

# Phenomenology of Supersymmetric Trinification resulting from the Dimensional Reduction of a $\mathcal{N} = 1, 10D, E_8$ Theory

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We present a supersymmetric extension of the Standard Model which originates from a  $10D, \mathcal{N} = 1, E_8$  gauge theory. The transition to four dimensions occurs after the dimensional reduction of the theory over the  $SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$  space. Making use of the Wilson flux breaking mechanism, the resulting  $4D$  theory is an  $\mathcal{N} = 1, SU(3)^3$  Grand Unified Theory with two  $U(1)$ s. After the symmetry breaking of the trinification gauge group, the model is viewed as a split-like supersymmetric version of the Standard Model with an extra gauge singlet that reminds of an NMSSM configuration. A preliminary 1-loop analysis shows great promise, predicting the unification (and first supersymmetry breaking) scale at  $\sim 10^{15}$  GeV with no proton decay and the (second) supersymmetry breaking scale in the region of a few TeV.

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## 1. Introduction

Our study is a realistic example, actually the best one found in long standing searches, of the fundamental and insightful works of Forgacs-Manton (F-M), the Coset Space Dimensional Reduction (CSDR) [1–3] and Scherk-Schwartz (S-S) [4], the group manifold reduction. The CSDR mechanism took into consideration the number of dimensions and the starting gauge group, as predicted by the heterotic string [6], with those two sharing the common ground that they lead to promising Grand Unified Theories (GUTs). Moreover, it is worth noting that the higher-dimensional theory is accompanied by the unification of the gauge and scalar sectors, with the scalars being identified as part of the extra-dimensional components of the vector fields (due to the constraints imposed). In case the higher-dimensional theory is assumed to be supersymmetric, fermions participate in the aforementioned unification, in the sense that they consist the fermionic counterpart of the gauge fields in a vector supermultiplet. Two remarkable features of the CSDR are that the fermionic terms of the higher-dimensional action lead to  $4D$  Yukawa interactions and that the reduced  $4D$  theories can be chiral, if necessary conditions are applied on the fermionic spectrum of the higher-dimensional theory [7]. However, the most powerful property of the CSDR mechanism is that it does not inherit the amount of supersymmetry to the  $4D$  theory, e.g.  $N = 4$  in the naive reduction or  $N = 1$  in the CY reduction. The CSDR leads either to non-supersymmetric theories when the reduction is done over symmetric coset spaces or to softly broken  $N = 1$ , at least if the higher-dimensional theory is defined in  $10D$ . [8–11] (see also [12]).

In our specific model, the initial, higher-dimensional theory is a  $10D$ ,  $N = 1$ ,  $E_8$  gauge theory whose spectrum is minimal, consisting solely of a vector supermultiplet. The CSDR mechanism is performed over the  $SU(3)/U(1)^2 \times \mathbb{Z}_3$ , which is a modification of the  $6D$  flag manifold  $SU(3)/U(1)^2$  (non-symmetric coset space), where the freely-acting  $\mathbb{Z}_3$  component has been introduced to enable the triggering of the Wilson flux mechanism, which causes a diminution of the produced gauge symmetry of the reduced (grand unified) theory to the  $SU(3)^3 \times U(1)^2$  [2, 8, 9, 13] (see also [14]). The produced GUT is also (softly broken)  $N = 1$  supersymmetric.

A specific choice of small compactification space radii breaks the  $SU(3)^3 \times U(1)^2$  gauge group at a unification scale  $\sim 10^{15}$  GeV, resulting in a (broken) split-like supersymmetric scenario, in which gauginos, Higgsinos (of the third generation), sleptons and a singlet field that originates from the higher-energy theory all acquire masses at the TeV scale, and the rest supersymmetric spectrum (together with the 'exotic' particles that come from the trinification multiplets and two more singlet fields) is superheavy. The heavy states are integrated out many orders of magnitude above the TeV scale, leading to additional interaction between the light states which will be taken into account. An early 1-loop phenomenological analysis gives masses for the light Higgs boson mass within the experimental boundaries, while the top and bottom quark masses are also in  $(2\sigma)$  agreement with experimental measurements. The full model, analysis and various remarks and other details can be found in our recent work [15, 16].

## 2. Dimensional Reduction of $E_8$ over $SU(3)/U(1)^2$

In this section we apply directly the CSDR in our specific case, that is the  $10D$ ,  $N = 1$ ,  $E_8$  Yang-Mills-Dirac theory with Weyl-Majorana fermions over the non-symmetric coset space  $SU(3)/U(1)^2$

[2, 8, 14, 17]. The produced  $4D$  action is:

$$S = C \int d^4x \operatorname{tr} \left[ -\frac{1}{8} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (D\phi_a)^2 \right] + V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi, \quad (1)$$

where  $V(\phi)$  is given as:

$$V(\phi) = -\frac{1}{8} g^{ac} g^{bd} \operatorname{tr} \left( f_{ab}^C \phi_C - ig[\phi_a, \phi_b] \right) \left( f_{cd}^D \phi_D - ig[\phi_c, \phi_d] \right) \quad (2)$$

and  $\operatorname{tr}(T^i T^j) = 2\delta^{ij}$ , where  $T^i$  are the  $E_8$  generators. Also,  $g$  is the coupling constant,  $C$  is the coset volume,  $D_\mu = \partial_\mu - igA_\mu$ ,  $D_a$  are the  $4D$  covariant derivative and the coset space covariant derivative, respectively and, last,  $g_{ab}$  is the metric of the coset space, given by  $g_{\alpha\beta} = \operatorname{diag}(R_1^2, R_1^2, R_2^2, R_2^2, R_3^2, R_3^2)$ .  $V(\phi)$  is only formal since  $\phi$  must satisfy  $f_{ai}^D \phi_D - [\phi_a, \phi_i] = 0$ . The  $4D$  gauge group is determined by the centralizer of  $U(1) \times U(1)$  in  $E_8$ :

$$H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B.$$

Moreover, the CSDR rules determine the representations of the particles that consist the particle spectrum of the  $4D$  theory (details in [2, 8, 13]). Specifically the surviving gauge fields (of  $E_6 \times U(1)_A \times U(1)_B$ ) fall into  $N = 1$  vector supermultiplets whereas the matter fields fall into six  $N = 1$  chiral ones, where three of the latter are  $E_6$  singlets carrying  $U(1)_A \times U(1)_B$  charges, while the rest have non-trivial transformation properties under the whole  $4D$  gauge group. In particular, the matter fields transform under  $E_6 \times U(1)_A \times U(1)_B$  as:

$$\alpha_i \sim 27_{(3, \frac{1}{2})}, \beta_i \sim 27_{(-3, \frac{1}{2})}, \gamma_i \sim 27_{(0, -1)}, \quad (3)$$

$$\alpha \sim 1_{(3, \frac{1}{2})}, \beta \sim 1_{(-3, \frac{1}{2})}, \gamma \sim 1_{(0, -1)}. \quad (4)$$

Regarding the potential of the theory, besides the terms identified as F and D-terms, the rest are interpreted as soft scalar masses and trilinear soft terms. As far as the gaugino mass,  $M$ , is concerned, it is given by the following relation [8]:

$$M = (1 + 3\tau) \frac{R_1^2 + R_2^2 + R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}, \quad (5)$$

where  $\tau$  signals the potential presence of torsion in the flag manifold. This expression implies that, in absence of torsion, the gauginos gain mass at the compactification scale [2]. This result can change in presence of torsion [8] as required in the split supersymmetry scenario, which requires a gaugino mass in the  $TeV$  scale.

### 3. Breaking by Wilson flux mechanism

In the previous section we demonstrated the case of applying the CSDR on an  $E_8$  gauge theory over the coset space  $SU(3)/U(1)^2$ . Nevertheless, the resulting  $4D$  gauge group,  $E_6 \times U(1)^2$  cannot be broken down to the gauge group of the Standard Model (SM) by the scalar Higgs accommodated in the 27 representation. Therefore, in order to end up with a different  $4D$  gauge group (with less

symmetry), the Wilson flux breaking mechanism is introduced [18–20]. In order that the above mechanism to get induced, the coset space must be modified from simply connected, that is the default case for  $SU(3)/U(1)^2$ , to multiply connected. To achieve this modification, the freely-acting discrete symmetry  $\mathbb{Z}_3$  on  $SU(3)/U(1)^2$  is employed, therefore the space on which the reduction is performed is now the  $SU(3)/U(1)^2 \times \mathbb{Z}_3$ . Each  $g \in \mathbb{Z}_3$  is mapped to an element  $U_g \in E_6$  (by a non-contractable to zero Wilson loop) and the set of these elements consists the image of  $\mathbb{Z}_3$ ,  $T^{E_6}$  in  $E_6$ . The above map turns out to be a homomorphism, and once it is determined,  $E_6$  breaks to the centralizer  $C_{E_6}(T^{E_6}) = SU(3)^3$  [13]. Also, the presence of the discrete symmetry functions as a filtering mechanism for the spectrum, i.e. only fields that are invariant under the action of  $\mathbb{Z}_3$  on both their gauge and geometric indices make it through to the resulting  $SU(3)^3$  gauge theory<sup>1</sup>. In the  $E_6$  phase, the matter fields were belonging to the trivial or 27 representations. For the trivial case, out of the three  $E_6$  singlets  $\alpha, \beta, \gamma$  of eq.(4) only one survives, specifically the  $\alpha \equiv \theta_{(3, \frac{1}{2})}$ . In turn, the  $SU(3)^3$  representations of the non-trivial surviving matter fields are obtained by the decomposition  $E_6 \supset SU(3)^3$ , that is  $27 = (1, 3, \bar{3}) \oplus (\bar{3}, 1, 3) \oplus (3, \bar{3}, 1)$  and are found to be the following:

$$\alpha_1 \equiv \Psi_1 \sim (1, 3, \bar{3})_{(3, \frac{1}{2})}, \beta_3 \equiv \Psi_2 \sim (\bar{3}, 1, 3)_{(-3, \frac{1}{2})}, \gamma_2 \equiv \Psi_3 \sim (3, \bar{3}, 1)_{(0, -1)}, \quad (6)$$

where the above are the parts of the three 27 chiral multiplets of  $\alpha_i, \beta_i, \gamma_i$  of eq.(3) and combined they form one complete generation. The reduction of the number of the generations is an unwelcome feature and in order to return to a spectrum of three ones, non-trivial monopole charges in the  $U(1) \times U(1)$  part of the coset needs to be introduced, leading to three identical instances of the above fields, recovering the desired number of generations [28].

The employment of the Wilson flux breaking mechanism affects the scalar potential as well, in the sense that it can be rewritten from the  $E_6$  language to the  $SU(3)_c \times SU(3)_L \times SU(3)_R$  one as [13]:

$$V_{sc} = 3 \cdot \frac{2}{5} \left( \frac{1}{R_1^4} + \frac{1}{R_2^4} + \frac{1}{R_3^4} \right) + \sum_{l=1,2,3} V^{(l)}, \quad (7)$$

in which  $V^{(l)} = V_{susy} + V_{soft} = V_D + V_F + V_{soft}$ , with  $l$  being a generation index which we drop in the ensuing (unless its presence is necessary), since we focus on the third generation for our analysis and calculations. The F-terms derive from the superpotential which is given by the following expression:

$$\mathcal{W} = \sqrt{40} d_{abc} \Psi_1^a \Psi_2^b \Psi_3^c, \quad (8)$$

the various D-terms are written as:

$$D^A = \frac{1}{\sqrt{3}} \langle \Psi_i | G^A | \Psi_i \rangle, \quad D_1 = 3 \sqrt{\frac{10}{3}} (\langle \Psi_1 | \Psi_1 \rangle - \langle \Psi_2 | \Psi_2 \rangle), \quad (9)$$

$$D_2 = \sqrt{\frac{10}{3}} (\langle \Psi_1 | \Psi_1 \rangle + \langle \Psi_2 | \Psi_2 \rangle - 2 \langle \Psi_3 | \Psi_3 \rangle - 2 |\theta|^2) \quad (10)$$

<sup>1</sup>For more details on the parametrization of the filtering procedure see the original work [15] but also [13, 21].

and, last, the soft supersymmetry breaking terms are:

$$\begin{aligned}
V_{soft} &= \left( \frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \langle \Psi_1 | \Psi_1 \rangle + \left( \frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \langle \Psi_2 | \Psi_2 \rangle \\
&+ \left( \frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) (\langle \Psi_3 | \Psi_3 \rangle + |\theta|^2) \\
&+ 80\sqrt{2} \left( \frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_1 R_2} \right) (d_{abc} \Psi_1^a \Psi_2^b \Psi_3^c + h.c.) \\
&= m_1^2 \langle \Psi_1 | \Psi_1 \rangle + m_2^2 \langle \Psi_2 | \Psi_2 \rangle + m_3^2 (\langle \Psi_3 | \Psi_3 \rangle + |\theta|^2) + (\alpha_{abc} \Psi_1^a \Psi_2^b \Psi_3^c + h.c.).
\end{aligned} \tag{11}$$

Following [22], the multiplets of the fields found in (6) can be nicely expressed in the  $SU(3)_c \times SU(3)_L \times SU(3)_R$  language as complex  $3 \times 3$  matrices according to the following assignment:

$$\Psi_2 \sim (\bar{3}, 1, 3) \rightarrow (q^c)_p{}^\alpha, \quad \Psi_3 \sim (3, \bar{3}, 1) \rightarrow (Q_\alpha^a), \quad \Psi_1 \sim (1, 3, \bar{3}) \rightarrow L_a^p, \tag{12}$$

which leads to the more legible and comprehensive form of the particle content of an enhanced version of the Minimal Supersymmetric Standard Model (MSSM):

$$q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \quad Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}, \quad L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}.$$

#### 4. Specification of parameters and GUT breaking

##### Choice of radii

Having established the theoretical frame, in order to advance to the phenomenological part, we proceed by making two important assumptions. First, the compactification scale is considered to be high<sup>2</sup> and second, the compactification and unification scales coincide,  $M_C = M_{GUT}$ , which means that the scale of the three radii of the compactification scale is  $R_l \sim \frac{1}{M_{GUT}}$ ,  $l = 1, 2, 3$ . Without any further assumption this would lead to a superheavy supersymmetric spectrum<sup>3</sup> (of  $\mathcal{O}(M_{GUT})$ ) and soft trilinear couplings. However, we can treat one of the radii, let us call the third, to be slightly different than the others. Under this assumption, inspection of the expression of the scalar potential, (11), leads to the understanding that the supersymmetric spectrum undergoes a separation (split-like scenario), with the squarks being superheavy but the sleptons gaining mass in the  $TeV$  energy regime.

##### The breaking of $SU(3)^3$

The breaking of the  $SU(3)_L$  and  $SU(3)_R$  parts of the gauge group can be triggered by the following vevs of the two families of  $L$ 's:

$$\langle L_s^{(3)} \rangle = \text{diag}(0, 0, V), \quad \langle L_s^{(2)} \rangle = \text{anti} - \text{diag}(0, 0, V),$$

<sup>2</sup>Working with high compactification scale, the Kaluza-Klein excitations can be ignored. In case the compactification scale was considered at the  $TeV$  scale, then the eigenvalues of the Dirac and Laplace operators of the  $SU(3)/U(1)^2$ , which are not known yet, would be necessary to be included in the calculation.

<sup>3</sup>Gauginos are not taken into consideration in this reasoning since they obtain mass in a geometric manner [8]

where the  $s$  index designates the scalar component of the supermultiplet<sup>4</sup>. These vevs are singlets under  $SU(3)_c$ , therefore they do not break the colour part of the total gauge group. Appropriate combination of the two vevs leads to the desired breaking, that is to the SM gauge group [23]:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y . \quad (13)$$

According to the configuration of the scalar potential, the above breaking gives vevs to the singlet of each family (not necessarily to all three), specifically in our case,  $\langle \theta^{(3)} \rangle \sim \mathcal{O}(TeV)$ ,  $\langle \theta^{(1,2)} \rangle \sim \mathcal{O}(M_{GUT})$ . As far as the two abelian symmetries are concerned, they break due to  $\langle \theta^{(1,2)} \rangle$  (in addition to  $\langle L_s^{(2,3)} \rangle$ ), but their global versions remain in the theory. Last, the electroweak breaking proceeds by the following vev configuration,  $\langle L_s^{(3)} \rangle = \text{diag}(v_d, v_u, 0)$  [24].

### Lepton Yukawa couplings and $\mu$ terms

Due to the presence of the aforementioned global symmetries, invariant lepton Yukawa terms are not allowed in the Yukawa sector. Nevertheless, according to [27], the  $4D$  theory can be considered as renormalizable, therefore below the unification scale an effective term can emerge in the form of higher-dimensional operator  $L\bar{e}H_d\left(\frac{\bar{K}}{M}\right)^3$  [13], where  $\bar{K}$  denotes the vev of the conjugate scalar component of any combination of  $S^{(i)}$ ,  $\nu_R^{(i)}$  and  $\theta^{(i)}$ . Similar argumentation may also allow mass terms for  $S^{(i)}$  and  $\nu_R^{(i)}$ , ending up to be superheavy. Moreover, appropriate higher-dimensional operators can be employed for the emergence of the  $\mu$ -term, one for each family  $H_u H_d \bar{\theta} \frac{\bar{K}}{M}$ . Due to the vev configuration, it is understood that the  $\mu$  terms corresponding to the Higgs doublets of the  $l = 1, 2$  generations will be supermassive, while that of the  $l = 3$  generation will be at the  $TeV$  scale. Thus, for consistency reasons we include all operators of dimension 5,6 and 7.

## 5. 1-loop Analysis

Since the dimensional reduction led to a GUT, it is understood that all gauge couplings are equal to  $g$  at  $M_{GUT}$ . Moreover, at the higher-dimensional level, there is a single coupling, therefore the (quark) Yukawa couplings have to be equal to  $g$  at  $M_{GUT}$ . Our phenomenological analysis is performed using 1-loop  $\beta$ -functions for all parameters involved. Below the unification scale they run using supersymmetric RGEs (squarks included) plus the 4 additional Higgs doublets (and their supersymmetric counterparts), down to an intermediate scale  $M_{int}$ , namely the scale below which all supermassive particles and parameters have decoupled. Below  $M_{int}$  the RGEs include only the 2 Higgs doublets that originate from the third generation, their corresponding Higgsinos, the gauginos, the sleptons and the extra singlet superfield. The fermionic part of the singlet, due to the higher-dimensional term mentioned above, is intertwined with the Higgs fields and is included in the neutralino rotation matrix, creating a scenario close to the NMSSM configuration. Last, below ( $M_{TeV}$ ) we have non-supersymmetric RGEs.

### Gauge unification

The first test for each GUT is to produce the prediction of the unification scale,  $M_{GUT}$ . We follow the straightforward methodology, namely the  $a_{1,2}$  are used for the  $M_{GUT}$  calculation and the

<sup>4</sup>There exist more vevs that can be added without affecting the breaking, see [23].

$a_3$  is used for confirmation. The 1-loop gauge  $\beta$ -functions are given by  $2\pi\beta_i = b_i\alpha_i^2$ , where the  $b_i$  coefficients vary for each of the three energy regions according to the corresponding particle spectrum [15]. Taking into account an uncertainty of 0.3% at the boundary of  $M_{GUT}$ , the various scales of our model are obtained:  $M_{GUT} \sim 1.7 \times 10^{15} GeV$ ,  $M_{int} \sim 9 \times 10^{13} GeV$ ,  $M_{TeV} \sim 1500 GeV$ . The calculation of the  $\alpha_s$  gives the following prediction:

$$a_s(M_Z) = 0.1218, \quad (14)$$

which is within  $2\sigma$  of the experimental value,  $a_s(M_Z) = 0.1187 \pm 0.0016$  [25]. But what happens with proton decay?

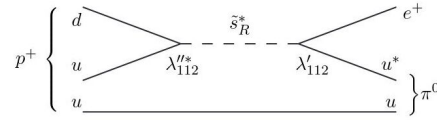
### Proton Decay

The dangerous processes that could fall under the experimental limits of the proton half-life are the decays to  $K^+\bar{\nu}$ ,  $\pi^0\mu^+$ ,  $\pi^0e^+$ ,  $\pi^+\bar{\nu}$  and  $K^0\mu^+$ . In the usual MSSM superfield notation the terms that can account for all the above processes are:

$$\bullet Lq^c Q + QLq^c + h.c. \quad (15)$$

$$\bullet QQQ + q^c q^c q^c + h.c. \quad (16)$$

From the combination of terms in Eq. (15) and Eq. (16) we can get diagrams like (but not limited to) the diagram below.



However, the presence of the two abelian symmetries forbids the terms of Eq. (16). Thus, proton decay cannot occur from such processes.

Let us be more thorough and discuss proton decay in the context of the MSSM as well. In the MSSM we have no *superfast* proton decay because it is protected by R-parity. However, the model discussed here features no such symmetry. Thus, one can have superfast proton decay (from the process of the above diagram) if  $L_i Q_j \bar{d}_k$  and  $\bar{u}_i \bar{d}_j \bar{d}_k$  are *both* present. However, neither term exists in the model, since the former does not appear in the superpotential (that is derived from the initial theory) and cannot appear as a higher dimensional operator because of restrictions by the abelian symmetries, while the latter is forbidden by the abelian symmetries and cannot appear as a higher dimensional operator for the same reason. Consequently, the proton is stable in our model.

### Further results

As mentioned above, the gauge and quark Yukawa interactions share the same coupling constant  $g$  due to unification and that property was used as a boundary condition in our calculations for the  $M_{GUT}$ . However, as commented in section 4, tau Yukawa terms are absent due to the presence of the

global symmetries and that is why they were introduced through higher-dimensional operators. This means that there exists much wider freedom of the corresponding coupling constant and therefore the boundary condition is not fixed to  $g$ . This property motivated us to pick the input in our model to be the tau lepton mass [25].

Also, we take into consideration uncertainties in the two important energy scales,  $M_{GUT}$  and  $M_{TeV}$ , due to threshold corrections (for details see [26]). In the current level of our analysis it was sufficient to consider all “light” supersymmetric particles on equal footing. The top and bottom Yukawa couplings come with the following uncertainties: 6% for the GUT boundary and 2% for the  $TeV$  boundary. As follows, the masses of the quarks of the third generation and that of the (light) Higgs are in agreement with their experimental values [25]. As explained above, all allowed Yukawa terms share the common value of the coupling constant,  $g$ , at the unification scale. For that reason, it is necessary to consider that the model exhibits a large  $\tan\beta \sim 48$  in order to recover the experimentally observed discrepancy of the fermion masses.

We currently work on a full 2-loop analysis that will produce the entire (TeV scale) supersymmetric spectrum (lifting its mass degeneracy of the previous analysis) and scan the available parameter space taking into account existing experimental constraints. We also work on a study that will determine to which extent dark matter can be explained in the context of the model, and a study for the potential discovery of the model in the next LHC run and/or a prospective 100TeV collider.

## 6. Conclusions

First we considered a  $10D$ ,  $\mathcal{N} = 1$ ,  $E_8$  Yang-Mills-Dirac theory with Weyl-Majorana fermions, constructed on the compactified spacetime of the form  $M_4 \times B_0/\mathbb{Z}_3$ , where  $B_0$  is the coset space  $SU(3)/U(1) \times U(1)$  and  $\mathbb{Z}_3$  is a discrete group which acts freely on  $B_0$ . In order to result with the promising  $4D$  (softly broken)  $\mathcal{N} = 1$ ,  $SU(3)^3$  GUT (plus two  $U(1)$ s), we employed two mechanisms: the CSDR and the Wilson flux breaking. The GUT breaking along with the assumption of a slight discrepancy between the radii of the coset led to a split-like supersymmetric scenario where the gauginos, third generation Higgsinos, sleptons and an extra singlet gain mass at the TeV scale, while the rest supersymmetric particles become supermassive ( $\sim M_{GUT}$ ). The model is proton-decay safe and the preliminary 1-loop analysis gives very promising results.

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