PROCEEDINGS OF SCIENCE

Cosmological parameters from different Baryon Acoustic Oscillations dataset

Denitsa Staicova[a,*]

[a] *Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria*

E-mail: dstaicova@inrne.bas.bg

Baryon Acoustic Oscillations (BAO) datasets offer interesting possibilities for inferring cosmological parameters, due to their unique properties of being relics of the pre-recombination Universe. Since the BAO measurement are done by tracking the spatial distribution of millions of galaxies, the statistical effects and the assumptions on the model play significant role when the BAO data is used to constrain cosmological parameters. In this proceeding, we review our recent articles in which we tried to minimize some assumptions when we used BAO datasets to fit different cosmological models. In one of them, we use a BAO dataset along with cosmic chronometers, supernovae, gamma-ray bursts and quasars but we take the sound horizon as a free parameter. In the other, we take a BAO dataset along with a CMB distance prior and we take the combination $H_0 r_d$ as a free parameter. We discuss the results, and also the difficulties and the advantages of these approaches.

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1. The attractivity of BAO

The epoch of recombination is the period which "initializes" the large scale structures we currently observe. In the pre-recombination Universe all the baryonic matter, the photons and the neutrinos are all trapped together in a hot dense plasma "soup", which oscillates under the attractive force of gravity and the repulsive force of radiative pressure. These pressure waves are called the Baryonic Acoustic Oscilations (BAO). The oscillations propagate in the Universe with a speed of sound about $\sim c/\sqrt{3}$ until the Universe expands and cools down enough that the photons decouple from the baryons (the decoupling epoch) and the baryons stop feeling the photons (drag epoch). When this happens, the BAO effectively freeze at the so called sound horizon, which since then expands with the expansion of the Universe. The sound horizon depends only on the ratio between the initial matter and radiative content of the universe and it is defined with a simple formula. It represents the radius at which a bump in the two-point correlation function of galaxies can be found and it can be used as a standard ruler to measure distances ([1]. Observing this effect requires a very precise spectroscopy and it was first observed in 2005 by the Sloan Digital Sky Survey. Since then, it has been observed trough numerous objects thanks to missions like SDSS, WiggleZ, DES, BOSS, eBOSS etc. They have been measured in clustering of galaxies and quasars, from the correlation function of the Lyα absorption lines in the spectra of distant quasars, in cross correlation with quasar positions and galaxies. The measurements, coming each year with higher precision and better covariance matrices, have been since used in numerous works in an attempt to help solve the problem of cosmological tensions ([2–7]).

The problem of BAO measurements is that in them, one always measures to combination $H_0 r_d$, where H_0 is the Hubble parameter measuring the expansion of the Universe and r_d is the mentioned sound horizon. This means that one should always use some kind of tight prior on one of these parameters, in order to be able to constrain the other. This can be either the Riess prior on H_0 coming from local Universe measurements or the Planck prior on H_0 or on r_d coming from the CMB epoch. In both cases, one makes certain assumptions on the model.

In this proceeding, we review the results from our recent articles on using BAO datasets along with other datasets to infer cosmological parameters [8–10] . In these works, we find different ways to reduce the dependence on the prior on r_d, so that one can disentangle the degeneracy between H_0 and r_d and to check if it aleviates the H_0-tension. We take two different BAO datasets and add to them different other datasets. In one of them, [8], we use a BAO dataset along with cosmic chronometers, supernovae, gamma-ray bursts and quasars but we take the sound horizon as a free parameter. In the other, [10], we take a BAO dataset along with a CMB distance prior and we take the combination $H_0 r_d$ as a free parameter. In both cases, we perform statistical analysis to find which model represents the best fit.

2. Numerical methods

We use a nested sampler as implemented within the open-source package *Polychord* ([11]) with the *GetDist* package ([12]) to present the results. The priors we use to obtain the results can be found in the respective articles [8, 10] and also in [9] where some further details are given.

We use two different datasets for the two works we discuss, but in both case the question of the possible covariance of the data arises. This happens because the BAO involve very precise measurements of millions of galaxies for which it is not possible to always know the covariance matrix that is related to the systematic errors of the instruments and the design of the measurement itself. This matrices are derived trough mock up simulations but they are not always publicly available. For this reason, we use another method to test our points for strong covariance. We perform a covariance analysis based on the one proposed in Ref. [13]. We add to the standard covariance matrix for uncorrelated points C_{ii} the C_{ij} random non-digaonal terms signifying the possible correlation.

$$C_{ii} = \sigma_i^2 \quad \longrightarrow \quad C_{ij} = \sigma_i^2 + 0.5\sigma_i\sigma_j \tag{1}$$

Here $\sigma_i\sigma_j$ are the published 1σ errors of the data points i, j. Then we check how our results change under different number of correlated terms. If they do not change much, we can use them for inferring cosmological parameters. With this approach we show that the effect of up to 25% random correlations with this magnitude results in less than 10% deviation in the final results, thus it is minimal and the points can be considered uncorrelated.

3. LCDM with BAO

The theoretical setup is based on the currently accepted best-fit model is the ΛCDM model. It assumes a Friedmann-Lemaître-Robertson-Walker metric with the scale parameter $a = 1/(1+z)$, where z is the redshift. For it one gets the following equation of state connecting different components of the universe:

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda, \tag{2}$$

where Ω_r, Ω_m, Ω_Λ, and Ω_k are respectively the fractional densities of radiation, matter, dark energy, and the spatial curvature at redshift $z = 0$. Here $E(z) = H(z)/H_0$, and $H(z) = \dot{a}/a$ is the Hubble parameter at z, while H_0 is the Hubble parameter today. The radiation density can be computed as $\Omega_r = 1 - \Omega_m - \Omega_\Lambda - \Omega_k$. To extend this to wCDM, the Friedmann equation is generalized to $\Omega_\Lambda \to \Omega_{DE}^0(1+z)^{-3(1+w)}$, while $\Omega_k = 0$ represents a flat universe and $\Omega_k \neq 0$ is Ω_kCDM.

The BAO measurements rely on different cosmological distances. The comoving angular diameter distance ([14])

$$D_M = \frac{c}{H_0} S_k\left(\int_0^z \frac{dz'}{E(z')}\right), \tag{3}$$

where one accounts for non-zero spatial curvature with:

$$S_k(x) = \begin{cases} \frac{1}{\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k}x\right) & \text{if} \quad \Omega_k > 0 \\ x & \text{if} \quad \Omega_k = 0 \\ \frac{1}{\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k}x\right) & \text{if} \quad \Omega_k < 0 \end{cases}. \tag{4}$$

The other distances we use are the Hubble distance $D_H(z) = c/H(z)$, the angular diameter distance $D_A = D_M/(1+z)$ and the volume averaged distance $D_V(z) \equiv [zD_H(z)D_M^2(z)]^{1/3}$.

The sound horizon r_d at the drag epoch ($z_d \approx 1060$) when photons and baryons decouple:

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \tag{5}$$

where $c_s \approx c/\sqrt{(3 + 9\rho_b/(4\rho_\gamma))}$ is the speed of sound in the baryon-photon fluid with the baryon $\rho_b(z)$ and the photon $\rho_\gamma(z)$ densities, respectively ([15]).

Since the actual measured quantities are the projections $\Delta z = r_d H/c$ and $\Delta\theta = r_d/(1+z)/D_A(z)$, where Δz and $\Delta\theta$ are the redshift and the angular separation, from BAO measurements can get information only about the quantity $r_d \times H$. For this reason, we look for different approaches to disentangle these two quantities. In this case, we take r_d as a free parameter and let the MCMC find it from the data. This, however, cannot work for the BAO dataset alone and it requires adding additional datasets.

The final dataset we use in [8] a set of uncorrelated data points from different **BAO** measurements: the Sloan Digital Sky Survey (SDSS), the WiggleZ Dark Energy Survey, Dark Energy Camera Legacy Survey (DECaLS), the Dark Energy Survey (DES), the 6dF Galaxy Survey (6dFGS) ([16–31]). To this dataset, we add **cosmic chronometers (CCs)** (30 uncorrelated CC measurements of $H(z)$ [32]), and **standard candles (SCs)** (the Pantheon Type Ia supernova dataset [33, 34]), and quasars [35] and gamma-ray bursts [36] .

The complete results can be found in [8]. Here we would like to summarize the results and highlight the important parts. On Fig. 1 one can see the posterior distributions for H_0, Ω_m, r_w, r_d.

- It was found that the BAO data alone cannot constrain the parameters enough. This is due to the degeneracy between H_0 and r_d that needs to be decoupled somehow. Adding a prior on H_0, like the Riess prior, and/or adding new datasets, like the CC improves significantly the fit.
- The result strongly depends on the prior on r_d with too small prior in any end of the interval forcing the inferred value of H_0 to be in agreement with it.
- As expected, using the Riess prior brings the results closer to the local H_0 measurement from LMC Cepheids ([37], ($H_0^{our} = 71.40 \pm 0.89 km/s/Mpc$), while no such prior makes the value closer to the Planck result ($H_0^{our} = 69.85 \pm 1.27 km/s/Mpc$) [38].
- The BAO + other datasets result alleviates the H_0-tension being very close to the measurement of the Tip of the Red Giants Branch $H_0 = 69.8 \pm 1.9 km/s/Mpc$ [39].
- The obtained spatial curvature is $\Omega_k = -0.076 \pm 0.017$, signifying a closed universe. This value is in agreement with a number of recent results, see [9] for more details.
- The predicted value of w in the wCDM model differs for BAO alone dataset and the combined dataset: $w = -1.067 \pm 0.065$ vs $w = -0.989 \pm 0.049$. This appears to be related to the choice of datasets we employ. Notably, the w parameter is well constrained even just from the BAO dataset.
- Finally we use the Akaike Information criteria to compare the different models and find that LCDM is the best model by this measure.

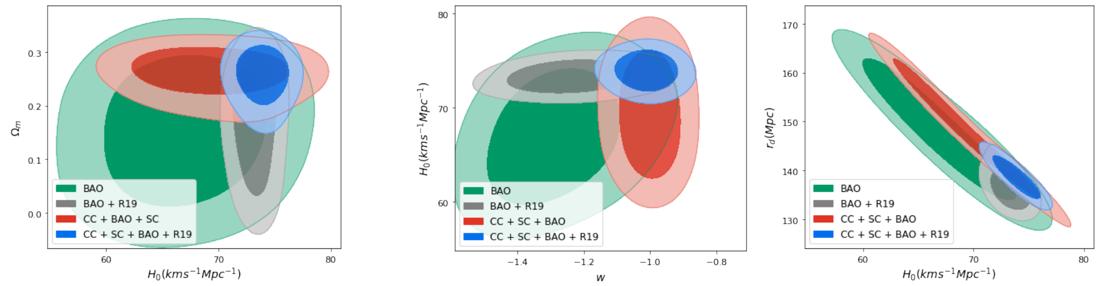

Figure 1: The posterior distribution for H_0, Ω_m, w, r_d, for different datasets/priors for the wCDM model

Notably, this work demonstrated the discussed in [40] "tensions in the r_d − H_0 plane.", which makes the parameters $H_0 - \Omega_m - r_d$ coupled to each other. An elaboration on the topic can be found in [9] but the implications are that one cannot solve the tension on one parameter, without taking care of the others.

4. DE models with BAO and distance priors

In this work, we expand on the theoretical background of FLRW discussed above with the Chevallier-Polarski-Linder (CPL) parametrization of wwaCDM [41, 42]:

$$\Omega_\Lambda(z) = \Omega_\Lambda^{(0)} \exp\left[\int_0^z \frac{3(1+w(z'))dz'}{1+z'}\right] \tag{6}$$

in which we consider three possible models:

$$w(z) = \begin{cases} w_0 + w_a z & \text{Linear} \\ w_0 + w_a \frac{z}{z+1} & \text{CPL} \\ w_0 - w_a \log(z+1) & \text{Log} \end{cases} \tag{7}$$

which recover ΛCDM for $w_0 = -1, w_a = 0$.

The distance priors provide effective information of CMB power spectrum in two aspects: the acoustic scale l_A characterizes the CMB temperature power spectrum in the transverse direction, and the "shift parameter" R influences the CMB temperature spectrum along the line-of-sight direction. Explicitly [43]:

$$l_\mathrm{A} = (1+z_*)\frac{\pi D_\mathrm{A}(z_*)}{r_s(z_*)}, \quad R \equiv (1+z_*)\frac{D_\mathrm{A}(z_*)\sqrt{\Omega_m}H_0}{c}, \tag{8}$$

where $z_* \approx 1089$ is the redshift at the photon decoupling epoch. r_s is the comoving sound horizon. Ref. [44] derives the distance priors in several different models using *Planck* 2018 TT,TE,EE + lowE which is the latest CMB data from the final full-mission Planck measurement [38]. Importantly, when working with the distance priors, one needs to account for the covariance matrix between l_A and R, which is model-dependent.

The final dataset we use in [10] consists of another combination of measurements by SDSS, WiggleZ and DES: The dataset we are using is a collection of points from different BAO observations

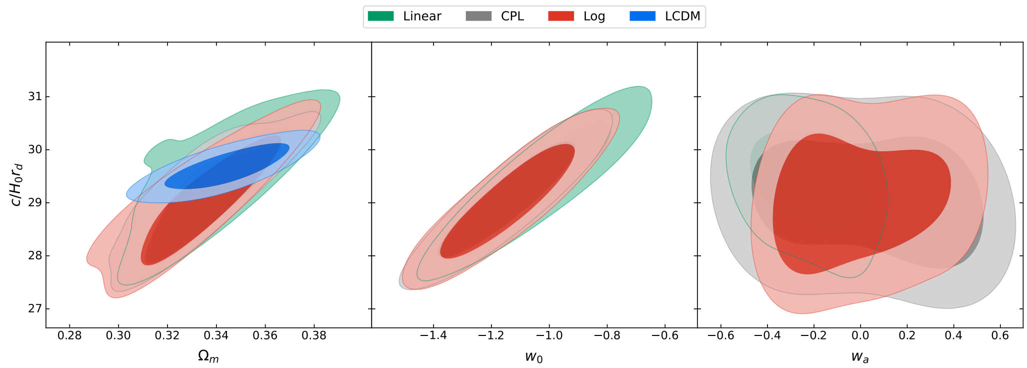

Figure 2: The posterior distribution for Ω_m, w_0, w_a, for different parametrization of DE.

[20, 21, 23, 25, 26, 45–52], to which we add the CMB distant prior [53]. The difference here is that the points are chosen such that to have only a D_A/r_d measurement which we can use to derive the quantity $H_0 \cdot r_d$. The latter is used as one parameter with a large prior so that it does not skew our results.

On Fig. 2 one can see excerpts from the posteriror distribution we obtain. Again, the full results can be found in [10], here we summarize the most important points.

- We are able to constrain Ω_m very well with this approach. Under a rather large prior, it is fitted to be $\Omega_m = 0.334 \pm 0.016$.
- The w_0 DE parameter is constrained to be $w < -1$ ($w_0 = [-1.034 \pm 0.16, -1.143 \pm 0.161, -1.14 \pm 0.154$ for Linear, CPL and Log respectively).
- The w_a DE parameter is not very well constrained: ($w_a = [-0.208 \pm 0.192, 0.056 \pm 0.393, -0.027 \pm 0.247]$ for Linear, CPL and Log respectively). We see that the error here is significant. This is not a feature only of this method, but it has been already observed in other studies.
- The curvature of the Universe is again negative ($\Omega_k = -0.028 \pm 0.052$) but this time, the error is much bigger, including the flat universe in its 68% CL.
- Statistically the best model in AIC and BIC is LCDM, while for DIC there is an almost weak evidence that the Linear model is best, while in BF, there is inconclusive evidence for the Log model. In any case, the evidence against LCDM, when it exists, is very weak confirming that LCDM is the best model.

From this study, one can see that one can avoid the degeneracy between H_0 and r_d on the price of increase in the errors of the inferred quantities. The advantage is that one can use this method as an additional tool to study different models and to check if they have some inherent biases.

5. Conclusion

The $H_0 - r_d$ degeneracy poses a lot of questions in front of modern cosmology. Usually, it is solved by either placing a prior on H_0 or on r_d. In series of works, we look for a way to disentangle it without the use of further assumptions. Here we summarize two of our works. In the first one, we choose r_d as a free parameter but include a number of additional datasets (SnIa, quasars, CC, GRBs) to help constraining it. We do it for 3 different models: ΛCDM, Ω_kCDM and wCDM, for which we perform statistical tests and see that ΛCDM is the best model for these datasets.

In the second article, we use the combined $H_0 r_d$ parameter instead of priors on each parameter. Then we use a new BAO dataset for which the use of the combined parameter is possible, plus the CMB distance prior and we add to the 4 standard models, the 3 parametrizations of the wwaCDM. While this approach is not new, this combination of datasets and models has not been used before. Our conclusions are that in this case, ΛCDM is still the best model but with signs of small preferences towards some DE models. Also in both papers, we see evidences of a closed universe, more significant in the first paper than in the second. It seems that the BAO datasets demonstrate a consistent signs of non-flat Universe and some statistically small to negligible preference for some DE models. Our results show that the BAO data needs to be improved before it can challenge statistically ΛCDM but yet it leaves some doors open for new DE models.

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