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Extracting Bigravity from String Theory

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Given the success of constructing quantum field theories for vector spin-1 theories numerous attempts have been made to do the same for theories involving spin-2 theories. The most successful of these attempts is String theory which, gives a consistent frame work to create a Hilbert space that includes a mssless spin-2. However, there are other issues to be addressed involving spin-2 theories it has been shown that there exists a unique way to couple a massive spin-2 to a gravity theory (massless spin-2) which is known as Bimetric gravity. Therefore, the natural question to ask is whether or not this theory comes from a string theory construct i.e. can one find a massive spin-2 state in string theory whose low energy limit coupled to massless case gives the Bimetric gravity? In the following, we will discuss our attempt to find such a state and the subtleties involving the construction of effective action.

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1. Introduction

There are different ways to UV complete gravity the most accessible and consistent method so far has been string theory. It is well known that Hilbert space of String theory contains a massless spin-2 both in bosonic and superstring theory [1]. In different papers [2],[3] it was shown that this state in low energy reproduces gravity. In order to see that one needs to define how to obtain the low energy limit of massless spin-2 string state to gravity. This question is going to be asked again during our discussion on Bimetric gravity. For the graviton, in short, one takes the Einstein-Hilbert action and perform a perturbation around a background metric $G_{\mu\nu}$. The pertrubative spin-2 field is called graviton:

$$\mathcal{L}_{GR} = m_g^2 \int d^4x \ \sqrt{g} R(g) \to g_{\mu\nu} = \hat{g}_{\mu\nu} + \kappa h_{\mu\nu} \tag{1}$$

Naturally, the graviton $h_{\mu\nu}$ has self interactions coming from the expansion of the Ricci tensor. Namely:

$$\mathcal{L} = \frac{1}{2} h^{\mu\nu} \, \mathcal{E}^{\rho\sigma}_{\mu\nu} \, h_{\rho\sigma} \tag{2}$$

where we have the operator

$$\mathcal{E}^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{2}\Big(\eta^{\rho}_{\mu}\eta^{\sigma}_{\nu}\partial^2 - \eta^{\sigma}_{\nu}\partial_\mu\partial^\rho - \eta^{\sigma}_{\mu}\partial_\nu\partial^\rho + \eta_{\mu\nu}\partial^\sigma\partial^\rho + \eta^{\rho\sigma}\partial_\mu\partial_\nu - \eta_{\mu\nu}\eta^{\rho\sigma}\partial^2\Big) \tag{3}$$

Now, from the string theory side first one has to calculate different n-point amplitudes involving the massless spin-2 state. In addition, since string theory is considered as the UV completion of GR one should take the low energy limit of string theory that can be summarized in $\alpha' \to 0$ limit which takes the string length and shrinks it to zero. Further, since the mass of n-th string state is given by $m^2 = \frac{n}{\alpha'}$, this limit makes all massive levels of string states decouple and pushed away to infinity as the denominator goes to zero. This procedure for reasons that we will see later (and it is not difficult to see) is not as straightforward for the massive string strings scattering.

There are many different ways to extend GR but one of the main extensions is to couple another spin-2 theory to gravity this task proved to be hard. Since it was shown that this coupling will create ghosts and instabilities. Hassan and Rosen manage to write down the unique ghost free theory involving two interacting spin-2 theories [8]-[13].

In addition to the theoretical interest in understanding this coupling of different spin-2 theories to each other, there are phenomenological reasons too. A massive spin-2 theory that only interacts with matter through gravity is considered a possible candidate for dark matter since it is not charged under the standard model gauge group it exhibits proper features for dark matter [16]-[18].

Our task in this project is to investigate the connection between string theory and Bimetric gravity. In order to do that we are going to first use that, in the proper basis, Bimetric gravity can be written as an interacting massless and massive spin-2 theory. In which, the massless theory has the Einstein-Hilbert and the massive has Fierz–Pauli actions. Therefore, we can assume the existing massless spin-2 closed string state as the massless spin-2 in the Bimetric. The important task is to find a candidate for massive spin-2 in string theory. String theory has an infinite tower of massive states that include different spins. We are going to select a massive open string spin-2 state and calculate its self-interaction as well as its interaction with the massless closed string. Finally, having this amplitude we need to define the low energy limit which as was described above for the massive string states is not as simple as just taking $\alpha' \to 0$. We are going to carefully analyze the amplitude and define the consistent limit of $\alpha' k^2 \to 0$ to obtain the low energy limit of the interaction. We are going to compare our string effective action with Bimetric up to the cubic order and discuss its implications.

2. Bi-metric Gravity

The goal of Bimetric gravity, as the name suggests, is to produce a consistent, ghost free theory that consists of two spin-2 fields (i.e. metrics). The action for ghost free Bimetric gravity was written by Hassan and Rosen as [8],[9]:

$$S = m_g^2 \int \mathrm{d}^4x \sqrt{g} R(g) + \alpha^2 m_g^2 \int \mathrm{d}^4x \sqrt{f} R(f) - 2\,\alpha^2 m_g^4 \int \mathrm{d}^4x \sqrt{g}\, V(S;\beta_n)\,. \tag{4}$$

Above, we have two independent Einstein-Hilbert kinetic terms for the two spin-2 fields $g_{\mu\nu}$ and $f_{\mu\nu}$ with two different Planck masses m_g and m_f (with $\alpha = \frac{m_f}{m_g}$). The important part of this action is the potential. This potential couples the two metrics and makes them have non-trivial self and mixed interactions. As mentioned this potential is uniquely ghost free and given by:

$$V(S;\beta_n) = \sum_{n=0}^{4} \beta_n e_n(S)\,, \tag{5}$$

where, e_n and $S^\mu{}_\nu$, are given by:

$$\begin{aligned} &e_n(S) = S^{\mu_1}{}_{[\mu_1} \dots S^{\mu_n}{}_{\mu_n]}\,. \\ &S^\mu{}_\nu S^\nu{}_\rho = g^{\mu\sigma} f_{\sigma\rho} \quad \text{namely} \quad S^\mu{}_\nu = (\sqrt{g^{-1}f})^\mu{}_\nu\,. \end{aligned} \tag{6}$$

In the action (4) we have two independent Ricci tensors associated with g and f therefore, we are going to have two independent diffeomorphisms transformations that keep each of the two Ricci tensors invariant. This symmetric structure is important when we are going to discuss the mass eigenbasis of these fields.

Having the action we can now perform the straightforward but tedious method of quantizing this theory. This involves defining background values for both $\hat{g}_{\mu\nu}$ and $\hat{f}_{\mu\nu}$ and perturbing the fields around them. There are different choices for these values possible in fact Bimetric gravity admits all known background solutions in GR [13]. We are going to choose the simplest case for now, namely the proportional background, which relates the two backgrounds as $\hat{f}_{\mu\nu} = c^2 \hat{g}_{\mu\nu}$ and further we are going to choose flat background for $g_{\mu\nu}$ i.e. $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$. Hence, we can write the perturbation expansion for each metric as:

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu} \quad , \quad f_{\mu\nu} = \eta_{\mu\nu} + \delta f_{\mu\nu}\,. \tag{7}$$

within the action (4) and adapt all relevant formulae in our references for the particular case of Minkowski backgrounds. The above perturbation will lead to complicated coupling of $\delta g_{\mu\nu}$ and $\delta f_{\mu\nu}$ and two coupled Einstein equations for $g_{\mu\nu}$ and $f_{\mu\nu}$ fields [13]. Nevertheless, in order to have structural similarities to string theory we need to have a massless graviton plus a spin-2 field.

Therefore, we need to go to the mass eigenstate of these coupled system. In order to diagonalize the mass matrix, namely the potential (5), one may define [19]

$$G_{\mu\nu} \equiv m_g\,(\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}) \quad , \quad M_{\mu\nu} \equiv \alpha m_g\,(\delta f_{\mu\nu} - \delta g_{\mu\nu})\,, \tag{8}$$

Plugging this back into the action (4) we obtain the quadratic order together and equations of motion:

$$\begin{aligned} &\mathcal{L}^{(2)}(G)\frac{1}{2}G^{\mu\nu}\,\mathcal{E}^{\rho\sigma}_{\mu\nu}G_{\rho\sigma} = 0 \to \Box h_{\mu\nu} = 0 \\ &\mathcal{L}^{(2)}(M)\frac{1}{2}M^{\mu\nu}\,\mathcal{E}^{\rho\sigma}_{\mu\nu}M_{\rho\sigma} + \frac{m^2_{FP}}{4}([M^2] + [M]^2) = 0 \to (\Box - m^2_{FP})M_{\mu\nu} = 0 \end{aligned} \tag{9}$$

mass parameters are defined with respect to original Bimetric parameters in the following way:

$$m^2_{\rm FP} \equiv m^2_{\rm Pl}\,(\beta_1 + 2\beta_2 + \beta_3) \quad , \quad m^2_{\rm Pl} \equiv m^2_g(1+\alpha^2)\,. \tag{10}$$

As intended we have a propagating massless spin-2 field (2 propagating degrees of freedom) and another spin-2 field that has a Fritz-Pauli mass term which one can call massive spin-2 (5 propagating degrees of freedom).

In addition, there are higher order contributions in the expansion of the potential. For the purposes of our goal we only need the cubic interactions which only involve M^3, GM^2 terms. The cubic expansion terms are very long and involved as an example for the GM^2 one gets [17]:

$$\begin{aligned} \mathcal{L}_{\rm GM^2} \;=\; & \tfrac{m_{\rm Pl}}{8}(\beta_1 + 2\beta_2 + \beta_3)\left[[G][M]^2 - 4[M][GM] - [G][M^2] + 4[GM^2]\right] \\ & + \tfrac{1}{4m_{\rm Pl}}\Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\mu[M]\partial_\nu[M] + 2\partial_\nu[M]\partial_\rho M^\rho_\mu \\ & + 2\partial_\nu M^\rho_\mu\partial_\rho[M] - 2\partial_\rho[M]\partial^\rho M_{\mu\nu} + 2\partial_\rho M_{\mu\nu}\partial_\sigma M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu \\ & - 2\partial_\rho M_{\nu\sigma}\partial^\sigma M^\rho_\mu + 2\partial_\sigma M_{\nu\rho}\partial^\sigma M^\rho_\mu\big) + \tfrac{1}{2}[G]\big(\partial_\rho[M]\partial^\rho[M] \\ & - \partial_\rho M_{\mu\nu}\partial^\rho M^{\mu\nu} - 2\partial_\rho[M]\partial_\mu M^{\mu\rho} + 2\partial_\rho M_{\mu\nu}\partial^\nu M^{\mu\rho}\big)\Big] \\ & + \tfrac{1}{2m_{\rm Pl}}\Big[M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\mu[G]\partial_\nu[M] + \partial^\rho G_{\rho\mu}\partial_\nu[M] \\ & + \partial_\nu G_{\mu\rho}\partial^\rho[M] - \partial_\rho G_{\mu\nu}\partial^\rho[M] + \partial_\rho G^{\rho\sigma}\partial_\sigma M_{\mu\nu} - 2\partial_\mu G^{\rho\sigma}\partial_\sigma M_{\nu\rho} \\ & + \partial_\mu[G]\partial^\rho M_{\rho\nu} + \partial^\rho G_{\mu\nu}\partial^\sigma M_{\rho\sigma} - 2\partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma} - 2\partial^\rho G_{\mu\sigma}\partial^\sigma M_{\nu\rho} \\ & + 2\partial^\rho G_{\mu\sigma}\partial_\rho M^\sigma_\nu + \partial^\rho[G]\partial_\nu M_{\mu\rho} - \partial^\rho[G]\partial_\rho M_{\mu\nu}\big) + \tfrac{1}{2}[M]\big(\partial_\rho[G]\partial^\rho[M] \\ & - \partial_\rho G_{\mu\nu}\partial^\rho M^{\mu\nu} - \partial_\rho[G]\partial_\sigma M^{\rho\sigma} - \partial_\rho G^{\rho\sigma}\partial_\sigma[M] + 2\partial_\rho G_{\mu\nu}\partial^\nu M^{\mu\rho}\big)\Big] \end{aligned} \tag{11}$$

It is clear that this expansion must be taken very carefully since we still have symmetries in the theory that can be used to gauge fix both fields $G_{\mu\nu}$ and $M_{\mu\nu}$. Motivated by string theory on-shell conditions and using the linearized diffeomorphism and equations of motion we fix the gauge so that we have transverse and traceless properties for both of the fields meaning:

$$\begin{aligned} &\partial^\mu G_{\mu\nu} = 0, \qquad \partial^\mu M_{\mu\nu} = 0 \\ &G^\mu_\mu = 0, \qquad M^\mu_\mu = 0 \end{aligned} \tag{12}$$

Imposing these conditions on the cubic order expansions of the Bimetric theory will reduce the terms significantly and we have the following for the cubic interactions[20]:

$$\begin{aligned}\mathcal{L}_{\mathrm{GM}^2} &= \tfrac{1}{m_g\sqrt{1+\alpha^2}}\Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big) \\ &\qquad +2M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\big)\Big]\end{aligned} \tag{13}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{M}^3} &= \tfrac{(-\beta_1+\beta_3)\,(1+\alpha^2)^{3/2} m_g}{6\,\alpha}\,[M^3] \\ &\quad +\tfrac{(1-\alpha^2)}{m_g\,\alpha\sqrt{1+\alpha^2}}\,M^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 2\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big)\,,\end{aligned} \tag{14}$$

where, the braket $[X]$ means trace of the (composite) field inside.
Therefore, we have constructed a setup for Bimetric theory to be able to compare it to string theory. As we are going to see our goal now is to compute an effective action from string theory and reproduce the above gauge fixed Bimetric Lagrangian.

3. String Scattering Amplitude

String theory is a 2-dimensional CFT therefore its scattering amplitude is defined as the integration of vertex operators over the associated Riemann surface. Using operator state correspondence vertex operators are defined as the state at infinite conformal time. So schematically, in order to compute the scattering amplitude of state $|a\rangle$ off of state $|b\rangle$ we have:

$$\begin{aligned}&|a\rangle \leftrightarrow V_a(z), \qquad |b\rangle \leftrightarrow V_b(z) \\ &\mathcal{A}(a,b) = \int_{\mathcal{M}} \mathcal{D}[z]\ :V_a(z)::V_b(z):\end{aligned} \tag{15}$$

Here there are several points to be discussed:

- Since the CFT is defined over Riemann surfaces the variables are complex $z \in \mathbb{C}$ and the manifold $\mathcal{M}$ is the world sheet of the theory.

- Depending on the existence of boundary for the surface the manifold $\mathcal{M}$ can have boundary. In case we are using pure closed string there are no boundaries e.g sphere. Otherwise, when we have open strings with or without closed string we have a manifold with boundary e.g. disk.

- The measure $\mathcal{D}$ is defined as quotient by Conformal killing group (CKG). This group for $\mathcal{M}$ with boundary is $SL(2,\mathbb{R})$ and for $\mathcal{M}$ without a boundary is $SL(2,\mathbb{C})$. We use this quotient to fix the position of the three real/complex variables of the measure respectively.

Having all this in mind our first task is to define the states which we are going to use to describe the Bimetric gravity. As was mentioned before, one has infinitely many massive states with different spins in string theory. We need to define two states one associated with the massless spin-2, $G_{\mu\nu}$

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and one for the massive spin-2, $M_{\mu\nu}$. In the case of the massless spin-2 we have the ground states of the closed string Type II:

$$\begin{aligned} &\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1}|0\rangle \leftrightarrow V(z,\bar{z})^{(-1,-1)}_{G}(z,\bar{z},\varepsilon,q) \\ &V(z,\bar{z})^{(-1,-1)}_{G}(z,\bar{z},\varepsilon,q) = -\frac{2g_c}{\alpha'}\,\varepsilon_{\mu\nu}\,\widetilde{\psi}^{\mu}(\bar{z})\psi^{\nu}(z)\,e^{iqX(z,\bar{z})}\,, \end{aligned} \tag{16}$$

This vertex operator is given in the $(-1,-1)$ BRST ghost charge. The total charge of an amplitude independently for left and right movers (i.e. holomorphic and anti holomorphic) should be (-2). This was the easier of the two tasks now we need to find a good candidate for the massive spin-2 field $M_{\mu\nu}$. Looking at the work done in [21],[22] we can organize the open string massive states together with the closes massless spin-2 state in bosonic and superstring. After doing so, we are going to choose the massive spin-2 open string state for two main reasons. First, it is the first massive spin-2 that appears at the lowest level. It has a simpler vertex operator structure. Second, the Bimetric theory in terms of string theory was also discussed in [23] and in that paper this state was recognized as a candidate state for Bimetric. So having defined the state we have the vertex operator in -1 ghost charge as:

$$V^{(-1)}_{M}(x,\alpha,k) = \frac{g_o}{(2\alpha')^{1/2}}\,T^a\,e^{-\phi(x)}\,\alpha_{\mu\nu}\,i\partial X^{\mu}(x)\psi^{\nu}(x)\,e^{ikX(x)}\,, \tag{17}$$

Both of these states satisfy string on-shell transverse and traceless conditions.

4. Effective action

Here we are going to define the method that we are going to use in order to produce effective action from string theory it is done in a few deceivingly simple steps:

- First, we define our candidate states and then calculate the amplitude involving these states. The structure of the amplitude i.e. number of legs and type of states scattering depends on the terms that we want to reproduce from the Bimetric.
- Second having the integrated expression of the amplitude we are going to have an expansion in α'. Now we take the low energy limit.

Now, we have all the prerequisites to calculate the amplitude and compare it to the Bimetric gauged fixed Lagrangian (14). We have two types of amplitudes GM^2 and M^3. For GM^2 we have open closed mixed amplitude and for the M^3 we have pure open string amplitude. So for both of these cases at tree level we have $\mathcal{M} = D^2$ and the amplitudes. The amplitudes can be written as:

$$\begin{aligned} \mathcal{A}(G,M,M) \;=\; &\int_{\mathbf{R}}\int_{\mathcal{H}_+}\frac{\mathrm{d}x_1\mathrm{d}x_2\,\mathrm{d}^2z}{V_{\mathrm{CKG}}} \\ &\langle :V^{(-1)}_{M}(x_1,\alpha_1,k_1) :: V^{(-1)}_{M}(x_2,\alpha_2,k_2) :: V^{(0,0)}_{G}(z,\bar{z},\varepsilon,q) :\rangle_{\mathbb{D}_2} \end{aligned} \tag{18}$$

and

$$\begin{aligned}\mathcal{A}(M,M,M) &= \int_{\mathbf{R}} \frac{\prod_{i=1}^{3} \mathrm{d}x_i}{V_{\mathrm{CKG}}} \langle : V_M^{(-1)}(x_1,\alpha_1,k_1) :: V_M^{(-1)}(x_2,\alpha_2,k_2) :: V_M^{(0)}(x_3,\alpha_3,k_3) :\rangle_{\mathbb{D}_2} \\ &\quad +(2 \leftrightarrow 3)\,,\end{aligned} \tag{19}$$

For the first amplitude after fixing the CKG we are going to have one real integral left (depending on the choice of the fixing it can be the position of different vertex operators). Normally, these amplitude integrals will be given in terms of gamma functions and in our case calculating these integrals involved a lot of tedious bookkeeping of terms and contractions. For calculation purpose we can organize the terms in side the first amplitude in terms of the power of $\frac{1}{(x_1-x_2)^i}$ and write:

$$\mathcal{A}(2,1) = \frac{g_c}{\alpha'^2} Tr\big(T^a T^b\big) \sum_{i=1}^{4} \mathbf{A}_i\,, \tag{20}$$

As an example we can write the result for $\mathbf{A}_4$ after integration as:

$$\mathbf{A}_4 = \frac{1}{16} 4^s \left\{ 2A\, \frac{\sqrt{\pi} 2^{-s} s\, \Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} - (C+\Delta)\, \frac{\sqrt{\pi} 2^{-s} \Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} + (E-F)\, \frac{(s-1)\left[\Gamma\left(\frac{s-1}{2}\right)\right]^2}{4\Gamma(s)} \right\}, \tag{21}$$

where $s = -2 + 2\alpha' k_1 \cdot k_2$.

Crucially, we can see that taking the low energy limit of this amplitude is not straight forward. Since taking the α' to zero will lead and expansion in $s|_{\alpha'}$ variable with the limit:

$$s|_{\alpha'} = \alpha'\, k_1 \cdot k_2 \overset{\alpha' \to 0}{\longrightarrow} 1 \tag{22}$$

as it can be seen clearly a series involving $\alpha'\, k_1 \cdot k_2$ cannot be truncated since its value is not small. However, we can use conservation of momentum and rewrite s

$$s = -2\alpha'\, k_1 \cdot q \overset{\alpha' \to 0}{\longrightarrow} 0\,. \tag{23}$$

Therefore, in this part of the kinematic space, both α' and s go to zero simultaneously and truncation of the expansion of the Gamma functions is possible. This is the consistent way of taking the low energy limit of the amplitude which takes into account the fact that we have massive external states. Having defined the limit we can now take it for the mixed GM^2 amplitude and obtain the low energy terms then we construct the effective Lagrangian via replacement:

$$\varepsilon_{\mu\nu} \to G_{\mu\nu}, \qquad \alpha^{1,2}_{\mu\nu} \to M_{\mu\nu}, \qquad k_\mu\,, q_\mu \to i\partial_\mu \tag{24}$$

So after all this in the leading order of α' we have:

$$\begin{aligned}\mathcal{L}^{\mathrm{eff}}_{\mathrm{GM}^2} = g_c \Big[& G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big) \\ & + M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\big)\Big]\,.\end{aligned} \tag{25}$$

In contrast, for the second amplitude involving 3 open string states we can fix all three positions so we will have no integrals and we do not need to worry of the gamma function expansion in kinematic space. we will get after field replacement (24):

$$\mathcal{L}^{\text{eff}}_{\text{M}^3} = \frac{g_o}{\alpha'} \Big\{ [M^3] + 2\alpha' M^{\mu\nu} \big[\partial_\mu M_{\rho\sigma} \partial_\nu M^{\rho\sigma} - 3\, \partial_\nu M_{\rho\sigma} \partial^\sigma M^\rho_\mu \big] + 4\alpha'^2 \, \partial^\mu \partial^\nu M_{\rho\sigma} \partial^\rho M^\kappa_\nu \partial^\sigma M_{\mu\kappa} \Big\} . \tag{26}$$

Now, if we compare these two effective Lagrangians with (13) and (14) respectively we see that although we have all the same terms in both cases there are numerical differences. We have indicated these differences with red color in both (25) and (26).

5. Conclusion

As we discussed at the beginning we set out to find a construction in string theory to produce Bimetric gravity. We planned the task in two parts first took the Bimetric Lagrangian and used the diagonalization of the fields (perturbations around Minkowski) to write it in terms of mass eigenbasis. Then we took the resulting (expanded) Lagrangian in the cubic order (e.g. (11)) and brought on-shell/gauge fixed. These Lagrangian terms were our goal to reproduce by string theory as an effective action. This brings us to the second part namely writing an effective action for interacting massless and massive spin-2 fields using string states. To achieve this goal we had to choose candidate states for the two fields. We chose closed string ground state for the massless spin-2 and the first massive open string state in spin-2 representation for the massive mode. Then we had to calculate the effective action for these states which involved 2 steps: First, we had to calculate the amplitude including the same number of fields as our desired Lagrangian terms. Second, we had to define a consistent way to expand and truncate the result in orders of α'. For the GM^2 case we have also the expansion of Gamma functions in terms of Mandelstam variables. Using the proper α' scaling of the massive momenta state we defined a proper and consistent low energy limit.

We managed to produce all terms that are present in GM^2 and M^3 Bimetric gauge fixed Lagrangian. However, there are numerical discrepancies that break the ideal embedding. This can be due to the fact that the state we chose for the massive spin-2 field does not have the symmetries, in particular linearized diffeomorphism, of the second spin-2 field before the mass diagonalization. We also tried to use the bosonic string state which did not solve the issue.

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