

## Testing curvature-induced in-vacuo dispersion with gamma-ray-bursts

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We report the results found in [1], where we explore the phenomenological viability of scenarios, suggested by different approaches to quantum spacetime, in which quantum-gravity effects in the propagation of particles are triggered by spacetime curvature/expansion. We rely on a toy model of curvature-induced Lorentz violation for a preliminary exploration, and we find that, differently from what commonly believed, the double suppression due to Planck-length and spacetime curvature is compensated by the high energies and the long (cosmological) distances traveled of the gamma-ray-burst photons.

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It is widely believed, based on fundamental approaches to quantum gravity, that at an effective theory level our standard (classical, continuous) description of spacetime should be abandoned in favor of a “quantum” spacetime, (whose specific features depend on the different quantum gravity scenarios). Particularly intriguing is the hypothesis that measurements may be affected by some sort of quantum-gravity driven “Heisenberg uncertainty principle” involving spacetime only, when approaching the sensibility of Planck scale ( $L_p \sim 10^{-33} \text{ cm} \equiv 1/E_p \sim 10^{-19} \text{ GeV}^{-1}$ ), characterized by the emergence of a minimum observable scale  $\lambda$  proportional to  $1/E_p$ . Of particular interest for quantum gravity phenomenology is the possibility that such scale manifests itself in a modification of some relativistic kinematical laws, leading in principle to observable traces in particle propagation. Among the different scenarios, one of the most promising is when  $\lambda$  parametrizes a modification of the dispersion relation such that photons (as well as ultra-relativistic particles) are affected by “in-vacuo” dispersion, i.e. their speed in vacuum differs (systematically or/and statistically) from  $c$  by correction terms proportional to (powers of) the ratio  $\lambda E \sim E/E_p$ , so that  $c$  characterizes the speed of only the “lowest energy” photons.

While the tremendous Planckian suppression seems to leave no hope for an actual (direct) observation of the effect, transient astrophysical sources, and in particular gamma-ray bursts, may present the opportunity to circumvent this obstruction: the incredibly high energies involved, together with the (cosmological) distances traveled by photons, may provide the sought amplification of the tiny Planckian effect, that would accumulate through propagation, so that the expected time-delay between the most energetic photons and the nearly simultaneously<sup>1</sup> emitted low energy signal could be within the reach of sensitivity of detection [4]. This kind of indirect observations, via an amplification of the effect, is at the base of several examples of quantum gravity phenomenological opportunities [2, 3], and analyses of this kind have proven to be able, in the past ten/fifteen years, to provide the first evidences that Planck-scale sensitivity can indeed be reached [9–11], stimulating and motivating further efforts in this direction [3, 12–15].

Due to the distances involved in these observations, where the typical sources of gamma-ray bursts relevant for the analysis may have redshift  $z$  significantly greater than 1, the analysis of time delays must take into account of the contribution of spacetime curvature, so that the distance factor becomes redshift dependent. A formula for time delays that includes the curvature contribution has been determined, within the LIV (Lorentz Invariance Violation) scenario, after several stages of assessment, to be the one proposed in [8]

$$\Delta t = \lambda \Delta p \int_0^{z_{em}} \frac{dz}{H(z)} (1+z). \quad (1)$$

Here  $p$  is the modulus of the (comoving, constant) momentum measured by an observer at the detector (which coincides with the redshifted photon energy measured at the detector),  $\Delta p$  is the difference between the comoving (constant) momentum of two (simultaneously-emitted) particles whose times of arrival are being compared, and  $\lambda$  is a phenomenological length scale characterizing the quantum-spacetime effects, to be determined experimentally.  $H(z)$  is the Hubble parameter, for which in the analyses here reported we adopted the favored description of current cosmological models, i.e.  $H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$  where  $H_0$ ,  $\Omega_m$  and  $\Omega_\Lambda$  are respectively the Hubble

<sup>1</sup>Within the assumptions of the astrophysical model [2, 3].

constant, the matter and the cosmological constant density parameters (to which we assign values according to Ref. [23]).

A feature of this formula, is that the quantum spacetime effect is still present in absence of (macroscopic) spacetime curvature, whose contribution then has the role of a correction to the one obtained already in a flat spacetime framework. The fact that the formula behaves like this, can be explained on one side for obvious “historical” reasons: one first looks for quantum spacetime modifications in the simpler case of a flat spacetime framework, and then adds the complexity of curvature (in the sense of universe expansion), aiming to reproduce the previous results in the flat limit. More importantly, one expects that the double suppression due to both Planck scale and spacetime curvature leaves no hope for the effect to be detectable. This is probably the main reason for which the quite large literature (see, e.g., Ref. [4–15]) devoted to the study of these phenomenological opportunities have been based so far only on models of this kind, and particularly exclusively on the time-delay formula.

However, on the theory side, there is a certain interest in scenarios where quantum spacetime effects emerge in conjunction with, and triggered by, (macroscopic) spacetime curvature. Models of quantum spacetime based on Hopf algebras typically are derived in frameworks where the quantum gravity deformation requires the presence of both curvature and Planck scales [16–18], and it has been argued how the permanence of the effect in the flat limit may require specific fine-tuning [19, 20]. Moreover, some studies based on the loop-quantum-gravity perspective suggest that the two scales may be linked together so that the relevant quantum-gravity effects should be triggered by curvature [21].

Motivated by these considerations, we reconsidered in [1] the argument against the phenomenological viability of scenarios in which in-vacuo dispersion is triggered by spacetime curvature. We had noticed indeed that it is reasonable to expect that curvature appears in the relevant formulas always multiplied by the distance traveled by the particle, compensating for its smallness, so that such terms may end up to not dampen the overall effect. We based our first exploratory analysis on a toy model inspired by some previous studies [22] where we were looking for a generalization of the formula (1). Our starting point is the dispersion relation

$$E = \frac{p}{a(t)} \left( 1 - \frac{\lambda}{2} \frac{p}{a(t)} - \frac{\lambda'}{2} p a(t) \right), \quad (2)$$

where  $E$  is the “comoving-time energy” ( $E \sim \partial_t$  [22]),  $a(t)$  is the scale factor in FRLW spacetime. When  $\lambda' = 0$ , the dispersion relation is the one used in [8] for deriving formula (1). A possible different dependence on the scale factor was already mentioned in [6]. In [22] we studied the dependence on these factors systematically.

However, when  $\lambda' = -\lambda$  one has no dispersion in absence of curvature. A first evidence that this is the case comes from the expression of the (comoving-time) particle velocity deriving from (2):

$$v(t) = \frac{1}{a(t)} \left( 1 - \lambda \frac{p}{a(t)} - \lambda' p a(t) \right). \quad (3)$$

This can be usefully rewritten as a relationship between velocity and redshift  $z = 1/a(t) - 1$ , working with a scale factor such that  $a(0) = 1$ , *i.e.* the scale factor is 1 at the time of detection.

One easily finds that

$$v(z) = 1 + z - \left( \lambda + \lambda' + 2\lambda z \left( 1 + \frac{1}{2}z \right) \right) p. \quad (4)$$

The fact that for  $\lambda' = -\lambda$  the correction is proportional to the redshift signals that indeed for  $\lambda' = -\lambda$  the dispersion effects are a purely ‘‘curvature-induced’’ correction. Here we already can see the main feature that we expected when reconsidering curvature-induced dispersion: the curvature-induced effects on velocity are much smaller than the ones found when  $\lambda' \neq -\lambda$  only when distances/redshift are small. However, the relevant phenomenology focuses on GRBs at redshifts of order unity or bigger, and in that case the magnitude of our curvature-induced effects is comparable to that of the effects produced when  $\lambda' \neq -\lambda$ .

Further insight can be gained by rewriting the formula for the velocity using the relation between the redshift and the Hubble constant  $H_0 = \dot{a}/a|_{t=0}$ , derived from Taylor expanding the redshift equation for small distances (i.e. small times  $t$ ),  $z \simeq 1/(1 + \dot{a}t) - 1 \simeq -H_0 t$ . This gives

$$v(t) \simeq 1 - H_0 t - ((\lambda + \lambda') - 2\lambda H_0 t) p. \quad (5)$$

Here again we see that the dispersion effects disappear if  $\lambda' = -\lambda$  and there is no curvature ( $H_0 = 0$ ); moreover, the curvature-induced effects are only significant for large distances (large values of  $t$ ).

Integrating equation (4) from the redshift of the source  $z_{em}$  to  $z_{now} = 0$ , considering that  $dt = -dz/(H(z)(1+z))$ , we find the time-delay formula

$$\Delta t = \Delta p \int_0^{z_{em}} \frac{dz}{H(z)} \left( \lambda(1+z) + \frac{\lambda'}{(1+z)} \right), \quad (6)$$

which of course for  $\lambda' = -\lambda$  describes effects that are very small at small redshift:

$$\Delta t|_{\lambda'=-\lambda} = 2\lambda \Delta p \int_0^{z_{em}} \frac{dz}{H(z)} \frac{z + z^2/2}{1+z}. \quad (7)$$

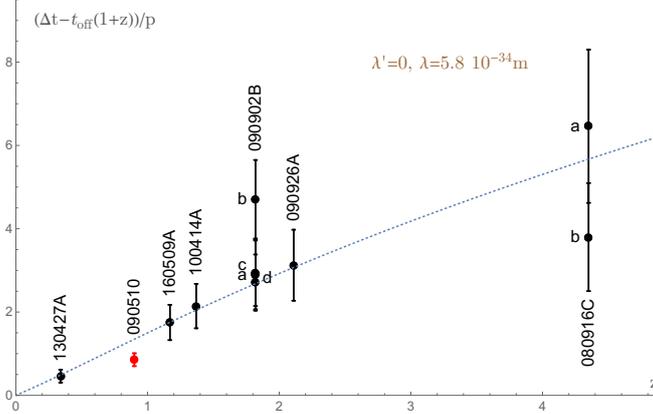
In order to test our toy model and compare it with the  $\lambda' = 0$  scenario, we considered the set of data that was at the basis of previous analyses reported in Refs.[14, 15], whose selection criteria led the authors to focus on 11 GRB photons of particularly high energy observed by the Fermi telescope. The key properties of those 11 photons are shown in table 1. A detailed description of the selection criteria can be found in Refs.[14, 15], with the most noteworthy among them being those concerning the requirements on energy of the photon and the time window allowed for considering a photon as possibly being emitted in coincidence with the first peak: the selected photons should be such that their energy at emission (factoring in redshift) is greater than 40 GeV, and should be compatible with an intrinsic time lag (‘‘inferred offset at the source’’) with respect to the first low-energy peak of the GRB of no more than 20 seconds.

The results of the analysis of Refs. [14, 15] were that, after fitting on data just two parameters ( $t_{off}$ , accounting for an average time offset at the source [14, 15, 24], and  $\lambda$ ), one finds that 8 out of the 11 data points show a delay of arrival, with respect to the first peak of the relevant GRB, consistent with the  $\lambda' = 0$  scenario. The fact that 3 out of the 11 data points do not fit the model is not necessarily concerning, since the assumption that the highest-energy photons be emitted in coincidence with the first bright low-energy peak can at best work most of the time, but surely not

event	$z$	$E_{\text{obs}}$ [GeV]	$\Delta t$ [s]
130427A	0.34	77.1	18.10
090510	0.90	29.9	0.86
160509A	1.17	51.9	62.59
100414A	1.37	29.8	33.08
090902Ba	1.82	14.2	4.40
090902Bb	1.82	15.4	35.84
090902Bc	1.82	18.1	16.40
090902Bd	1.82	39.9	71.98
090926A	2.11	19.5	20.51
080916Ca	4.35	12.4	10.56
080916Cb	4.35	27.4	34.53

**Table 1:** Data taken from the Fermi gamma-ray space observatory, publicly available at <https://fermi.gsfc.nasa.gov/ssc/data/access/>, concerning the 11 photons featured in our figures. The first column reports the name of the events, specified by the GRB name followed by a (lowercase) letter in case of multiple photons associated to the same GRB. The second column reports the redshift of the relevant GRB. The third column reports the (observed) value of events energy at the detector. The last column reports the difference in times of observation between the photon and first low-energy peak of the GRB.

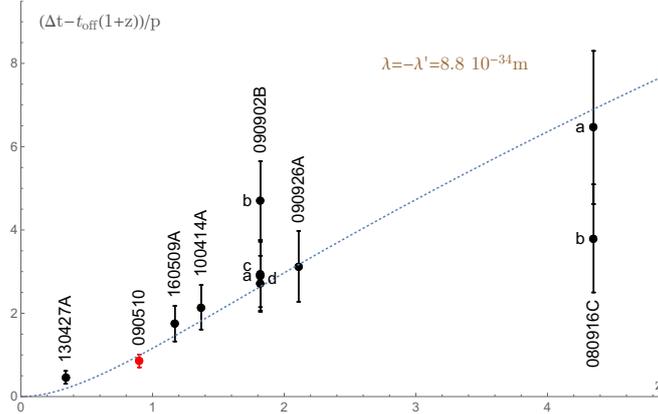
always. In particular, the fact that the data point 090902Bb reflects a larger delay than predicted by the model might just mean that it was emitted in coincidence with a later peak, not the first peak. The data point 080916Cb reflects a smaller delay than predicted by the model (and emission by peaks later than the first one can only inflate the delay estimate, not reduce it); however, the data point 080916Cb is affected by rather large uncertainties. The main, and significant, challenge to the interpretation of Fig.1 in connection with the  $\lambda' = 0$  scenario resides in the data point 090510, whose delay with respect to the time of observation of the first low-energy peak is much smaller than predicted by the  $\lambda' = 0$  scenario, and it is a data point with very little uncertainty. The data point 090510 cannot be described in any way within the  $\lambda' = 0$  scenario.



**Figure 1:** The 11 photons already considered in Refs. [14, 15], here characterized in terms of the values of their  $(\Delta t - t_{\text{off}}(1+z))/p$  and their redshift  $z$ , two variables which, according to Eq. (6), should be linked by the dotted blue curve, if  $\lambda' = 0$  and  $\lambda = 5.8 \cdot 10^{-34}m$ .

Fig.2 reports the results for the same analysis redone for our  $\lambda' = -\lambda$  scenario with curvature-triggered effects. First of all one should notice that, in agreement with our main thesis, for comparable values of  $\lambda$  our novel  $\lambda' = -\lambda$  scenario and the much studied  $\lambda' = 0$  scenario both produce effects at the level needed for this sort of analysis: there is no overall suppression of the

effects due to the smallness of spacetime curvature. Intriguingly, the  $\lambda' = -\lambda$  scenario predicts much weaker in-vacuo-dispersion effects only for small redshifts, with an impact on the data analysis which however is encouraging: as seen by comparing Fig.2 to Fig.1, the  $\lambda' = -\lambda$  scenario provides a description of the data which overall is as good as that of the  $\lambda' = 0$  scenario, and the  $\lambda' = -\lambda$  scenario handles nicely also the 090510 data point, the one which is instead very problematic for the  $\lambda' = 0$  scenario.



**Figure 2:** Same as fig.1, but here the dotted blue curve is for Eq. (6) in the “curvature-induced case”  $\lambda' = -\lambda$ , with  $\lambda = 8.8 \cdot 10^{-34} m$ .

While the results reported in [1] are based on a specific toy model, they show the viability of a phenomenology based on curvature-induced quantum gravity effects, and we expect that some of the features uncovered with our  $\lambda' = -\lambda$  scenario (particularly the slow onset of effects at small redshifts) are likely to be shared by other scenarios for curvature-triggered quantum-gravity effects.

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