PROCEEDINGS OF SCIENCE

Blackhole information

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Information of a blackhole inside has a long history. After mentioning the original information idea, we show that local gauge invariance can connect the inside information to the outside surface area due to the Sperner's lemma.

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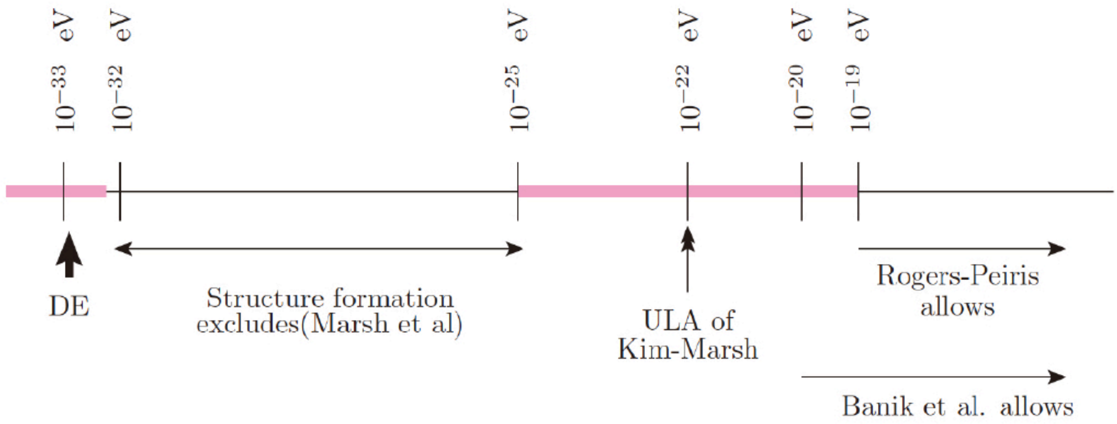

Figure 1: The allowed mass range of ULA. The lavender area is forbidden. There remains a small gap near 10^{-32} eV.

1. Introduction

In this talk, I apply a simple mathematical lemma to the blackhole information. The similarities of blackhole and thermodynamics was noted by Beckenstein [1]:

- A large blackhole increases its mass by absorbing nearby mass that can be compared to thermodynamic system losing more information as time passes by, and
- Combining two same mass blackholes, the surface area increses, that can be compared to the entropy increase in thermodynamics.

For a blackhole radius $r_h = 2GM$, the surface area is $A = 16\pi G^2M^2$. Expressing A as a dimensionless surface area, the information count is

$$S = \frac{M_{\rm ir}^2}{2M_{\rm P}^2} \tag{1}$$

where $M_{\rm P}$ is the reduced Planck mass 2.43×10^{18} GeV and $M_{\rm ir}$ is the irreducible mass [1, 2].

A massless particle has two polarizations. Massive one has 3. Digitization of mass information shows that for blackhole mass $M_{\rm ir} = 10^{-5}$ gram distinguishes mass information over the particle information. For our discussion, therefore, we consider blackholes with $M_{\rm ir} > 10^{-5}$ gram.

If our visible universe is a blackhole, then the dimensionless information (or $\rightsquigarrow$ entropy with $k_B = 1$) of our universe must be about $10^{122-123}$ [3]. Reference [3] excludes the visible universe filled with primodial blackholes. If a fixed mass is composed of very light particles, it carries a lot of information. Such a particle ultra light axion (ULA) is some barometer counting a lot of information. Reference [3] argues that ULA of mass order 10^{-32} eV can provide the needed information. Figure 1 shows a small gap allowed near the ULA of mass order 10^{-32} eV. In this talk, irrespective of these, we look for a logical possibility, "Can there be a method to see the information inside a blackhole?"

Suppose that we punctuate a hole of diameter ℓ in the blackhole surface as shown in the LHS figure of Fig. 2. Waves with wavelength $\lambda > \ell$ cannot come out from the puncture while waves of

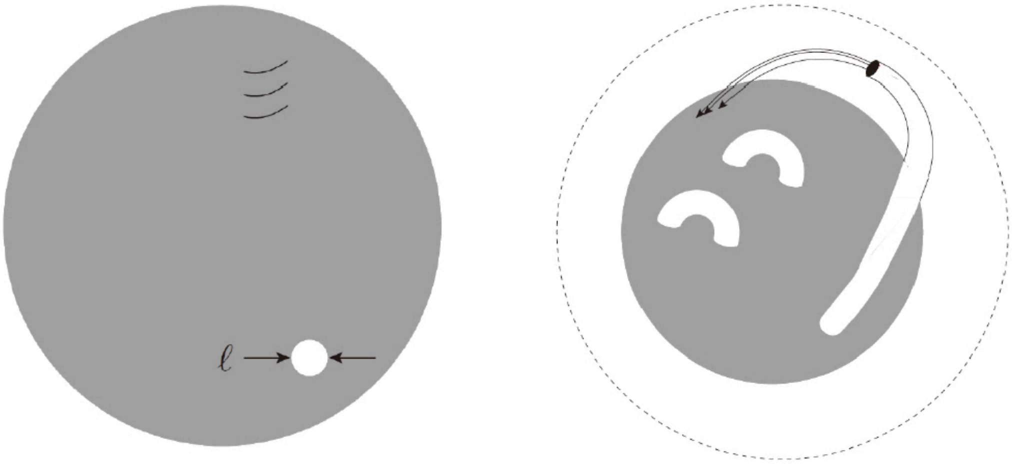

Figure 2: Punctures in the blackhole surface. The RHS figure has five punctures.

$\lambda < \ell$ can come out. But we will ignore these quantum waves for getting information inside the blackhole. We are interested in the flux lines of the classical concept. Flux lines are topological in the sense that they have no thickness, unlike the quanta.

Consider the dashed surface in the RHS figure of Fig. 2. Follow the flux lines into the blackhole inside. Then, it is the surface integral of the inside sphere, and by the Gauss theorem we obtain the total inside charge. So, through this puncture we obtain the information of the blackhole inside. Following the flux lines to outside, they must land on some place that is possible only on the surface. Thus, the surface(area) is the information the blackhole possesses inside. This is an equality, *i.e.* $M_{\rm ir}$ becoming the reducible mass $M_{\rm r}$ [4]. It is because the flux lines can go and come through the puncture. Blackhole entropy is the afore-mentioned dimensionless surface area, or information,

$$S = \frac{M^2}{2M_{\rm P}^2} \tag{2}$$

where M is the blackhole mass.

2. Sperner's Lemma

A triangle is topologically the same as a disk. Anyway a triangle rubber cloth can cover the surface of a sphere. So does a triangle rubber band. In lattice gauge theory, the line integral is equivalent to the plaquet which is the surface intgral of $F^a_{\mu\nu}F^{a\,\mu\nu}$ over the surface that is enclosed by the closed curve. The plaquet is a gauge singlet. Figure 3 shows three line integrals. In this talk, we are not doing lattice gauge theory. To prove Sperner's lemma [5], just we integrate some quantity over the surface enclosed by a closed curve.

Let us consider Fig. 4 which is the triangle (pqr) enclosed by the line integral in Fig. 3 (c). Within (pqr), the gauge invariance requires that a specific quantity denoted by the adjoint index a can pass through this lavender triangle, where a is determined once (pqr) is chosen. So, we consider the surface integral of the quantity,

$$F^{a\,\mu\nu} \tag{3}$$

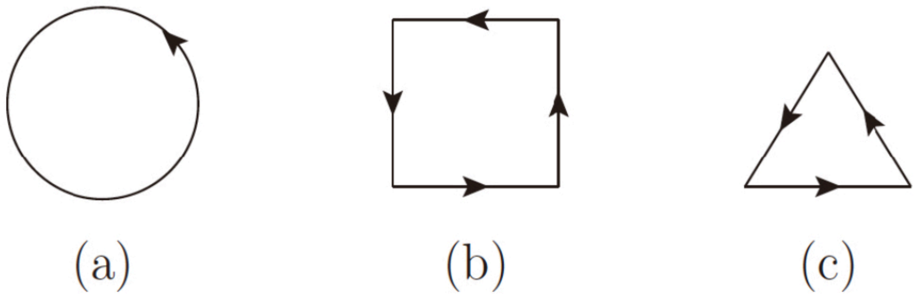

Figure 3: The counterclockwise line integral: (a) circle, (b) square, (c) triangle.

Depending on p, q, and r of Fig. 4, a of Eq. (3) is determined. Consider SU(3) gauge theory. Then, for $p = 1$(or A), q = 2(or B), r = 3(or C), a becomes an SU(3) singlet direction.

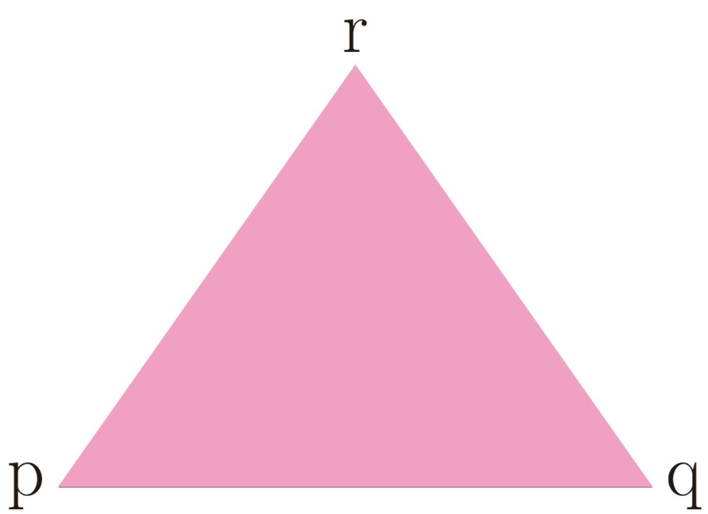

Figure 4: Triangle (pqr).

An easy proof of Sperner's lemma [5] is given in [6]. It is also given in [7] together with the history on the Brouer Fixed Point Theorem since Sperner's lemma is a part of the theorem. A large triangle (pqr) is divided into small triangles (ijk), as shown in Fig. 5. At each vertex of small triangles, an index (i) is assigned. It can be a color index if QCD is considered. Considering triangles is general enough. For example, if we consider squares, we can divide the squares to triangles and we go back to the case of Fig. 5. On the outside segment (pq), the vertices of small triangles contain only p or q. The same assignments apply to the outside segments (qr) and (rp) also.

The proof is composed of two parts. First count the number of small segments on one outside segment. Second, count the number of small triangles. Some scores are assigned for the indices (ij) on the outside segment (pq). Also, some scores are assigned for small triangles (ijk). In these cases, the scores are shown odd numbers. For example, on (AB) in the LHS of Fig. 5 there are three (ab), one (aa) and one (bb). For triangle (ABC) in the LHS of Fig. 5 there are three (123),

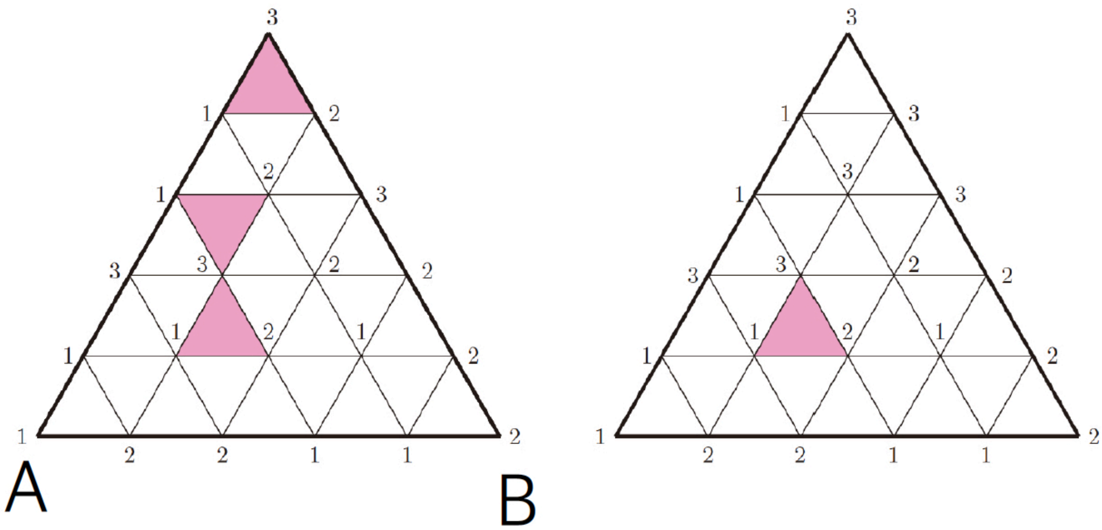

Figure 5: Triangle (ABC) is divided into small triangles (ijk). In the LHS figure, there are three (123)'s. In the RHS figure, a different labeling on BC allows just one (123).

Table 1: Scores for the cases of short segments (ij) and small triangles (ijk).

Base line (AB)		Triangle (ABC)	
Segments (ij)	Score	Small triangles (ijk)	Score
External (ab)	1	(abc)	1
Internal (ab)	2	every internal (aab) and (abb)	2
All others	0	All others	0

eight (122), five (223), five (112), one (113), two (133), and one (111). For each of these, scores are given in Table 1. The score of (ab) is 3, and the score of (abc) is 3. So, it is easier to count the score of triangles by counting the score of (ab) on (AB). It is easy to see that the score of (ab) is always odd on (AB).

Let us apply the lemma to QCD. There are three colors 1, 2, and 3. The number of triangle (123) is odd. In Fig. 6, we colored all small triangles according to the colors made by three fundamental colors of light: red, blue and green. For a typical choice of (ijk), there remains only one white triangle. For other choices, the number of white triangles must be odd, and removing triangles pairwise, there remains one white triangle. Then, we go back to Fig. 2, and information of the blackhole inside is related the surface information of the blackhole.

Acknowledgments

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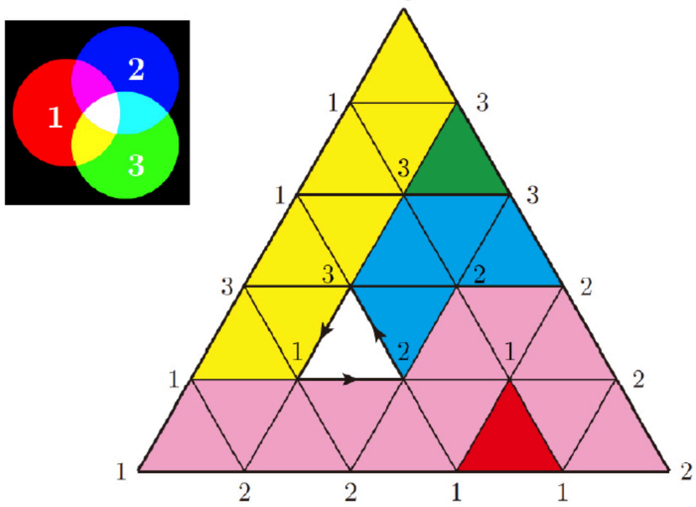

Figure 6: Labels 1, 2, 3 are three colors of QCD. Triangle (123) allows only SU(3) invariant quantity, which is colored white. The other triangles are colored, which is equivalent to saying that only colored fluxes can pass through those colored triangles.

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