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Supersymmetry breaking, brane dynamics and the swampland

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Breaking supersymmetry in string theory entails a number of instabilities, both of perturbative and non-perturbative. This casts doubt on the consistency of string theory upon supersymmetry breaking, at least at the level of low-energy effective field dynamics. In order to explore this issue, in this contribution we review the connection between vacuum (in)stabilities and the interactions between branes of various dimensions, both charged and uncharged, in three models where supersymmetry is broken at the string scale. These are the $USp(32)$ and $U(32)$ orientifold projections of the type IIB and type 0B strings and the $SO(16) \times SO(16)$ heterotic model. We describe the force exerted by branes in the probe regime and via a string amplitude computation. Whenever they can be compared, they show qualitative agreement despite no (manifest) supersymmetric protection takes place. We discuss a deeper connection with the weak gravity conjecture for charged branes, which is satisfied in a novel way due to a renormalization of the effective charge-to-tension ratio.

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1. Introduction

The problem of supersymmetry breaking is one of the fundamental obstacles that string theory ought to overcome in order to connect to realistic phenomenology. The absence of supersymmetry typically entails numerous problems in low-energy effective field theory (EFT), and elucidating their relation with microscopic physics is a stepping stone toward a deeper understanding of the issue. A natural starting point is string-scale supersymmetry breaking, since it is simple to concoct at the level of string perturbation theory. Despite its deceptive simplicity, the resulting models exhibit dynamical tadpoles, and the ultimate fate of the vacuum remains unclear.

Here we discuss the $SO(16) \times SO(16)$ heterotic model of [1, 2] and two orientifold models: the $U(32)$ "type 0′B" model of [3, 4] and the $USp(32)$ model of [5], which exhibits the peculiar phenomenon of "brane supersymmetry breaking" (BSB) [6–9]. Recent considerations in the context of the swampland program [10–13] also support the naturalness of string-scale supersymmetry breaking. In particular, the gravitino mass appears to play an important role, and resonates with the phenomenon of BSB, as discussed in [14].

In this paper we review the connection between brane interactions, vacuum stability and the weak gravity conjecture (WGC) discussed in [15] (see also [16]), in order to shed light on the nature and consistency of instabilities induced by dynamical tadpoles. Concretely, we present the computation of static interaction potentials between stacks of parallel branes of various charges and dimensions. In the case of charged, extremal branes, these potentials turn out to be repulsive [15, 17], which highlights the connections between top-down microscopic physics and bottom-up swampland considerations [18].

This contribution is organized as follows. In Section 2 we present the string models, their brane content and their low-energy EFT description. In Section 3 we discuss the backgrounds sourced by parallel stack of branes, including uncharged D8-branes and charged D1, D3 and NS5-branes. At extremality, the latter source near-horizon Anti-de Sitter (AdS) throats. However, a large, but finite number of D3-branes sources a geometry that deviates from AdS approaching the branes [19, 20]. In Section 4 we introduce interaction potentials in the probe regime, in which one of the two stacks of branes is heavy, and in Section 4.2 we discuss the string amplitude regime, in which both are light. Finally, we comment on the holographic regime arising from the world-volume gauge theory of D1-branes in the $USp(32)$ model of [5].

The upshot of our analysis is that branes with the same charges always repel, corroborating the weak gravity conjecture (WGC) [21] in a non-supersymmetric context. The qualitative agreement across parameter space is non-trivial, suggesting a deeper connection between microscopic physics and bottom-up swampland considerations.

2. Branes and gravitational tadpoles

We begin reviewing the non-supersymmetric string models that we have mentioned in the preceding section, focusing on their brane content. The perturbative spectra of the two orientifold models contain several charged and uncharged branes [22]. When many branes are superimposed, they backreact on the background and their effect can be studied via the gravitational bulk action. In the opposite limit, one can compute the force mutually exerted by brane in terms of string amplitudes. Similarly, the heterotic NS5-branes in the $SO(16) \times SO(16)$ model can be studied in the probe regime, although their amplitude counterpart is not well understood at present.

The ten-dimensional orientifold models can be constructed imposing the consistency of one-loop vacuum amplitudes [23–30] (see also [31–34] for reviews). The $USp(32)$ model of [5] involves an O9-plane with positive tension and charge together with $\overline{\text{D9}}$-branes. The R-R tadpole vanishes, as required by anomaly cancellation, but the NS-NS tadpole does not. As a result, supersymmetry is preserved in the closed-string sector, but it is non-linearly realized in the open-string sector, a peculiar phenomenon dubbed "brane supersymmetry breaking" (BSB) [6–9].

The low-energy physics of this model (see [35, 36] for more details) includes the string-frame runaway exponential potential

$$T \int d^{10}x \sqrt{-g_s}\, e^{-\phi} \,, \tag{1}$$

and its Einstein-frame counterpart in ten dimensions is

$$T \int d^{10}x \sqrt{-g}\, e^{\gamma\phi} \,, \qquad \gamma = \frac{3}{2} \,. \tag{2}$$

The $U(32)$ type $0'$B model of [3, 4] arises instead from a projection of the type 0B model contains the same potential, albeit with halved coefficient due to the zero-tension O9-plane [32].

Finally, in the heterotic model the one-loop vacuum energy does not vanish[1]. The resulting string-frame cosmological constant once again corresponds to a runaway exponential potential

$$T \int d^{10}x \sqrt{-g}\, e^{\gamma\phi} \,, \qquad \gamma = \frac{5}{2} \tag{3}$$

in the Einstein frame.

The D-brane content of the orientifold models was determined in detail in [22] via the consistency of one-loop vacuum amplitudes. The upshot of this analysis is that, analogously to the type I superstring, charged D1-branes and D5-branes appear in the perturbative spectrum of the $USp(32)$ orientifold model. The respective world-volume gauge groups are symplectic and orthogonal, in contrast to the type I case, due to the difference in O9-planes. The remaining branes for $p \neq 3$ are uncharged, and stacks of such branes exhibit unstable tachyonic modes in the open-string spectrum. However, a single D3-brane and a single D4-brane are free of tachyons in this model (see [15, 22] for more details). The type $0'$B model, studied in [22, 43], is qualitatively different: Dp-branes with p odd are charged, while Dp-branes with p even are uncharged, and the world-volume gauge groups are unitary.

Analogously to the supersymmetric case, the force between parallel, extremal Dp-branes vanishes at leading order [22], as determined by the annulus amplitude. The full interaction involves the O9-plane and the $\overline{\text{D9}}$-branes, which can be treated in the probe limit in which one of the two stacks of Dp-branes is parametrically heaver than the other. As a result,one finds a repulsive force and the WGC holds due to a peculiar tadpole-driven mechanism [17].

[1] In a similar fashion, one can concoct models with vanishing or suppressed leading contributions to the vacuum energy [37–42].

2.1 Low-energy effective description

The (bosonic) low-energy dynamics of the three models that we have discussed in the preceding section includes a dilaton ϕ and a $(p+2)$-form field strength $H_{p+2} = dB_{p+1}$. It can be encompassed by the family of Einstein-frame actions

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} \left(R - \frac{4}{D-2} (\partial\phi)^2 - V(\phi) - \frac{f(\phi)}{2(p+2)!} H_{p+2}^2 \right) . \tag{4}$$

The string models that we discuss live in $D = 10$ spacetime dimensions, and in these cases

$$V(\phi) = T\, e^{\gamma\phi} , \qquad f(\phi) = e^{\alpha\phi} \tag{5}$$

as we have anticipated in the preceding section. In particular,

$$D = 10 , \quad p = 1 , \quad \gamma = \frac{3}{2} , \quad \alpha = 1 \tag{6}$$

for the orientifold models, while

$$D = 10 , \quad p = 1 , \quad \gamma = \frac{5}{2} , \quad \alpha = -1 , \tag{7}$$

for the heterotic model. Equivalently, the Kalb-Ramond form may be dualized, so that

$$D = 10 , \quad p = 5 , \quad \gamma = \frac{5}{2} , \quad \alpha = 1 \tag{8}$$

in the dual frame. In all models, T is a known number of order one in string units [44].

3. Geometries sourced by branes

Starting from the effective action in (4), one can study the gravitational backreaction of branes imposing the relevant isometries on the spacetime fields. From the resulting solutions, one can compute the force exerted by a heavy stack of branes, which sources the geometry, on light probe branes.

To begin with, we present the solutions sourced by charged, extremal branes that were found in [17]. Although these solutions cannot be expressed in closed form, extremal p-branes leave an unbroken $SO(1,p) \times SO(q)$ isometry which severely constrains the field equations. In a suitable coordinate system, the most general solution takes the form

$$\begin{aligned} ds^2 &= e^{\frac{2}{p+1}v - \frac{2q}{p}b}\, dx_{1,p}^2 + e^{2v - \frac{2q}{p}b}\, dr^2 + e^{2b}\, R_0^2\, d\Omega_q^2 , \\ \phi &= \phi(r) , \\ H_{p+2} &= \frac{n}{f(\phi)(R_0\, e^b)^q}\, \text{Vol}_{p+2} , \qquad \text{Vol}_{p+2} = e^{2v - \frac{q}{p}(p+2)b}\, d^{p+1}x \wedge dr , \end{aligned} \tag{9}$$

where r is a transverse radial coordinate and R_0 is an arbitrary reference radius, introduced for dimensional reasons. The equations satisfied by the unknown functions of r stem from a constrained Toda-like system [43, 45], where the action

$$S_{\text{red}} = \int dr \left[\frac{4}{D-2} (\phi')^2 - \frac{p}{p+1} (v')^2 + \frac{q(D-2)}{p} (b')^2 - U \right] \tag{10}$$

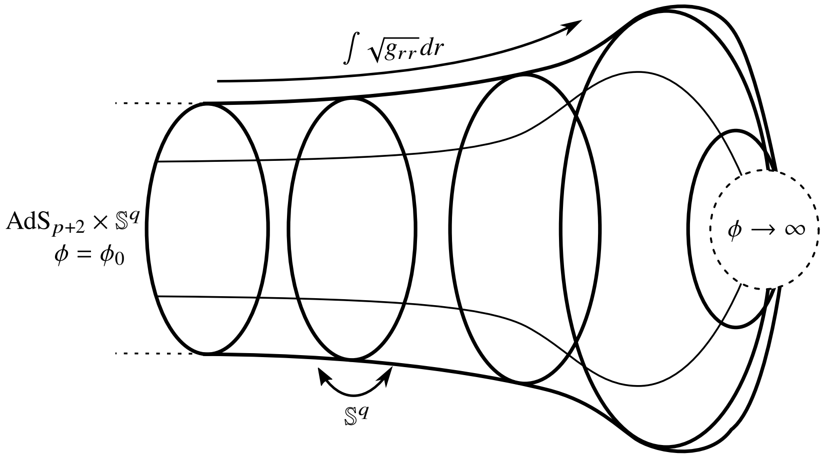

Figure 1: a schematic depiction of the geometry transverse to the branes, interpolating between the near-horizon throat and the pinch-off singularity.

is supplemented with the Hamiltonian constraint

$$\frac{4}{D-2}\,(\phi')^2 - \frac{p}{p+1}\,(v')^2 + \frac{q(D-2)}{p}\,(b')^2 + U = 0\,. \tag{11}$$

Finally, the potential reads

$$U = -T\,e^{\gamma\phi+2v-\frac{2q}{p}b} - \frac{n^2}{2R_0^{2q}}\,e^{-\alpha\phi+2v-\frac{2q(p+1)}{p}b} + \frac{q(q-1)}{R_0^2}\,e^{2v-\frac{2(D-2)}{p}b}\,. \tag{12}$$

The ansatz for the field strength is of electric type, and the corresponding flux n, proportional to the number N of branes, is

$$n = \frac{1}{\Omega_q}\int_{\mathbb{S}^q} f \star H_{p+2} \tag{13}$$

with Ω_q the volume of the unit q-sphere.

This ansatz contain the $\mathrm{AdS} \times \mathbb{S}$ Freund-Rubin solutions of [46], which in this coordinate system are described by

$$\begin{aligned} \phi &= \phi_0\,, \\ e^v &= \frac{L}{p+1}\left(\frac{R}{R_0}\right)^{\frac{q}{p}}\frac{1}{-r}\,, \\ e^b &= \frac{R}{R_0}\,, \end{aligned} \tag{14}$$

where $r < 0$. However, similarly to more familiar cases, in the present setting this solution is only the near-horizon regime $r \to -\infty$ of the full profile generated by the branes.

While in this regime the solution appears parametrically under control, the full geometry ends at a finite transverse geodesic distance from the branes. The EFT predicts a pinch-off singularity where strong-coupling effects become relevant, as depicted in fig. 1. The existence of the pinch-off singularity appears to be universal [47], and has been recently connected to some swampland conjectures [48–50] regarding (the absence of non-trivial) cobordism classes.

One can rewrite the AdS $\times\, \mathbb{S}$ near-horizon solution of eq. (14) in the coordinate-free form

$$\begin{aligned} ds^2 &= L^2\, ds^2_{\text{AdS}_{p+2}} + R^2\, d\Omega_q^2\,, \\ H_{p+2} &= c\, \text{Vol}_{\text{AdS}_{p+2}}\,, \\ \phi &= \phi_0\,, \end{aligned} \tag{15}$$

where the curvature radii L, R and the string coupling $g_s = e^{\phi_0}$ are given by

$$\begin{aligned} c &= \frac{n}{g_s^{\alpha} R^q}\,, \\ g_s^{(q-1)\gamma-\alpha} &= \left(\frac{(q-1)(D-2)}{\left(1+\frac{\gamma}{\alpha}\,(p+1)\right)T}\right)^q \frac{2\gamma T}{\alpha n^2}\,, \\ R^{2\frac{(q-1)\gamma-\alpha}{\gamma}} &= \left(\frac{\alpha+(p+1)\,\gamma}{(q-1)(D-2)}\right)^{\frac{\alpha+\gamma}{\gamma}} \left(\frac{T}{\alpha}\right)^{\frac{\alpha}{\gamma}} \frac{n^2}{2\gamma}\,, \\ L^2 &= R^2 \left(\frac{p+1}{q-1} \cdot \frac{(p+1)\,\gamma+\alpha}{(q-1)\,\gamma-\alpha}\right). \end{aligned} \tag{16}$$

Here $ds^2_{\text{AdS}_{p+2}}$ is the spacetime metric of unit curvature radius, and $\text{Vol}_{\text{AdS}_{p+2}}$ is the canonical volume form on AdS_{p+2} with curvature radius L. This solution exists if and only if the parameters $\gamma\,, \alpha$ in eq. (5) satisfy

$$\alpha > 0\,, \qquad q > 1\,, \qquad (q-1)\,\gamma > \alpha\,. \tag{17}$$

An important feature of this solution is that the large-n limit corresponds both to small string couplings and small curvatures, and thus one can expect that the EFT description encoded in eqs. (6) and (8) be reliable in this regime. While there is no scale separation, consistently with expectations from the swampland and other directions [51–53], the behavior of the Kaluza-Klein tower of states is nicely compatible with the distance conjecture (SDC) [54, 55] and AdS distance conjecture (ADC) [51]. Intriguingly, the mechanism behind this behavior also underlies the realization of the dS conjecture [55–57], as shown in [18]. Nevertheless, not all hope is lost for realistic cosmological models in these settings, since dS braneworlds arise spontaneously from brane nucleation. As a final comment, let us mention that in these settings the emergent string scenario of [58, 59] also arises via a novel mechanism [16].

The extremal near-horizon throats that we have described pertain to D1-branes in the orientifold models and NS5-branes in the heterotic model. Let us emphasize that, in general, AdS $\times\, \mathbb{S}$ throats of this type arise when branes are placed in a smooth region, but in these non-supersymmetric settings perturbative instabilities are present [60]. However, eq. (16) easily extends to any compact Einstein manifold, or even orbifold (insofar as the group action has no fixed points). Exploting this fact, in the heterotic model one can perform an antipodal $\mathbb{Z}_2$ orbifold of the internal $\mathbb{S}^3$ to project the Breitenlohner-Freedman tachyons out of the spectrum.

Perturbative instabilities notwithstanding, these solutions also undergo flux tunneling [17][2], as predicted by the argument of [54]. To wit, charged branes nucleate with a decay rate per unit volume which is exponentially suppressed in the large-flux limit. As we shall see, nucleated branes are repelled by the stack, which highlights the emergence of the WGC in this setting.

[2]For more details, see [61–67].

3.1 Static Dudas-Mourad solutions as 8-branes

We not describe the geometries sourced by 8-branes, which are uncharged. In contrast to the general case, these solutions can be expressed in closed form. In particular, the transverse space is an interval of finite proper length, due to the pinch-off singularity and the absence of a flux-driven throat.

Parametrized the interval by a coordinate y, the Einstein-frame solution is given by

$$\begin{aligned} ds^2_{\text{orientifold}} &= \left|\alpha_O\, y^2\right|^{\frac{1}{18}} e^{-\frac{\alpha_O y^2}{8}}\, dx^2_{1,8} + e^{-\frac{3}{2}\Phi_0} \left|\alpha_O\, y^2\right|^{-\frac{1}{2}} e^{-\frac{9\alpha_O y^2}{8}}\, dy^2\,, \\ \phi &= \frac{3}{4}\,\alpha_O\, y^2 + \frac{1}{3}\,\log\left|\alpha_O\, y^2\right| + \Phi_0 \end{aligned} \tag{18}$$

for the orientifold models, where we defined the $(p+1)$-dimensional Minkowski metric

$$dx^2_{1,p} \equiv \eta_{\mu\nu}\, dx^\mu\, dx^\nu\,. \tag{19}$$

The physical range of the internal coordinate is $y \in (0,\infty)$. In the heterotic model the solution takes a similar form qualitatively, although we shall not need it in the following. As anticipated, in both cases the internal length

$$R_c \equiv \int_0^\infty \sqrt{g_{yy}}\, dy < \infty\,, \tag{20}$$

is finite, and for $g_s \equiv e^{\Phi_0} \ll 1$ the interior of the parametrically wide interval is weakly coupled.

3.2 D3-branes in the type $0'$B model

As we have briefly mentioned in the preceding sections, extremal D3-branes in the type $0'$B model are an exception to the exact AdS $\times$ $\mathbb{S}$ throats, since the parameter $\alpha = 0$ violates the existence requirement. The corresponding near-horizon geometry was studied in [19, 20, 43], and it features non-homogeneous deviations from which are suppressed, but not uniformly so, in the large-flux limit. Choosing local coordinates such that the (string-frame) metric takes the form [20]

$$ds^2 = R^2(u)\, \frac{du^2}{u^2} + \frac{\alpha'^2\, u^2}{R^2(u)}\, dx^2_{1,3} + \widetilde{R}^2(u)\, d\Omega_5^2\,, \tag{21}$$

the curvature radii $R(u)\,,\widetilde{R}(u)$ and the dilaton $\phi(u)$ now run with the energy scale u. In the large-flux limit,

$$\begin{aligned} \frac{R^2(u)}{R^2_\infty} &\sim 1 - \frac{3}{16}\, g_s\, \alpha' T\, \log\left(\frac{u}{u_0}\right)\,, \\ \frac{\widetilde{R}^2(u)}{R^2_\infty} &\sim 1 - \frac{3}{16\sqrt[4]{8}}\, g_s^2\, N\, \alpha' T\, \log\left(\frac{u}{u_0}\right)\,, \\ \frac{1}{N}\, e^{-\phi} &\sim \frac{1}{g_s\, N} + \frac{3}{8\sqrt[4]{8}}\, g_s\, \alpha' T\, \log\left(\frac{u}{u_0}\right)\,, \end{aligned} \tag{22}$$

where u_0 is a reference scale and $R^2_\infty = \sqrt{4\pi\, g_s\, N}$ is the supersymmetric value of both radii. This solution can be used to compute the force exerted by D3-branes in the type $0'$B model in the probe regime.

4. Brane interactions and the weak gravity conjecture

In this section we study in detail the interactions between the branes that we have discussed. Despite the absence of supersymmetry, we find qualitative agreement: extremal branes of equal dimension strictly repel, as predicted by the WGC, while Dp-branes and Dq repel or attract, depending on p and q, both in the probe regime and in the string amplitude regime at large separations.

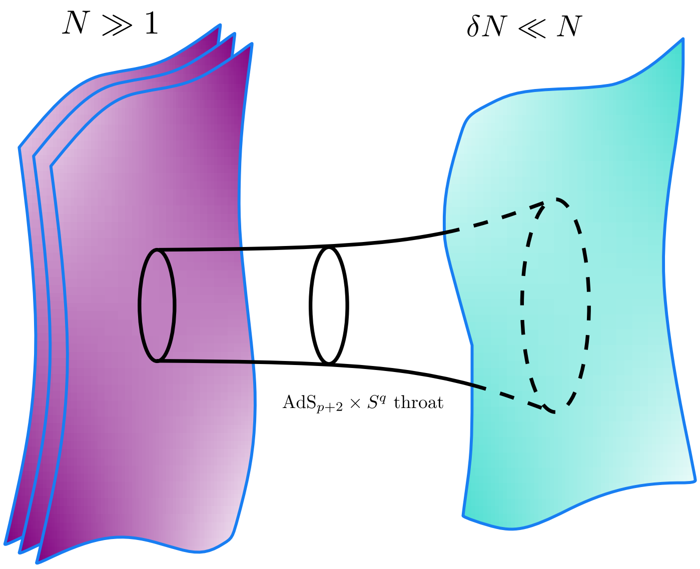

Figure 2: a depiction of the interaction between a heavy stack of $N \gg 1$ branes and $\delta N << N$ probe branes. The heavy stack sources the AdS $\times$ $\mathbb{S}$ throat probed by the light stack.

4.1 Probe potentials and the WGC

Let us begin with the probe regime, focusing on the near-horizon AdS $\times$ $\mathbb{S}$ throats on the Dudas-Mourad geometry. In order to encompass all the relevant cases, we shall consider a string-frame world-volume action of the form

$$S_p = -T_p \int d^{p+1}\zeta \, \sqrt{-j^* g_S} \, e^{-\sigma\phi} + \mu_p \int B_{p+1} \, , \tag{23}$$

where j is the embedding of the world-volume coordinates ζ in space-time. Its Einstein-frame expression reads

$$S_p = -T_p \int d^{p+1}\zeta \, \sqrt{-j^* g} \, e^{\left(\frac{2(p+1)}{D-2} - \sigma\right)\phi} + \mu_p \int B_{p+1} \, , \tag{24}$$

and $\sigma = 1\,, 2$ for D-branes and NS5-branes respectively.

The probe potential for extremal branes in AdS $\times$ $\mathbb{S}$ throats can be simplified working in Poincaré coordinates, where the Einstein-frame metric of the AdS $\times$ $\mathbb{S}$ throat takes the form

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + dx^2_{1,p}\right) + R^2 \, d\Omega_q^2 \, . \tag{25}$$

The world-volume embedding that describes probes parallel to heavy stack is

$$j \; : \quad x^\mu = \zeta^\mu \,, \qquad z = Z(\zeta) \,, \qquad \theta^i = \theta^i_0 \,, \tag{26}$$

with θ^i_0 fixed coordinates on $\mathbb{S}^q$. The probe action then becomes

$$S_p = -\tau_p \int d^{p+1}\zeta \left(\frac{L}{Z}\right)^{p+1} \left[\sqrt{1 + \eta^{\mu\nu}\, \partial_\mu Z \, \partial_\nu Z} - \frac{c\, L}{p+1} \, \frac{\mu_p}{\tau_p}\right] , \tag{27}$$

where the dressed tension

$$\tau_p \equiv T_p \, g_s^{-\frac{\alpha}{2}} \, . \tag{28}$$

Therefore, the probe potential is simply

$$\begin{aligned} V_{\text{probe}}(Z) &= \tau_p \left(\frac{L}{Z}\right)^{p+1} \left[1 - \frac{c\, L\, g_s^{\frac{\alpha}{2}}}{p+1} \, \frac{\mu_p}{T_p}\right] \\ &= \tau_p \left(\frac{L}{Z}\right)^{p+1} \left[1 - v_0 \, \frac{\mu_p}{T_p}\right] , \end{aligned} \tag{29}$$

where the $O(1)$ constant $v_0 > 1$ in our string models [17]. Crucially, this implies that extremal probes $\mu_p = T_p$ are *indeed repelled by the stack*, driven toward $Z \to 0$. In the orientifold models the picture is intuitive: D1-branes would be mutually BPS, but their interaction with the supersymmetry-breaking $\overline{\text{D9}}$-branes and O9-plane renormalizes the charge-to-tension ratio, as depicted in fig. 3. Similarly, in the heterotic model the force is mediated, at leading order, by the quantum-corrected vacuum energy. The WGC is thus realized a non-trivial way, even for extremal branes.

Finally, we turn to D3-branes, for which now the relevant geometry is described by eqs. (21) and (22). Once again, we embed the probe world-volume parallel to the x^μ coordinates, according to

$$j \; : \quad x^\mu = \zeta^\mu \,, \qquad u = U(\zeta) \,, \qquad \theta^i = \theta^i_0 \,, \tag{30}$$

where the coordinate u is to be interpreted as an energy scale.

The self-dual R-R field strength F_5[3] reads

$$\begin{aligned} F_5 &= (1 + \star) \, f_5 \, N \, \text{vol}_{\mathbb{S}^5} \\ &= f_5 \, N \, \text{vol}_{\mathbb{S}^5} + \frac{f_5 \, N}{\widetilde{R}(u)^5} \left(\frac{\alpha' \, u}{R(u)}\right)^3 \, d(\alpha' u) \wedge d^4 x \, , \end{aligned} \tag{31}$$

and the corresponding flux quantization condition

$$\frac{1}{2\kappa_{10}^2} \int_{\mathbb{S}^5} F_5 = \mu_3 \, N \tag{32}$$

fixes the parameter

$$f_5 = \frac{2\kappa_{10}^2 \, \mu_3}{\Omega_5} \, . \tag{33}$$

[3]Since the orientifold projection removes the Kalb-Ramond form, no additional terms appear in the Bianchi identity.

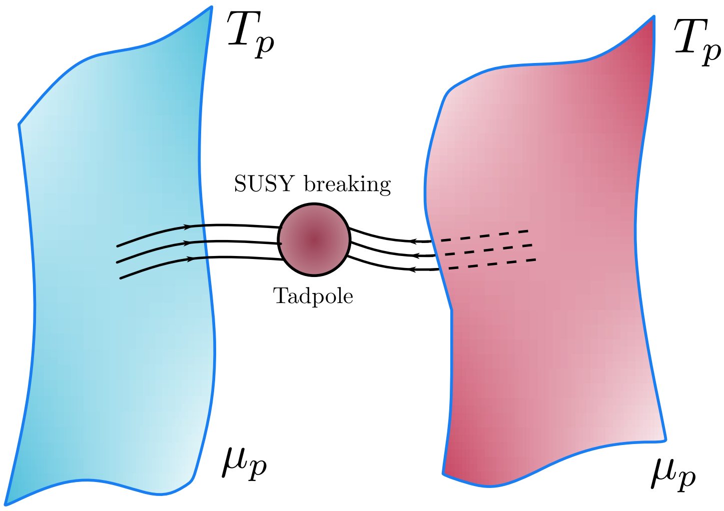

Figure 3: a depiction of the interaction between extremal branes mediated by supersymmetry breaking, reflecting the renormalization of the effective charge-to-tension ratio v_0 of eq. (29).

Therefore, the contribution to the potential C_4 that appears in the probe potential is

$$C_4 = c_4(u)\, d^4x + \ldots \tag{34}$$

where $dC_4 = F_5$ implies

$$\frac{c_4'(u)}{\alpha'} = \frac{f_5\, N}{\widetilde{R}(u)^5} \left(\frac{\alpha'\, u}{R(u)}\right)^3 . \tag{35}$$

Putting everything together, the probe potential evaluates to

$$V^{\text{D3}}_{\text{probe}}(U) = T_3 \left(\frac{\alpha'\, U}{R(U)}\right)^4 e^{-\phi(U)} - \mu_3\, c_4(U)\,, \tag{36}$$

which in the EFT limit $g_s\,, g_s^2\, N \ll 1$, $g_s\, N \gg 1$ further simplifies to

$$\begin{aligned} \frac{V^{\text{D3}}_{\text{probe}}(U)}{U^4} \sim \frac{16\,\pi\,\alpha'^2\, T_3 - f_5\,\mu_3}{64\,\pi^2\, g_s^2\, N} + \frac{15\, f_5\,\mu_3\,\alpha' T}{8192\sqrt[4]{8}\,\pi^2} \\ + \frac{3\left(64\,\pi\,\alpha'^2 T_3 - 5\, f_5\,\mu_3\right)\alpha' T}{2048\sqrt[4]{8}\,\pi^2} \log\left(\frac{U}{u_0}\right) . \end{aligned} \tag{37}$$

As expected, substituting the supersymmetric values

$$2\kappa_{10}^2 = (2\pi)^7\,\alpha'^4\,, \qquad T_3 = \mu_3 = \frac{N_3}{(2\pi)^3\,\alpha'^2} \tag{38}$$

for $N_3 \ll N$ probes, and using eq. (33), the leading term vanishes, on account of the BPS property, while the remaining sub-leading terms reflect supersymmetry breaking and their U-dependence takes the simple form

$$V^{\text{D3}}_{\text{sub-leading}}(U) \propto U^4 \left[5 - 4 \log\left(\frac{U}{u_0}\right)\right] . \tag{39}$$

Once again the resulting force is repulsive, as depicted in fig. 4.

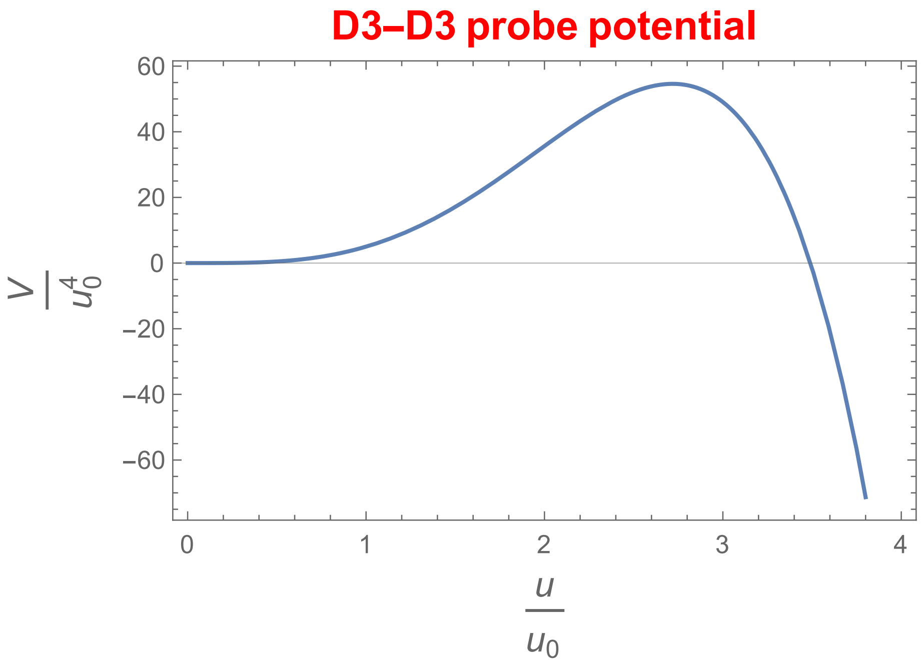

Figure 4: the normalized probe potential in eq. (37) in units of the reference scale u_0.

4.2 Uncharged branes and string amplitudes

As we have discussed in the preceding sections, a string-amplitude counterpart to the probe potentials between branes sharing charges would be considerably involved, since the would-be leading annulus amplitude vanishes. In the cases in which the two stacks do not share charges, or when at least one is uncharged, one can compute potentials both in the string amplitude regime and in the probe regime, the when the heavy stack sources a Dudas-Mourad geometry in particular. The other controlled back-reacted geometry in this setting corresponds to D1-branes, and D8-branes are the only other probes whose potential can be reliably computed in this case, since they can wrap the $\mathbb{S}^7$ in the near-horizon $\text{AdS}_3 \times \mathbb{S}^7$ throat. Similar considerations apply to the heterotic model. The resulting potentials are attractive for 8-branes and fundamental strings, as in the orientifold models, and repulsive for NS5-branes. As a final comment, in the heterotic model one can also compute the potential for probe F1-strings, extended along one of the directions parallel to the NS5-branes. However, the Kalb-Ramond form B_2 vanishes upon pull-back on the string world-sheet, and thus the resulting force is attractive.

To begin with, we consider a stack of N_p Dp-branes probing the orientifold Dudas-Mourad geometry sourced by D8-branes. In order to simplify the expressions, we work in the string frame in units where $\alpha_{\text{O}} = 1$. Since the boundary of the interval spanned by the coordinate y hosts two singularities, we expect this configuration to be under control insofar as the (string-frame) geodesic coordinate

$$r \equiv \frac{1}{\sqrt{g_s}} \int_0^y \frac{du}{u^{\frac{1}{3}}} \, e^{-\frac{3}{8}u^2} \tag{40}$$

is far away from its endpoints $r = 0$, $r = R_c$. This overlap regime indeed exists, provided that $g_s \equiv e^{\Phi_0} \ll 1$.

Letting the (string-frame) warp factors $A(y)$, $B(y)$ of [68] be defined according to

$$ds^2_{10} = e^{2A(y)} \, dx_9^2 + e^{2B(y)} \, dy^2 \, , \tag{41}$$

the probe action for Dp-branes reads

$$\begin{aligned} S_p &= - N_p \, T_p \int d^{p+1}x \, e^{(p+1)A(y) - \Phi(y)} \\ &\equiv - N_p \, T_p \int d^{p+1}x \, V_{p8} \, , \end{aligned} \tag{42}$$

so that the potential per unit tension is

$$V_{p8} = g_s^{\frac{p-3}{4}} \, y^{\frac{2}{9}(p-2)} \, e^{\frac{p-5}{8}y^2} \, , \tag{43}$$

whose behavior is shown in figs. 5 and 6. If the potential drives probes toward $y \to \infty$ it is repulsive, since this is where the pinch-off singularity lies [17]. All in all, for $p < 3$ probes are repelled by the D8-branes, while for $p > 4$ they are attracted to the D8-branes. The cases $p = 3$, 4 exhibit unstable equilibria, but at large separations the potentials are repulsive. This is the regime that we shall now compare with a string amplitude computation.

Finally, we consider N_8 D8-branes probing the near-horizon geometries sourced by $N_1 \gg N_8$ extremal D1-branes or $N_3 \gg N_8$ extremal D3-branes in the orientifold models. For completeness we shall also consider 8-branes probing the $\text{AdS}_7 \times \mathbb{S}^3$ throat sourced by $N_5 \gg N_8$ NS5-branes in the heterotic model. These cases are particularly simple to address because 8-branes can wrap the internal spheres without collapsing in a vanishing cycle, while leaving enough dimensions to be parallel to the heavy stack. Furthermore, these are also the only cases where computations can be compared to the results in the preceding sections, which hold in the opposite regime N_1 , N_3 , $N_5 \ll N_8$. The respective potentials V_{81} , V_{83} , V_{85} are

$$\begin{aligned} V_{81} &\propto N_8 \, T_8 \, R^7 \left(\frac{L}{Z}\right)^2 \, , \\ V_{85} &\propto N_8 \, T_8 \, R^3 \left(\frac{L}{Z}\right)^6 \end{aligned} \tag{44}$$

for the $\text{AdS} \times \mathbb{S}$ throats of eq. (15), up to an irrelevant (positive) constant, while

$$V_{83} \sim \sqrt{2} \, \pi^{\frac{1}{4}} \, \alpha'^{\frac{9}{2}} \, g_s^{-\frac{3}{4}} \, N_3^{\frac{1}{4}} \, N_8 \, T_8 \, U^4 \left(1 + \frac{3}{8} \, g_s \, T \, \log\left(\frac{U}{u_0}\right)\right) \tag{45}$$

for branes probing the geometry of eq. (21) sourced by D3-branes in the type 0′B model. These potentials are attractive, which may at first glance appear in contradiction with the results in the

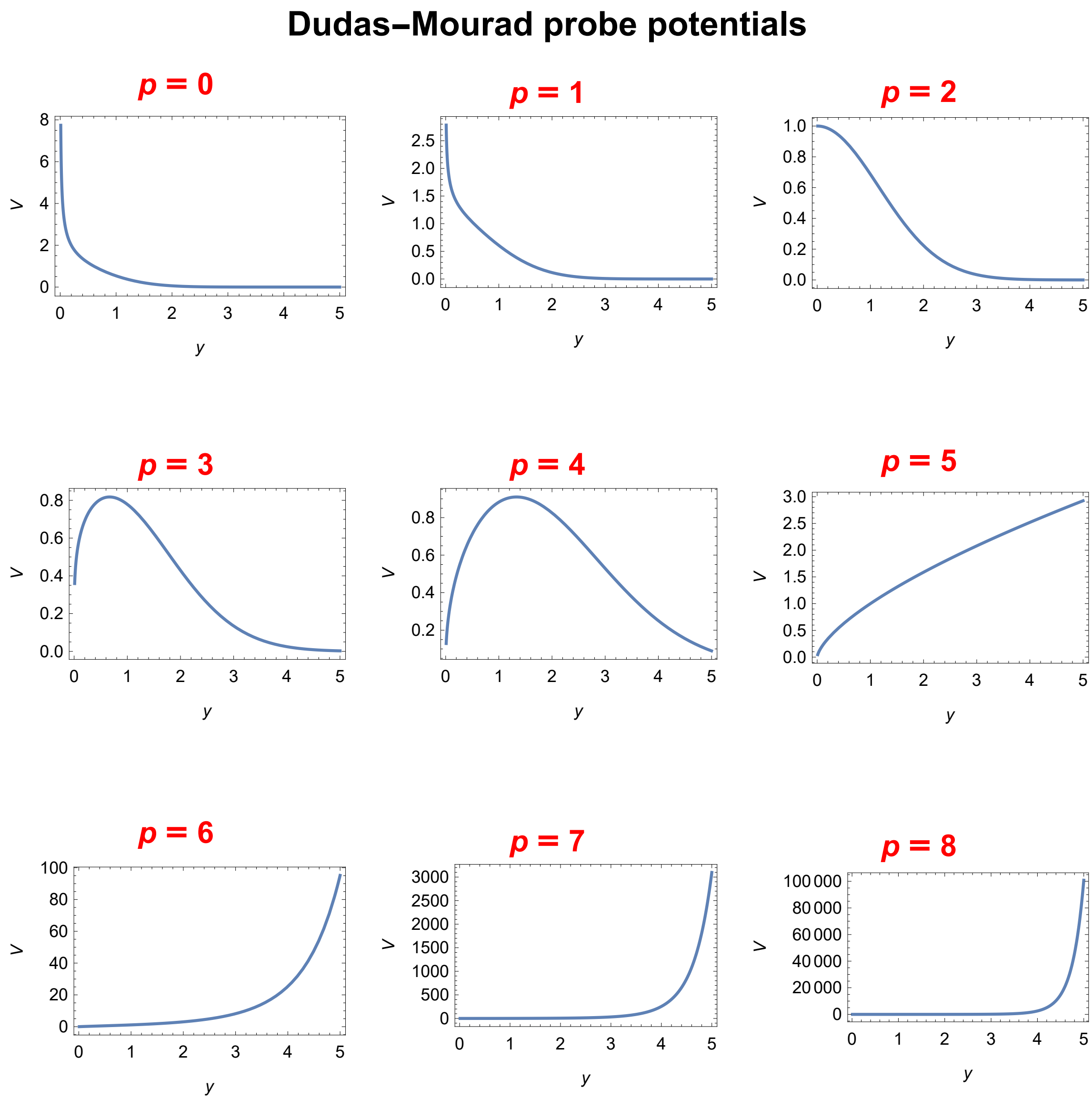

Figure 5: probe potentials for $g_s = 1$ and $p \leq 8$. For $p < 3$ the probe stack is repelled by the D8-branes, while for $p > 4$ it is attracted to the D8-branes. A string amplitude computation yields a qualitatively similar behavior, despite the string-scale breaking of supersymmetry.

following section. However, notice that at large separation wrapped 8-branes wrapped behave as uncharged 1-branes, 5-branes and 3-branes respectively, which is indeed consistent with an attractive potential between branes of equal dimension.

In order to see this more clearly, let us now compare these to a string amplitude computation. The leading-order amplitude encoding the interaction between parallel stack of N_p Dp-branes and N_q Dq-branes, with $p < q$ for definiteness, corresponds to the annulus. The transverse-channel

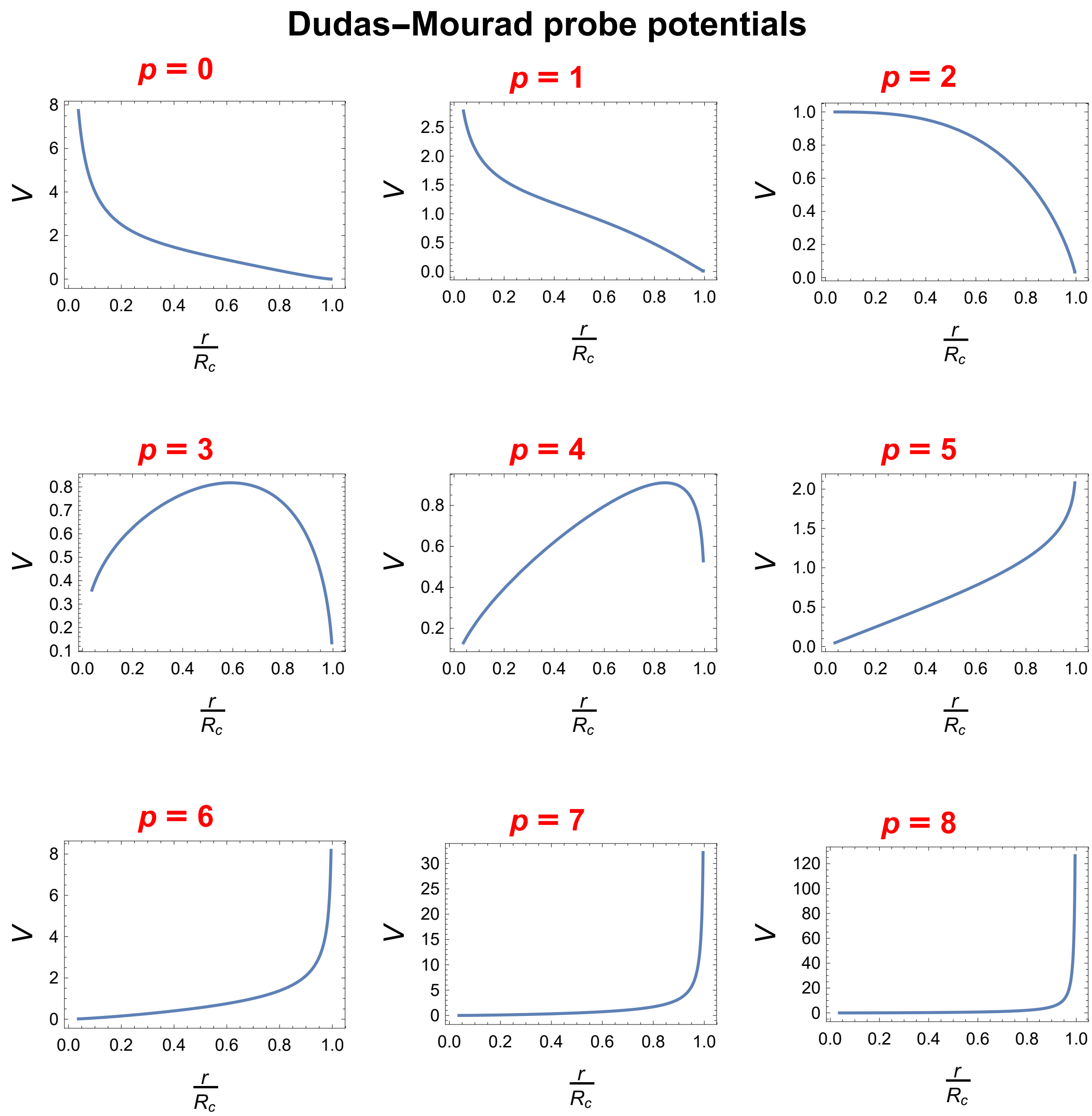

Figure 6: probe potentials for $g_s = 1$ and $p \leq 8$, plotted as functions of the geodesic coordinate along the compact direction.

integrand in the present cases takes the form [22]

$$\widetilde{\mathcal{A}}_{pq} \propto N_p\, N_q \left(V_{8-q+p}\, O_{q-p} - O_{8-q+p}\, V_{q-p}\right) \tag{46}$$

up to a (positive) normalization, where the characters are evaluated at $\mathfrak{q} = e^{-2\pi\ell}$. In suitable units for the transverse separation r, the potential V_{pq} is then given by

$$V_{pq} \propto -\, N_p\, N_q \int_0^\infty \frac{d\ell}{\ell^{\frac{9-q}{2}}}\, \frac{\widetilde{\mathcal{A}}_{pq}}{\eta^{8-q+p}} \left(\frac{2\eta}{\vartheta_2}\right)^{\frac{q-p}{2}} e^{-\frac{r^2}{\ell}}\,. \tag{47}$$

For large r, the integral is dominated by the large-ℓ region. In this region eq. (46) is asymptotic to $\mathfrak{q}^{-\frac{1}{3}}\,\tilde{A}_{pq}$, with

$$\begin{aligned}\tilde{A}_{pq} &\propto V_{8-q+p}\,O_{q-p} - O_{8-q+p}\,V_{q-p} \\ &\sim 2\,(4-q+p)\,\mathfrak{q}^{\frac{1}{3}}\,,\end{aligned} \tag{48}$$

so that for $q < 7$ one finds

$$V_{pq} \propto (q-p-4)\,\frac{N_p\,N_q}{r^{7-q}}\,. \tag{49}$$

This potential is repulsive for $p < q-4$ and attractive for $p > q-4$. The integral in eq. (47) diverges for $q \geq 7$, but the correct distributional results for $q = 7\,,\,8$ are potentials proportional to $(p-3)\log(r)$ and $(p-4)\,r$ respectively. The cases that can be compared to probe-brane computations are thus $p = q$, which leads to an attractive potential compatible with eqs. (44) and (45), and $q = 8$, which leads to a potential proportional to $(p-4)\,r$. Therefore, the latter interaction is repulsive for $p < 4$ and attractive for $p > 4$, consistently with the results in the preceding section.

Let us remark that with broken supersymmetry this agreement, while qualitative, is quite non-trivial, and along with the numerous connections to swampland conjectures it seems to point to deeper principles akin to the WGC encompassing also uncharged branes. All in all, the results that we have presented in this contribution indicate that brane dynamics can be robust probe for microscopic physics even in the absence of (linear) supersymmetry, at least to some extent. Indeed, deeper connections with holography and swampland conjectures have been explored recently [16, 50] from this perspective, complementing the bottom-up considerations of [69, 70] (see also [48, 49]).

References

[1] L. Alvarez-Gaume, P. H. Ginsparg, G. W. Moore and C. Vafa, Phys. Lett. B **171** (1986), 155-162 doi:10.1016/0370-2693(86)91524-8

[2] L. J. Dixon and J. A. Harvey, Nucl. Phys. B **274** (1986), 93-105 doi:10.1016/0550-3213(86)90619-X

[3] A. Sagnotti, [arXiv:hep-th/9509080 [hep-th]].

[4] A. Sagnotti, Nucl. Phys. B Proc. Suppl. **56** (1997), 332-343 doi:10.1016/S0920-5632(97)00344-7 [arXiv:hep-th/9702093 [hep-th]].

[5] S. Sugimoto, Prog. Theor. Phys. **102** (1999), 685-699 doi:10.1143/PTP.102.685 [arXiv:hep-th/9905159 [hep-th]].

[6] I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B **464** (1999), 38-45 doi:10.1016/S0370-2693(99)01023-0 [arXiv:hep-th/9908023 [hep-th]].

[7] C. Angelantonj, Nucl. Phys. B **566** (2000), 126-150 doi:10.1016/S0550-3213(99)00662-8 [arXiv:hep-th/9908064 [hep-th]].

[8] G. Aldazabal and A. M. Uranga, JHEP **10** (1999), 024 doi:10.1088/1126-6708/1999/10/024 [arXiv:hep-th/9908072 [hep-th]].

[9] C. Angelantonj, I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, Nucl. Phys. B **572** (2000), 36-70 doi:10.1016/S0550-3213(00)00052-3 [arXiv:hep-th/9911081 [hep-th]].

[10] N. Cribiori, D. Lust and M. Scalisi, JHEP **06** (2021), 071 doi:10.1007/JHEP06(2021)071 [arXiv:2104.08288 [hep-th]].

[11] A. Castellano, A. Font, A. Herraez and L. E. Ibáñez, JHEP **08** (2021), 092 doi:10.1007/JHEP08(2021)092 [arXiv:2104.10181 [hep-th]].

[12] G. Dall'Agata, M. Emelin, F. Farakos and M. Morittu, JHEP **10** (2021), 076 doi:10.1007/JHEP10(2021)076 [arXiv:2108.04254 [hep-th]].

[13] N. Cribiori, [arXiv:2203.15449 [hep-th]].

[14] T. Coudarchet, E. Dudas and H. Partouche, JHEP **07** (2021), 104 doi:10.1007/JHEP07(2021)104 [arXiv:2105.06913 [hep-th]].

[15] I. Basile, JHEP **10** (2021), 080 doi:10.1007/JHEP10(2021)080 [arXiv:2106.04574 [hep-th]].

[16] I. Basile, [arXiv:2201.08851 [hep-th]].

[17] R. Antonelli and I. Basile, JHEP **11** (2019), 021 doi:10.1007/JHEP11(2019)021 [arXiv:1908.04352 [hep-th]].

[18] I. Basile and S. Lanza, JHEP **10** (2020), 108 doi:10.1007/JHEP10(2020)108 [arXiv:2007.13757 [hep-th]].

[19] C. Angelantonj and A. Armoni, Nucl. Phys. B **578** (2000), 239-258 doi:10.1016/S0550-3213(00)00136-X [arXiv:hep-th/9912257 [hep-th]].

[20] C. Angelantonj and A. Armoni, Phys. Lett. B **482** (2000), 329-336 doi:10.1016/S0370-2693(00)00475-5 [arXiv:hep-th/0003050 [hep-th]].

[21] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, JHEP **06** (2007), 060 doi:10.1088/1126-6708/2007/06/060 [arXiv:hep-th/0601001 [hep-th]].

[22] E. Dudas, J. Mourad and A. Sagnotti, Nucl. Phys. B **620** (2002), 109-151 doi:10.1016/S0550-3213(01)00552-1 [arXiv:hep-th/0107081 [hep-th]].

[23] A. Sagnotti, [arXiv:hep-th/0208020 [hep-th]].

[24] G. Pradisi and A. Sagnotti, Phys. Lett. B **216** (1989), 59-67 doi:10.1016/0370-2693(89)91369-5

[25] P. Horava, Nucl. Phys. B **327** (1989), 461-484 doi:10.1016/0550-3213(89)90279-4

[26] P. Horava, Phys. Lett. B **231** (1989), 251-257 doi:10.1016/0370-2693(89)90209-8

[27] M. Bianchi and A. Sagnotti, Phys. Lett. B **247** (1990), 517-524 doi:10.1016/0370-2693(90)91894-H

[28] M. Bianchi and A. Sagnotti, Nucl. Phys. B **361** (1991), 519-538 doi:10.1016/0550-3213(91)90271-X

[29] M. Bianchi, G. Pradisi and A. Sagnotti, Nucl. Phys. B **376** (1992), 365-386 doi:10.1016/0550-3213(92)90129-Y

[30] A. Sagnotti, Phys. Lett. B **294** (1992), 196-203 doi:10.1016/0370-2693(92)90682-T [arXiv:hep-th/9210127 [hep-th]].

[31] E. Dudas, Class. Quant. Grav. **17** (2000), R41-R116 doi:10.1088/0264-9381/17/22/201 [arXiv:hep-ph/0006190 [hep-ph]].

[32] C. Angelantonj and A. Sagnotti, Phys. Rept. **371** (2002), 1-150 [erratum: Phys. Rept. **376** (2003) no.6, 407] doi:10.1016/S0370-1573(02)00273-9 [arXiv:hep-th/0204089 [hep-th]].

[33] J. Mourad and A. Sagnotti, [arXiv:1711.11494 [hep-th]].

[34] A. Sagnotti and J. Mourad, LHEP **2021** (2021), 219 doi:10.31526/lhep.2021.219 [arXiv:2107.04064 [hep-th]].

[35] E. Dudas and J. Mourad, Phys. Lett. B **514** (2001), 173-182 doi:10.1016/S0370-2693(01)00777-8 [arXiv:hep-th/0012071 [hep-th]].

[36] G. Pradisi and F. Riccioni, Nucl. Phys. B **615** (2001), 33-60 doi:10.1016/S0550-3213(01)00441-2 [arXiv:hep-th/0107090 [hep-th]].

[37] K. R. Dienes, Phys. Rev. Lett. **65** (1990), 1979-1982 doi:10.1103/PhysRevLett.65.1979

[38] K. R. Dienes, Phys. Rev. D **42** (1990), 2004-2021 doi:10.1103/PhysRevD.42.2004

[39] S. Kachru, J. Kumar and E. Silverstein, Phys. Rev. D **59** (1999), 106004 doi:10.1103/PhysRevD.59.106004 [arXiv:hep-th/9807076 [hep-th]].

[40] C. Angelantonj and M. Cardella, Phys. Lett. B **595** (2004), 505-512 doi:10.1016/j.physletb.2004.06.058 [arXiv:hep-th/0403107 [hep-th]].

[41] S. Abel, K. R. Dienes and E. Mavroudi, Phys. Rev. D **91** (2015) no.12, 126014 doi:10.1103/PhysRevD.91.126014 [arXiv:1502.03087 [hep-th]].

[42] S. Abel and R. J. Stewart, Phys. Rev. D **96** (2017) no.10, 106013 doi:10.1103/PhysRevD.96.106013 [arXiv:1701.06629 [hep-th]].

[43] E. Dudas and J. Mourad, Nucl. Phys. B **598** (2001), 189-224 doi:10.1016/S0550-3213(00)00781-1 [arXiv:hep-th/0010179 [hep-th]].

[44] I. Basile, Riv. Nuovo Cim. **44** (2021) no.10, 499-596 doi:10.1007/s40766-021-00024-9 [arXiv:2107.02814 [hep-th]].

[45] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B **546** (1999), 155-181 doi:10.1016/S0550-3213(99)00041-3 [arXiv:hep-th/9811035 [hep-th]].

[46] J. Mourad and A. Sagnotti, Phys. Lett. B **768** (2017), 92-96 doi:10.1016/j.physletb.2017.02.053 [arXiv:1612.08566 [hep-th]].

[47] R. Antonelli,

[48] G. Buratti, M. Delgado and A. M. Uranga, JHEP **06** (2021), 170 doi:10.1007/JHEP06(2021)170 [arXiv:2104.02091 [hep-th]].

[49] G. Buratti, J. Calderón-Infante, M. Delgado and A. M. Uranga, JHEP **10** (2021), 037 doi:10.1007/JHEP10(2021)037 [arXiv:2107.09098 [hep-th]].

[50] R. Angius, J. Calderón-Infante, M. Delgado, J. Huertas and A. M. Uranga, [arXiv:2203.11240 [hep-th]].

[51] D. Lüst, E. Palti and C. Vafa, Phys. Lett. B **797** (2019), 134867 doi:10.1016/j.physletb.2019.134867 [arXiv:1906.05225 [hep-th]].

[52] D. Lüst and D. Tsimpis, JHEP **07** (2020), 060 doi:10.1007/JHEP07(2020)060 [arXiv:2004.07582 [hep-th]].

[53] G. B. De Luca and A. Tomasiello, JHEP **12** (2021), 086 doi:10.1007/JHEP12(2021)086 [arXiv:2104.12773 [hep-th]].

[54] H. Ooguri and C. Vafa, Adv. Theor. Math. Phys. **21** (2017), 1787-1801 doi:10.4310/ATMP.2017.v21.n7.a8 [arXiv:1610.01533 [hep-th]].

[55] H. Ooguri, E. Palti, G. Shiu and C. Vafa, Phys. Lett. B **788** (2019), 180-184 doi:10.1016/j.physletb.2018.11.018 [arXiv:1810.05506 [hep-th]].

[56] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, [arXiv:1806.08362 [hep-th]].

[57] S. K. Garg and C. Krishnan, JHEP **11** (2019), 075 doi:10.1007/JHEP11(2019)075 [arXiv:1807.05193 [hep-th]].

[58] S. J. Lee, W. Lerche and T. Weigand, JHEP **02** (2022), 190 doi:10.1007/JHEP02(2022)190 [arXiv:1910.01135 [hep-th]].

[59] S. J. Lee, W. Lerche and T. Weigand, JHEP **02** (2022), 096 doi:10.1007/JHEP02(2022)096 [arXiv:1904.06344 [hep-th]].

[60] I. Basile, J. Mourad and A. Sagnotti, JHEP **01** (2019), 174 doi:10.1007/JHEP01(2019)174 [arXiv:1811.11448 [hep-th]].

[61] J. D. Brown and C. Teitelboim, Phys. Lett. B **195** (1987), 177-182 doi:10.1016/0370-2693(87)91190-7

[62] J. D. Brown and C. Teitelboim, Nucl. Phys. B **297** (1988), 787-836 doi:10.1016/0550-3213(88)90559-7

[63] J. M. Maldacena, J. Michelson and A. Strominger, JHEP **02** (1999), 011 doi:10.1088/1126-6708/1999/02/011 [arXiv:hep-th/9812073 [hep-th]].

[64] N. Seiberg and E. Witten, JHEP **04** (1999), 017 doi:10.1088/1126-6708/1999/04/017 [arXiv:hep-th/9903224 [hep-th]].

[65] J. J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, JCAP **12** (2009), 006 doi:10.1088/1475-7516/2009/12/006 [arXiv:0904.3106 [hep-th]].

[66] A. R. Brown and A. Dahlen, Phys. Rev. D **82** (2010), 083519 doi:10.1103/PhysRevD.82.083519 [arXiv:1004.3994 [hep-th]].

[67] A. R. Brown and A. Dahlen, Phys. Rev. D **84** (2011), 023513 doi:10.1103/PhysRevD.84.023513 [arXiv:1010.5241 [hep-th]].

[68] E. Dudas and J. Mourad, Phys. Lett. B **486** (2000), 172-178 doi:10.1016/S0370-2693(00)00734-6 [arXiv:hep-th/0004165 [hep-th]].

[69] S. Lanza, F. Marchesano, L. Martucci and I. Valenzuela, JHEP **02** (2021), 006 doi:10.1007/JHEP02(2021)006 [arXiv:2006.15154 [hep-th]].

[70] S. Lanza, F. Marchesano, L. Martucci and I. Valenzuela, JHEP **09** (2021), 197 doi:10.1007/JHEP09(2021)197 [arXiv:2104.05726 [hep-th]].