Disagreement of DAMA and Xenon-experiments.

Most underground experiments are designed to look for dark matter particles hitting the nuclei in the experimental apparatus, which is then scintillating so that such nuclear recoil events can be seen. But our pearls are excited in such a way that they send out energetic *electrons* (rather than nuclei), and thus does not match with what is looked for, except in the DAMA-LIBRA experiment, in which the only signal for events coming from dark matter is a seasonal variation due to the Earth running towards or away from the dark matter flow. Recently the Xenon1T experiment has actually observed an excess of electron recoil events, which we suggest are coming from the decay of our excited dark matter pearls.

4.3.1 Mass bound from need for Seasonal Oscillations in DAMA

In order that our model shall be able to fully produce the DAMA-effect from the decay of our excited bubbles of new vacuum (or the like), it is needed that the bubbles or whatever, which produce the electron or gamma signal detectable by DAMA, come down to the experiment in less than a year. Otherwise of course the seasonal variation would be washed out and only a constant signal seen by the DAMA-LIBRA experiment.

Now it is hard to estimate the terminal velocity for the bubbles penetrating solids, but we believe we can estimate the terminal velocity for the bubbles going through the liquid xenon in the Xenon1T experiment due to the gravitational attraction of the Earth. In fact there is a balance between the gravitational force gM and the Stoke's law force $6\pi\eta Rv_{terminal Xe}$, so that in the fluid xenon the terminal velocity is given by

$$Mg = 6\pi\eta Rv_{terminal Xe}. \quad (71)$$

In our earlier article [5] we took the viscosity of the liquid xenon to be

$$\eta = \eta_{liquid\ gas} \approx 100\mu Pa\ s = 0.1mPa\ s, \quad (72)$$

and obtained a limit

$$M \geq 2.1 * 10^{-15} kg = 1.2 * 10^{12} GeV. \quad (73)$$

In fact we here used the experimental fact that there are 250 times more events seen in the DAMA experiment per kg scintillator than in the Xenon1T excess of electron recoil events. This we take to mean that the terminal velocity in the NaI of the DAMA experiment - and we extend it to all solids - is 250 times smaller than in the fluid xenon.

Using equations (71) and (72) together with

$$M = \frac{4\pi}{3}\rho_B R^3, \quad (74)$$

and the density of the bubble $\rho_B = 5.2 * 10^{11} kg/m^3$ taken from equation (38), we obtain

$$v_{terminal Xe} = \frac{gM}{6\pi\eta_{fluid\ gas}R} \quad (75)$$

$$= \frac{2g\rho_B R^2}{9\eta_{fluid\ gas}} \quad (76)$$

$$= 1.13 * 10^{16} m^{-1} s^{-1} * R^2. \quad (77)$$

Taking the terminal velocity in solid material to be 250 times smaller

$$v_{terminal\ solid} \approx \frac{v_{terminal\ Xe}}{250} \quad (78)$$

$$\text{we get } v_{terminal\ solid} \approx 4.5 * 10^{13} s^{-1} m^{-1} * R^2. \quad (79)$$

Taking the depth penetrated into the earth so as to reach the DAMA experiment to be 1400 m, the pearls need to come down in less than a year and thus to have a (terminal) velocity larger than 1400m/1 year, i.e.

$$v \geq \frac{1400m}{365 * 24 * 60 * 60s} \quad (80)$$

$$= 4.4 * 10^{-5} m/s, \quad (81)$$

the requirement on the radius would be

$$R^2 \geq \frac{4.4 * 10^{-5} m/s}{4.5 * 10^{13} s^{-1} m^{-1}} \quad (82)$$

$$= 1.0 * 10^{-18} m^2 \quad (83)$$

$$\text{giving } R \geq 1.0 * 10^{-9} m \quad (84)$$

The corresponding condition on the mass $M = \frac{4\pi}{3} \rho_B R^3$ is

$$M \geq \frac{4\pi}{3} * 5.2 * 10^{11} kg/m^3 * (10^{-9} m)^3 \quad (85)$$

$$\text{giving } M \geq 2.1 * 10^{-15} kg = 1.2 * 10^{12} GeV. \quad (86)$$

This agrees with the bound (73) from the foregoing article [5].

It would be reasonable to correct this bound, obtained assuming that the whole impact of dark matter is seasonally varied, to instead using the more realistic assumption that only about 10% of the incoming dark matter is varying, since the ratio of the earth velocity 30 km/s to the velocity of the solar system moving through the average dark matter 300 km/s is about 10% . The electron recoil excess in the Xenon1T experiment of course contains both the varying and the constant part. So the factor 250, which we used above and required the ratio of the terminal velocities of the dark matter in the two experiments to be, should rather have been 250/10% = 2500. We require the speed in the solid $v_{terminal\ solid}$ to be unaltered - it is given by the one year requirement. So we have to increase the $v_{terminal\ Xe}$ by a factor 10 more. That would then increase the boundary value for the radius of the bubble R by a $\sqrt{10} = 3.1$ and the boundary mass M by a factor $10^{3/2} = 31$. So the corrected bounds become:

$$R \geq 3.1 * 1.0 * 10^{-9} m = 3.1 * 10^{-9} m \quad (87)$$

$$\text{and } M \geq 31 * 2.1 * 10^{-15} kg = 6.5 * 10^{-14} kg \quad (88)$$

$$= 3.9 * 10^{13} GeV. \quad (89)$$

4.4 The 3.5 keV X-ray Radiation

- **The Intensity of 3.5 keV X-rays from Clusters etc.** We fit the very photon-energy 3.5 keV and the overall intensity from a series of clusters, a galaxy, and the Milky Way Center with one parameter $\frac{\xi_{fS}^{1/4}}{\Delta V} = 0.6 MeV^{-1}$ [12, 13].

Even though we only need the one parameter $\frac{\xi_{fS}^{1/4}}{\Delta V} = \frac{2}{p_f}$, it is nice to know the notation:

$$\begin{aligned} \Delta V &= \text{“ difference in potential for a nucleon} \\ &\quad \text{between inside and outside the central part of the pearl”} \\ &\approx 2.5 MeV \end{aligned} \tag{90}$$

$$\xi_{fS} = \frac{R}{R_{crit}} \text{ estimated } \approx 5 \tag{91}$$

$$\begin{aligned} \text{where } R &= \text{“actual radius of the new vacuum part”} \\ &\approx r_{cloud 3.3 MeV} \end{aligned} \tag{92}$$

$$\begin{aligned} \text{and } R_{crit} &= \text{“ Radius when pressure so high} \\ &\quad \text{that nucleons are just about being spit out”} \end{aligned} \tag{93}$$

4.5 About the Xenon1T and DAMA-rates:

We estimate the event rates seen by DAMA-LIBRA and in the Xenon1T electron recoil excess using energy considerations based on the kinetic energy of the incoming dark matter as known from astronomy. In order to explain these estimates it is necessary to know how we imagine the dark matter to interact and get slowed down in the air or in the earth shielding the experiments, and how the dark matter particles get excited and emit 3.5 keV radiation or electrons.

Our estimate of the absolute rates for the two experiments are very very crude because we assume that the dark matter particles - in our model small macroscopic systems with of the order of 10^{12} of nuclei inside them - can have an exceedingly smooth distribution of lifetimes on a logarithmic scale. These calculations are discussed in [5]. Let us shortly review them here:

The dark matter pearls come into the atmosphere with high speed (galactic velocity) and are likely to be essentially stopped in the air, meaning they have slowed down to a speed ~ 49 km/s below which collisions with nuclei can no longer excite the 3.5 keV excitations. We consider the alternative scenario where the stopping takes place in the Earth in section 5.2.

Because the atmosphere roughly scales in density, rising by a factor e for each 7 km one goes down, the order of magnitude of the “stopping length” (meaning the distance over which the essential stopping of a pearl in our model takes place) cannot be much different from 7 km. Thus the corresponding “stopping time”, when the incoming velocity is ~ 300 km/s, is given in order of magnitude by 0.023 s.

We assume that there is a wide range of different 3.5 keV excitations possible with different decay rates distributed smoothly in the logarithm of the decay time. It follows that the decay rates dominating for pearls that survive for a time t after their excitation before they are seen to decay will correspond to a life time of order t . Because of the tightness of the bound (86) for the mass M from the requirement of the pearl reaching down faster than in a year and the estimated mass (8) from the dwarf galaxy self interaction estimate for low velocity, we must believe that the passage time t is close order of magnitudewise to 1 year.

Thus we have

$$\text{“stopping time”} \approx \frac{7 \text{ km}}{300 \text{ km/s}} \quad (94)$$

$$= 0.023s \quad (95)$$

$$\text{“passage time”} \approx 1 \text{ year} = 3 * 10^7 s \quad (96)$$

By essentially a dimensional argument, supported by assuming that excitations surviving longer before decaying also take more time to get excited, we get that there is a suppression factor *suppression* for getting the long living excitations excited and given by

$$\text{suppression} \approx \frac{\text{“stopping time”}}{\text{“passage time”}} \quad (97)$$

$$\approx \frac{0.023s}{3 * 10^7 s} \quad (98)$$

$$= 6 * 10^{-10}. \quad (99)$$

Note that this theoretical *suppression* ratio is independent of the pearl mass. It is to be compared to the ratio of the energy deposition rate in DAMA relative to the power from the kinetic energy of the dark matter, making the crude assumption that it is distributed uniformly over the matter down to the 1400 m depth of the DAMA experiment. The energy deposition rate per kg of scintillation material is calculated to be “deposited rate” = $2.7 * 10^{-22} W/kg$, assuming that all the DAMA-LIBRA events are due to decays with decay energy 3.5 keV. Divided into a layer of 1400 m thickness the kinetic energy rate we found in [5] to be “power to deposit” = $1.7 * 10^{-12} W/kg$. We then obtain the experimental suppression rate

$$\text{suppression}_{DAMA} = \frac{\text{deposited rate}}{\text{“power to deposit”}} \quad (100)$$

$$= \frac{1.7 * 10^{-12} W/kg}{2.7 * 10^{-22} W/kg} \quad (101)$$

$$= 1.6 * 10^{-10}. \quad (102)$$

This agrees surprisingly well with the crude theoretical *suppression* factor (99) above.

Similarly for Xenon1T

$$\text{suppression}_{Xenon1T} = 6 * 10^{-13}. \quad (103)$$

Below in (113, 114) we find the stopping times for the presumably less realistic case that the stopping first takes place in the stone. These stopping times are of the order of $10^{-6} s$ and thus shorter than the one based on the stopping in the air by about 4 orders of magnitude, i.e. they are 10^4 times smaller, and thus will give 10^4 times less rate prediction in DAMA.

The ratio of the rates in the two experiments - Xenon1T electron recoil excess and DAMA - should in principle be very accurately predicted in our model, because they are supposed to see exactly the same effect just in two different experiments in the same underground laboratory below the Gran Sasso mountain! One would therefore expect the rates to be the same, but the Xenon1T rate is about 250 times smaller than the DAMA rate. As mentioned in section 4.3.1, we interpret this to mean that the terminal velocity in the solid NaI of the DAMA experiment to be 250 or 2500 times smaller than in the fluid xenon.

4.6 Jeltema and Profumo's Observation

Let us briefly mention an at first very strange observation [36] from the point of view of the hypothesis that the 3.5 keV X-ray line observed astronomically should indeed come from dark matter: Jeltema and Profumo saw this 3.5 keV line in the X-ray spectrum from the Tycho supernova remnant! This is remarkable because otherwise one only sees this line coming from huge collections of stars such as galaxy clusters with supposedly a huge amount of dark matter, while the amount of dark matter in the supernova remnant is expected to be relatively minute.

In an earlier article [13] we assumed that our pearls could be excited by cosmic rays - of which there is an appreciable amount in supernova remnants - hitting the nuclei in the bubble of the new vacuum. If as we assumed the density and the size of this bubble is not so big that the nuclei significantly shadow for each other, the cross section for such a nucleus hitting collision divided by the mass will be just the value for this ratio obtained for a cosmic ray collision with a single nucleus. Then we took it that the energy collected from the cosmic ray by the dark matter in the neighbourhood of the supernova remnant would mainly go into 3.5 keV excitations, of which in turn about a fraction α (the fine structure constant) would be emitted as X-rays. Such a model gave good agreement with the radiation observed by Jeltema and Profumo.

5. Impact

5.1 Illustration of Interacting and Excitable Dark Matter Pearls

The dark matter pearls come in with high speed (galactic velocity), but get slowed down to a much lower speed by interaction with the atmosphere or the shielding mountains, whereby they also get excited to emit 3.5 keV X-rays or *electrons*. As mentioned above in section 4.5, we believe that the essential stopping of the pearls is most likely to take place in the atmosphere, but here we shall consider the possibility that it happens in the earth above the underground experiments.

Presumably already in the atmosphere the pearls with their dust around them will be heated so much that the dust grain will be burned off - much like the tail of a comet gets burned off by solar radiation. This means that a dark matter object penetrating into the Earth becomes much smaller than the object out in space for which Correa estimated the cross section over mass ratio from the dwarf galaxy studies.

The impact is illustrated in Figure 11.

5.2 Pearls Stopping and getting Excited in Earth Shield

What happens when the dark matter pearls, in our model of nano-meter size bubbles surrounded by dust grains, hit the earth shielding above the experimental halls of e.g. DAMA?

- **Stopping** For very large velocities v the dragging force acting on a pearl of radius R (say a naked bubble from our dark matter particles) in a medium of density ρ is

$$F_d = \frac{c_d}{2} \rho v^2 \pi R^2 \quad (104)$$

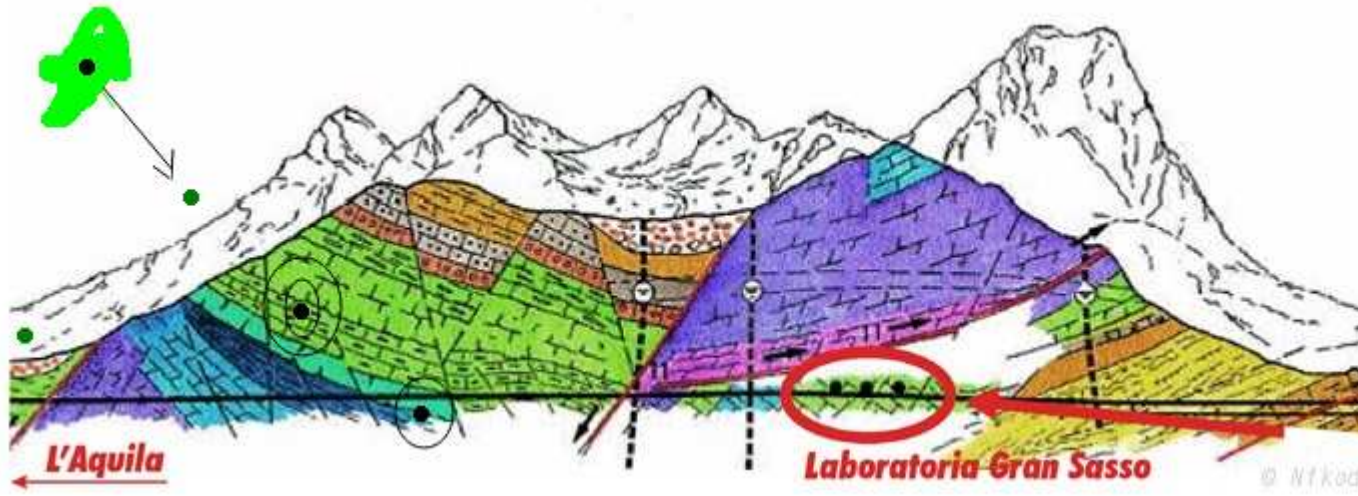


Figure 11: On this figure the motion of a single dark matter particle as time goes on and it comes deeper down towards the Earth is illustrated as a series of pictures of the particle. Out in space from where it comes it still has its grain of dust around it. Then that gets stripped off, and at the same time the “naked” bubble gets excited, first strongly but as it goes on it de-excites most of its excitations - i.e. most of its internal energy - and deeper in only a very little part is left. In most cases actually the excitation energy is used up rather high up in the shielding or in the air. But somewhat seldomly an excitation survives down to excite the DAMA counter, or make an event to be counted as an Xenon1T electron recoil excess event.

with c_d being the drag coefficient of order unity, and so with a mass M the velocity is diminished with the rate

$$-\frac{dv}{dt} = \frac{F_d}{M} \quad (105)$$

$$= \frac{c_d \rho v^2 \pi R^2}{2M} \quad (106)$$

giving $\frac{1}{v} = \int \frac{c_d \rho \pi R^2}{2M} dt$ (107)

$$= t \frac{c_d \rho \pi R^2}{2M} - \frac{1}{v_{start}}. \quad (108)$$

Here v_{start} is the velocity when the stopping of the pearl starts, say about 300 km/s for a pearl entering the Earth atmosphere and mountains. Formally the pearl never stops, but order of magnitudewise the stopping takes place in the time $t = \frac{M}{\rho \pi R^2 v_{start}}$ (after putting c_d and 2 to be of order unity). So the distance of stopping becomes

$$\text{“stopping length”} = t * v_{start} \quad (109)$$

$$= \frac{M}{\rho \pi R^2}. \quad (110)$$

Let us consider first the bubble inside a dust grain of radius $0.1 \mu m$ satisfying the low velocity limit $\frac{\sigma}{M}|_{v \rightarrow 0} = 15 m^2/kg$, which has mass $M = 3 * 10^{-15} kg$ (8), radius $R = 1.2 * 10^{-9} m$

(16) and $\frac{\pi R^2}{M} = 1.5 * 10^{-3} m^2/kg$. Taking for stone the density $\rho = 3000 kg/m^3$ we get

$$\text{“stopping length”} = \frac{1}{1.5 * 10^{-3} m^2/kg * 3000 kg/m^3} = 0.22m. \quad (111)$$

Similarly using the minimal size pearl which reaches down to 1400 m within one year, having mass $M = 6.5 * 10^{-14} kg$ (88), radius $R = 3.1 * 10^{-9} m$ (87) and $\frac{\pi R^2}{M} = 4.6 * 10^{-4} m^2/kg$, we get

$$\text{“stopping length”} = \frac{1}{4.6 * 10^{-4} m^2/kg * 3000 kg/m^3} = 0.72m. \quad (112)$$

The corresponding stopping times are with $v = 300 km/s$

$$t = \frac{0.22m}{3 * 10^5 m/s} = 7 * 10^{-7} s \quad (113)$$

and

$$t = \frac{0.72m}{3 * 10^5 m/s} = 2.4 * 10^{-6} s \quad (114)$$

respectively.

- **Excitation** As long as the velocity is still over ca. 49km/s, collisions with the nuclei in the shielding can excite the electrons inside the pearl by 3.5 keV or more and make pairs of quasi electrons and holes say. We expect that often the creation of (as well as the decay of) such excitations require electrons to pass through a (quantum) tunnel and that consequently there will be decay half lives of very different sizes. We hope even up to many hours or days or years...
- **Slowly sinking:** After being stopped in the of the order of $\frac{1}{4}m$ of the shielding the pearls continue with a much lower velocity driven by the gravitational attraction of the Earth. In section 4.3.1 we have discussed the minimal mass M required to get the pearl come down in less than a year so that the seasonal variation in the counting by the DAMA-experiment can be realized; we found that the boundary number is not so far order of magnitudewise from our estimate based on assuming a size for the typical dust grain. Thus the time for the passage down of the pearl really tends to be not so far from about a year. After the year-long passage down the 1400 m to the laboratories most of the pearls have returned to their ground states, but some exceptionally long living excitations may survive.

Note that the slowly sinking velocity is so low that collisions with nuclei cannot give such nuclei enough speed to excite the scintillation counters neither in DAMA nor in Xenon-experiments

- **Electron or γ emission** Typically the decay of an excitation could be that a hole in the Fermi sea of the electron cloud of the pearl gets filled by an outside electron under emission of another electron by an Auger-effect. The electron must tunnel into the pearl center. This can make the decay life time become very long and very different from case to case. That the decay energy is released most often as electron energy means that such events are discarded by most of the Xenon-experiments, which only expect the nucleus recoils to be dark matter events. This would explain the long standing controversy consisting in DAMA seeing dark matter with a much bigger rate than the upper limits from the other experiments.

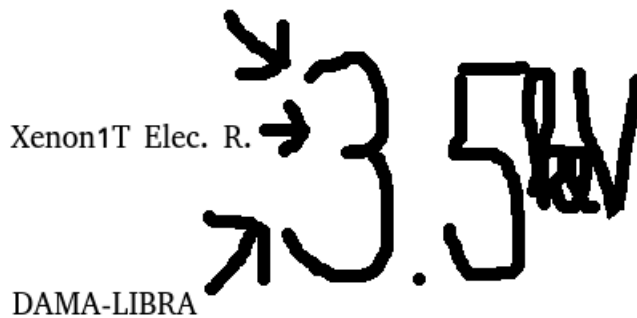
Using all the DAMA and DAMA-LIBRA data in the energy range 2 keV - 6 keV [23] gives an annual modulation amplitude of $0.0103 + /-0.0008 \text{ cpd/kg/keV}$ (cpd = counts per day).

Rather recently though Xenon1T looked for potential excess events among the *electron recoil events* and found about $16 \pm 5 \text{ events/year/tonne/keV}$ in the low energy region over a background of $(76 \pm 2) \text{ events/year/tonne/keV}$. This corresponds to a counting rate of about $0.00004 \text{ cpd/kg/keV}$. In our model this rate should be compatible with the above DAMA-LIBRA event rate. However they deviate by a factor of about 250. It therefore appears that we need the pearls to run much faster through the xenon apparatus than through the DAMA one, as assumed in section 4.3.1.

6. 3.5 keV

Order of magnitudewise we see 3.5 keV in 3 different places.

X-ray galaxy cl.



The energy level difference of about 3.5 keV occurring in 3 different places is important evidence motivating our model of dark matter particles being excitable by 3.5 keV:

- **The line** From places in outer space with a lot of dark matter, galaxy clusters, Andromeda and the Milky Way Center, an unexpected X-ray line with photon energies of 3.5 keV (to be corrected for Hubble expansion...) was seen.
- **Xenon1T** The dark matter searching Xenon1T did not find the standard nucleus-recoil events expected for dark matter, but found an excess of *electron-recoil* concentrated crudely around 3.5 keV.
- **DAMA** The seasonally varying component of their events lie in energy between 2 keV and 6 keV, not far from centering around 3.5 keV.

We take it seriously and not as an accident that both DAMA and Xenon1T see events with energies of the order of the controversial astronomical 3.5 keV X-ray line. We are thereby driven towards the hypothesis that the energies for the events in these underground experiments for the events are determined from a decay of an excited particle, rather than from a collision with a particle in the scintillator material. It would namely be a pure accident, if a collision energy should just coincide with the dark matter excitation energy observed astronomically.

So we ought to have decays rather than collisions! How then can the dark matter particles get excited ?

You can think of the dark matter pearls in our model hitting electrons and/or nuclei on their way into the shielding:

- **Electrons** Electrons moving with the speed of the dark matter of the order of 300 km/s toward the pearls in the pearl frame will have kinetic energy of the order

$$E_e \approx \frac{1}{2} * 0.5MeV * \left(\frac{300km/s}{3 * 10^5 km/s}\right)^2 = 0.25eV. \quad (115)$$

- **Nuclei** If nuclei are say Si, the energy in the collision will be $28 * 1900$ times larger $\sim 5 * 10^4 * 0.25eV \approx 10$ keV. That would allow a 3.5 keV excitation.

To deliver such ≈ 10 keV energy the nucleus should hit something harder than just an electron inside the pearls. It should preferably hit a nucleus, e.g. C, inside the pearl.

7. Direction of the Pull on Parameters from Fitting

The fitting of our model is not quite perfect although taking into account that it is so crude, what we assume, it really works reasonably well.

We can therefore seek to learn in what direction it should be improved to get a better agreement, i.e. we could ask the direction of the pull for our fitting.

The troubles really are:

- The combination $r_{dust}E$, where E is the elastic modulus of the dust grain and r_{dust} the size / radius of the dust pearl, should be very large to make the value of the parameter v_0 , which gives the characteristic velocity where the cross section to mass ratio $\frac{\sigma}{M}$ decreases significantly, agree with the value $v_0 \approx 220$ km/s suggested by fitting to the data of Figure 8.
- We have some lower limits for the mass M especially from the requirement that the pearls shall come sufficiently quickly through the shielding of the DAMA experiment, so as to be able to deliver the seasonal variation being used by DAMA to identify them as dark matter. This puts a constraint on r_{dust} when we use the relation

$$\frac{\sigma}{M}|_{v=0} = \frac{\pi r_{dust}^2}{M}, \quad (116)$$

and the zero velocity limit $\frac{\sigma}{M}|_{v=0}$ obtained from the extrapolation of Correa's fit to the data in Figure 8.

Both of these constraints push the dust radius r_{dust} upwards, but a factor of $2 * 10^{11}$ increase in r_{dust} from $10^{-7}m$ to $2 * 10^4m$, as would be required if we take the elastic modulus E to be just that for a normal material, would be too outrageous. However an increase in r_{dust} is what is called for.

We could attempt to justify such an increase of r_{dust} as an effect of the hardening of the material, which would make it less easy to split off pieces of the dust grain.

We also have the possibility to say, that the zero velocity cross section to mass ratio is largely only an extrapolation and not a true measurement. Thus we should rather say that we should only trust the combination of v_0 and $\frac{\sigma}{M}|_{v=0}$ which gives us the value of the ratio for the velocity region where it was truly measured by the dwarf galaxies essentially.

Estimating crudely seeking to fit the formula (37) by as small v_0 as possible we seemingly cannot get v_0 smaller than about 100 km/s. And if we seek this type of fit the zero-velocity ratio must be increased to about $1000 \text{ cm}^2/\text{g}$ rather than the used $150 \text{ cm}^2/\text{g}$. But this means that there is no chance for dramatically change the value for v_0 which we need.

It is thus hard to see how we can get a sensible fit to the velocity dependence of the cross section without the unbelievably large modulus, or very big dust grains.

8. Conclusion

We have described a seemingly very viable model for dark matter consisting of nano-meter size but macroscopic pearls. These pearls consist of a bubble of a new speculated type of vacuum containing some material - presumably carbon - under the high pressure of the skin (surface tension). It can contain about 10^{12} nucleons in the bubble of radius about a few nanometers.

The electrons in a pearl are partly pushed out of the genuine bubble of the new vacuum phase.

Further outside the bubble with its surrounding electron cloud there is an a priori usual dust grain, but it is modified somewhat in its density and strength - especially the elastic modulus - by the modification of the electron wave functions due to the for usual matter exceedingly big homolumo gap.

We have compared the model or attempted to fit:

- Astronomical suggestions for the self interaction of dark matter in addition to pure gravity
- The astronomical 3.5 keV X-ray emission line found by satellites, supposedly from dark matter.
- The underground dark matter searches.

We list below the quantities we have crudely estimated:

1. The low velocity cross section divided by mass is not predicted in our present model, but the pearl mass estimated using the value Correa obtained [15] from the dwarf galaxy data is only in very mild conflict with the mass constraints in our model - see Table 2 in section 8.2 below.
2. That the signal from Xenon1T and Dama should agree except for different scintillation efficiencies and notation, ... (*not working so well!*). However, we suppose it is solved by the pearls having higher terminal velocities in the *fluid* xenon, than in the solid NaI in DAMA.
3. The absolute rate of the two underground experiments.
4. The rate of emission of the 3.5 keV line from the Tycho supernova remnant [36] due to the excitation of our pearls by cosmic rays [13].

# & exp/th	Quantity	value	related Q.	value	sec.
1. exp th. th.	Dwarf Galaxies Velocity par. v_0 with hardening without hard.	220km/s 77km/s 0.7cm/s	$4r_{dust}E$ $4r_{dust}E$ $4r_{dust}E$	$8.1 * 10^{13}\text{kg/s}^2$ $1 * 10^{13}\text{kg/s}^2$ 400kg/s^2	4.1.4
2. exp th th	DAMA-LIBRA air stone	0.041cpd/kg 0.16cpd/kg $1.6 * 10^{-5}\text{cpd/kg}$	suppression	$1.6 * 10^{-10}$ $6 * 10^{-10}$ $6 * 10^{-14}$	4.5
3. exp th th	Xenon1T air stone	$2 * 10^{-4}\text{cpd/kg}$ 0.16cpd/kg $1.6 * 10^{-5}\text{cpd/kg}$	suppression	$6 * 10^{-13}$ $6 * 10^{-10}$ $6 * 10^{-14}$	4.5
4. exp th	Jeltema & P. counting rate	$2.2 * 10^{-5}\text{phs/cm}^2/\text{s}$ $3 * 10^{-6}\text{phs/s/cm}^2$	$\frac{\sigma}{M} _{\text{Tycho}}$ $1\% * \alpha * \frac{\sigma}{M} _{\text{nuclear}}$	$5.6 * 10^{-3}\text{cm}^2/\text{kg}$ $8 * 10^{-4}\text{cm}^2/\text{kg}$	4.6
5. exp th	Intensity 3.5 keV $\frac{N\sigma}{M^2}$	$10^{23}\text{cm}^2/\text{kg}^2$ $3.6 * 10^{22}\text{cm}^2/\text{kg}^2$	$\frac{\xi_{f.s}^{1/4}}{\Delta V}$	0.6MeV^{-1} 0.5MeV^{-1}	4.4
6. ast DAMA Xen.	Three Energies line av. en. av. en.	3.5keV 3.4keV 3.7keV			6

Table 1

5. Relation between the frequency 3.5 keV and the overall emission rate of this X-ray line observed from galaxy clusters etc.
6. Using a “dirty” story of hardening the dust grain, we can get the velocity scale parameter v_0 in our velocity dependence fit of the $\frac{\sigma}{M}$ data in Figure 8.
7. We have previously predicted the ratio of dark matter to ordinary matter in the Universe to be of the order of 5 by consideration of the binding energies per nucleon in helium and heavier nuclei, assuming that the ordinary matter at some time about 1 s after the Big Bang was expelled from the pearls under a fusion explosion from He fusing into say C [6].

8.1 Résumé of our Predictions

In Table 1 we summarize six of our predictions. In the first column is written the number in our listing and we use the symbols ”exp” = experiment meaning the number given in the third column corresponds to the experimentally observed number or is closely related to it, and ”th”

= theory meaning that the third column in that row contains our model prediction. The second column contains a short name for the quantity we attempt to predict plus in some cases a detail of the assumption in our model, such as “air” or “stone” specifying the material in which the dark matter particle is supposed to be stopped. In most cases we formulate the calculation as to be found in the section listed in the sixth column by calculating a quantity, related to the value measured and given in column 2, called “related quantity” = “related Q.” and specified in column 4, by its name or formula. The experimental and theoretical values of this related quantity “related Q.” are given in the appropriate “exp” and “th” lines of column 5, with the possible detailed choice of the version of our model spelled out by the words in column 2 like “air” or “stone”. In the second column of item 6 we use the shorthand “av. en.” to denote our best estimate of the average energy of the supposedly dark matter caused events.

Let us now briefly explain the 6 predictions in Table 1:

1. **Dwarf Galaxies, Velocity parameter v_0** We fit the velocity dependence as found by Correa [15] of the cross section to mass ratio $\frac{\sigma}{M}$ to a formula for relatively small velocities of the form

$$\frac{\sigma}{M} = \frac{\sigma}{M}|_{v \rightarrow 0} * (1 - v/v_0)^4. \quad (117)$$

The velocity parameter v_0 turned out to depend on the “related Q.” being $4r_{dust}E$, where r_{dust} is the radius of the dust particle around the bubble and E is the elastic modulus of this dust grain. In column 2 we distinguish two versions of our theoretical model: “with hardening” meaning that we use an effect which the very strong homolumo gap in the bubble is supposed to have on the dust around it. It makes the modulus E much bigger and we get the quantity $4r_{dust}E$ raised up to $1 * 10^{13} kg/s^2$ as written in column 5. Assuming no such effect we have the version of our model “without hard.” described in the third line of item 1.

2. **DAMA-LIBRA** The counting of the seasonal varying component of events in the DAMA-LIBRA experiment is given as number of counts per day “cpd” and taken per kg of scintillator material in which it is seen, so that the seasonal varying counting rate S_m (m for modulation) is given as $0.041cpd/kg$. We then predict this rate by an extremely crude argument first based on assuming that the energy from the kinetic energy of the incoming dark matter particles is equally distributed over all the matter in the Earth down to the 1400 m depth of the experiments. But we also expect a further suppression factor called *suppression* in column 4 arising from the fact that the energy observed by DAMA-LIBRA comes from decays of excitations of the dark matter pearls having exceptionally long living excitation states with lifetimes of the order of the passage time down to the experimental halls. For the fraction *suppression* of the kinetic energy surviving down to the depth of the experiment we take, by a dimensional argument, the ratio of the time of excitation (in air or in stone) relative to the passage time down to the experiment. The theoretical numbers $6 * 10^{-10}$ for air and $6 * 10^{-14}$ for stone are simply such ratios of excitation times to passage time.
3. **Xenon1T** This is just the same estimation of the rate from crude dimensional arguments as in the DAMA-LIBRA case. Here though one has no modulation involved. Rather the

Description	R	ΔR	M	ΔM	Sec.	formula
$\frac{\sigma}{M} _{v \Rightarrow 0} = 15 \frac{m^2}{kg}$	$1.2 * 10^{-9} m$	70%	$\approx 3 * 10^{-15} kg$	200%	4.1.1	(8), (16)
Faster than year	$\geq 1.0 * 10^{-9} m$		$\geq 2.1 * 10^{-15} kg$		4.3.1	(84),(86)
Corrected f.t.y.	$\geq 3.1 * 10^{-9} m$		$\geq 6.5 * 10^{-14} kg$		4.3.1	(87),(88)
Dust enough	$\geq 1.0 * 10^{-9} m$		$\geq 2 * 10^{-15} kg$		4.2	(70), (68)
Velocity dep. w. $E=400^4$	$\approx 10^{-8} m$ $10^{-10} m$	big	$\approx 10^{-13} kg$ $\approx 2 * 10^{-18} kg$	big	4.1.4	

Table 2

experimental numbers are extracted as the observed electron recoil events minus an estimated background.

4. **Jeltema & P.** Jeltema and Profumo[36] observed the 3.5 keV X-ray line supposedly coming from dark matter from the supernova remnants of the Tycho Brahe supernova. We suppose this to be due to 3.5 keV radiation from the relatively small amount of dark matter in the neighbourhood of the remnant being, however, excited by a very high intensity of cosmic rays, for which the dark matter shows an effective $\frac{\sigma}{M}$ as if it were just atomic nuclei. The fraction of order the fine structure constant α of the excitation energy is emitted as 3.5 keV X-rays, while the rest of the emission is dominated by electrons. The 1% in the fourth column alludes to the fraction of the supernova energy coming as cosmic rays, while the ratio $\frac{\sigma}{M}_{nuclear}$ is the above mentioned ratio taken assuming that, as far as the cosmic ray cross section is concerned, the dark matter pearl just behaves like a group of independent nuclei of too small a size to significantly shadow each other. In the experimental row we find the effective $\frac{\sigma}{M}$ that would provide the intensity of the 3.5 keV line observed by Jeltema and Profumo $2.2 * 10^{-5} phs/cm^2/s$ (where phs means photons), it is called $\frac{\sigma}{M}|_{Tycho}$.
5. **Intensity** We fitted both the overall intensity of the 3.5 keV X-ray line observed from galaxy clusters etc. as supposedly proportional to the local *square* of the dark matter density and the very 3.5 keV energy itself identified with the homolumo gap in the electron spectrum of the dark matter pearls. These two quantities depend on the same combination of parameters in our model $\frac{\xi_{fS}^{1/4}}{\Delta V}$ for which we found the values in the 5th column. Formally we here take the 3.5 keV homolumo gap used not as an experimental measurement, but just as a parameter used in the theory. But oppositely we somewhat arbitrarily considered the intensity measurement a genuine experiment formally.
6. **Three Energies** We here just emphasize the very important observation for our theory, that the energy number 3.5 keV per event pops up three times. Instead of formally considering one of the numbers as theory and another as experiment, we simply list in column 1 the name of the experiment giving the energy per event then listed in column 3.

8.2 Bounds and Estimates of Size: Mass or Radius

In Table 2 we summarize the various estimates and bounds on the mass and radius of our pearls, giving a reference to the relevant section and formula in each case.

The uncertainties given in this table are only extremely crude and supposed to be interpreted logarithmically, so that 200% means within about a factor 7 ($= e^2$) and 70% within a factor of order 2. The “big” means that the uncertainty is huge because the mass essentially comes in only via its sixth root $\sqrt[6]{M}$.

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