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Five questions about higher-spin holography in de Sitter space

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These notes reflect the contents of a discussion session on higher-spin dS/CFT, which was led by the author as part of the Corfu Summer Institute 2021 workshop "Quantum Features in a de Sitter Universe". I present and motivate some basic outstanding questions within the subject, and offer tentative answers. The presentation is heavily biased by my own work, which emphasizes the importance of extracting physics inside observable regions of de Sitter space, i.e. inside cosmological horizons. One may compare & contrast with my earlier views, expressed in arXiv:1710.05682. Since the writing of that document, I've reversed my position on the finiteness of the Hilbert space dimension, and become disillusioned with elliptic de Sitter space $dS/\mathbb{Z}_2$ as a promising direction of study. These are replaced by a focus on scattering amplitudes in the static patch.

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1. Introduction: why higher-spin dS/CFT

1.1 Hardships of $\Lambda > 0$ quantum gravity

A fundamental open problem in theoretical physics is to find a theory of quantum gravity that satisfies two basic macroscopic features of our Universe: $D = 4$ spacetime dimensions, and a positive cosmological constant $\Lambda > 0$. Within string theory, a possible solution is offered by the KKLT construction [1] and its cousins; however, their legitimacy is debated [2], and at any rate they appear dauntingly complicated to work with. In particular, a recurring issue with $\Lambda > 0$ quantum gravity in the abstract, and with string-theory models concretely, is that the simplest $\Lambda > 0$ spacetime – pure de Sitter space – appears to be unstable. All this is in contrast with AdS/CFT constructions [3], which are string theory's greatest success at complete & well-defined models of quantum gravity. Of course, these have the wrong sign of Λ, and 10 or 11 large spacetime dimensions.

Apart from the difficulty of formulating a model, there also seem to be deeper conceptual problems with $\Lambda > 0$ quantum gravity. For decades, string theory has effectively evaded the problems with quantum-fluctuating geometry by posing its questions at spatial (or lightlike) conformal infinity in asymptotically AdS (or flat) spacetimes. In asymptotically de Sitter spacetime, i.e. with $\Lambda > 0$, this is no longer possible. While such spacetimes still have a conformal boundary, this boundary isn't a physically accessible "place" at spatial infinity, but an *unobservable global time slice in the infinite future*. All observations are instead confined by cosmological horizons, within spatial boundaries of finite area. This reignites all the conceptual difficulties with quantum gravity: how to build an operator algebra and Hilbert space without the rigid scaffolding of a known geometry? How to construct commutation relations, when the Lorentzian causal structure is part of the quantum-fluctuating degrees of freedom? Or, in a more modern formulation (often voiced by Nima Arkani-Hamed): how to make sense of quantum probabilities, if *even the observer* is confined to a finite spatial region with its associated Bekenstein information bound, and thus can't record the outcomes of arbitrarily many repetitions of an experiment? There seems to be a link between the instability of quantum de Sitter space and this apparent incompleteness of quantum mechanics inside it. Should quantum mechanics itself be replaced by a more complete theory? As a personal speculation, I believe that this radical possibility is correct. By themselves, though, such beliefs are of little use. Is there some more "conservative" route to make progress?

1.2 The higher-spin model

In such a situation, one looks for a new deal to strike with the devil. Is there some working model of quantum gravity outside string theory, which may sacrifice some other aspects of real-world physics, but keep the real-world values of the spacetime dimension and cosmological constant? In [4], it was noticed that precisely such a model exists. It involves Vasiliev's theory of higher-spin (HS) gravity [5, 6] – the interacting theory of an infinite multiplet of gauge fields with increasing spins (one field for each spin, in the simplest version). This theory can be thought of as either a larger sibling of supergravity, or a smaller sibling of string theory. It was proposed in [7] that higher-spin theory in AdS_4 is holographically dual to a vector model on its 3d boundary. The remarkable observation of [4] is that the simplest case of this duality – between minimal type-A higher-spin theory and an $O(N)$ vector model of free[1] spin-0 fields – survives the analytic continuation from $\Lambda < 0$ to $\Lambda > 0$ with no immediately obvious pathology. In particular, the bulk fields all maintain positive norm, and the conformal weights of boundary operators remain real. This theory thus constitutes a working model of holographic duality in de Sitter space, i.e. of dS/CFT, in the real-world bulk spacetime dimension $D = 4$. It must be noted that the analytic continuation in [4] involves a violation of spin-statistics in the boundary theory: the vector model's fundamental spin-0 fields become *anticommuting* (the composite operators dual to bulk HS fields are quadratic in these, so they remain commuting). This violation is regarded by some as a deal-breaker, but we consider that an unjustified prejudice. Spin-statistics is a requirement for unitary Lorentzian theories, which is inherited by their Euclidean versions. However, the Euclidean CFT on the conformal boundary of de Sitter space is never intended to be continued into Lorentzian signature: in de Sitter, it is the bulk that is Lorentzian, while the boundary is intrinsically spacelike. There is no reason to expect that spin-statistics on the Euclidean boundary should be a consistency requirement for the Lorentzian bulk.

To dampen the excitement, we should make clear precisely why the higher-spin dS/CFT of [4] is a deal with the devil. String theory may have its discontents among the patriots of real-world physics, but it's uncontroversially *a theory of quantum gravity*: it is a quantum theory that reproduces General Relativity (more precisely, supergravity) in an appropriate limit. This limit has to do with a separation of scales: at distances much larger than the string length, the higher string modes become very massive, and only supergravity remains. In contrast, in Vasiliev's HS gravity, we have infinitely many fields that are *all massless*, with no obvious separation of scales that could leave us with just the spin-2 graviton and lower-spin matter. Thus, while this *is* a theory with a massless spin-2 field, that field isn't singled out like it is in GR, and its interactions are inevitably different. Intriguingly enough, there does appear to be a way around this issue: an appropriately extended HS theory (with supersymmetry and a color group) appears to be holographically dual [8] to ABJ theory [9], just like string/M-theory. Thus, this extended version of HS theory *does* reduce to supergravity in an appropriate regime, through some strong-coupling effects. Unfortunately, this is of no immediate help for the dS/CFT duality of [4]: there, it's important that the HS theory is in its minimal form, and in particular that it's *bosonic*, with no supersymmetry. This minimal HS theory does *not* reduce to General Relativity. Thus, the dS/CFT model of [4] is only a "quantum sort-of-gravity" with $\Lambda > 0$. While this is obviously bad news for real-world application, it also makes for a more tractable model, as we will argue in section 3.1.

1.3 Focus on the de Sitter static patch

As argued above, higher-spin dS/CFT is an intriguing and valuable toy model for $\Lambda > 0$ quantum gravity. A small dedicated group of researchers has worked on it continuously since its inception – see e.g. [10–12]. However, this has largely been in the context of a particular conceptual framework for dS/CFT. In this framework (originated by Maldacena [13]), the CFT partition function is interpreted as the Hartle-Hawking wavefunction of the de Sitter universe, evaluated on the global time slice at future infinity $\mathcal{I}^+$. This is perfectly well-motivated, and has

[1]The analytic continuation of [4] works also for an *interacting* $O(N)$ vector model of spin-0 fields on the boundary. For simplicity, we will stick to the free boundary theory. The interacting one can be obtained from it by a Legendre transform [7].

clear application in the context of inflation, where we assume a *temporary, approximate* de Sitter phase in the past, whose would-be $\mathcal{I}^+$ is now observable to us through e.g. the Cosmic Microwave Background. However, this framework is inadequate for describing physics in an actual $\Lambda > 0$ universe. In fact, it misses the key feature of such a universe: that only finite regions of space are observable. In particular, a de Sitter observer can only see *a single point* of $\mathcal{I}^+$, namely the asymptotic endpoint of her future horizon.

Thus, the focus of my own research program (which heavily biases the present document) is on identifying and extracting *physics inside observable regions* within the higher-spin dS/CFT model. Specifically, we'll focus on *pure 4-dimensional de Sitter space dS_4*, and the largest observable region within it, namely the so-called *static patch*. This is the region bounded by a pair of cosmological horizons: a *past horizon*, i.e. the future lightcone of some point on the past conformal boundary $\mathcal{I}^-$, and a *future horizon*, i.e. the past lightcone of some point on the future conformal boundary $\mathcal{I}^+$. The boundary endpoints of the two horizons can be thought of the initial & final endpoints of the observer's worldline. For a more detailed discussion of this causal structure, including why an "observable region" should care about future lightcones at all, see e.g. section 2B of [14].

There is much more to say on the subject of higher-spin dS/CFT, and many possible objections to our introduction above. However, we now switch to the question-and-answer format of the presentation at the Corfu 2021 workshop. The patient reader may find that some of her concerns are eventually addressed below.

2. Questions

The following is a list of questions regarding higher-spin dS/CFT, the answers to which are either unknown, uncertain, or simply not common knowledge. In this section, we state and motivate the questions. Tentative answers will be given in section 3.

2.1 Is the static-patch Hilbert space finite-dimensional?

In general, one expects that the Bekenstein-Hawking entropy of the de Sitter horizon should set a finite dimension for the Hilbert space in its interior. As discussed in section 1.1, this expectation touches on the most fundamental difficulties with $\Lambda > 0$ quantum gravity. However, it is really an expectation for *General Relativity and sufficiently similar theories*. In 3 spacetime dimensions, higher-spin theory *is* rather similar to GR – both are essentially Chern-Simons theories. Accordingly, BTZ black holes in 3d HS theory seem to respect a suitable generalization of the Bekenstein-Hawking formula – see e.g. [18]. What should we expect in 4 spacetime dimensions?

2.2 What's the appropriate observable in the de Sitter static patch?

In asymptotically flat spacetime, the well-defined observable for (perturbative) quantum gravity is the S-matrix of scattering amplitudes. In asymptotically AdS, it's the partition function, or correlators, on the boundary. For de Sitter space, it has been argued [15] that the only "observable" is just the Hilbert space. The $T\bar{T} + \lambda$ literature on dS_3/CFT_2 (see e.g. [16]) effectively sets as its observable the spectrum of states, i.e. the Hilbert space + Hamiltonian. The present author has previously leaned in a similar direction [14, 17]. On the other hand, as discussed in section 1.3, the inflation-inspired literature (which includes the main literature on the higher-spin model) speaks instead about the Hartle-Hawking state. Given this proliferation of partially related options/opinions, *what is the correct observable for quantum HS gravity inside a cosmological horizon in dS_4?*

2.3 How to jump from the conformal boundary to the cosmological horizon?

This question simply makes explicit the challenge of extracting physics inside a cosmological horizon out of dS/CFT. In holography as usually understood, the boundary theory is a CFT on the conformal boundary of spacetime. For de Sitter space, this is the global time slice in the infinite future $\mathcal{I}^+$ (and/or the infinite past $\mathcal{I}^-$). Maldacena's framework [13] for dS/CFT equates the CFT partition function with the Hartle-Hawking wavefunction on this boundary. Our ambition, on the other hand, is to tackle physics in an observable region, whose boundary is not $\mathcal{I}^\pm$, but rather a pair of cosmological horizons. Thus, we are faced with a *highly non-local task*: how to translate degrees of freedom and other quantities (such as the CFT partition function) between the unobservable conformal boundary and a cosmological horizon? There is a sense that this translation should occur *in one go*: though one can interpolate between $\mathcal{I}^\pm$ and the horizon with spacelike bulk hypersurfaces, this doesn't seem like a good idea. Indeed, $\mathcal{I}^\pm$ is special as the conformal boundary at infinity, the usual seat of a non-fluctuating geometry in holography. The horizon is more problematic, being a bulk hypersurface, but at least it's special on account of being lightlike, and anchored at a point of $\mathcal{I}^\pm$; maybe if we are smart and lucky enough, we can do quantum gravity working with such a hypersurface. But using *less special* bulk hypersurfaces as an interpolation of the two feels like a losing strategy. Is there a way, then, to jump from $\mathcal{I}^\pm$ to the horizon directly?

2.4 What about the unwanted maxima of the CFT partition function?

Our final two questions can also be classified as confessions. Our narrative so far implied that HS gravity and its dS/CFT holographic duality are both well-established, and the challenge is "merely" to learn how to extract from them some physics in observable regions of de Sitter space. In fact, the situation is more precarious.

We mentioned in section 1.2 that the authors of [4] demonstrated that higher-spin AdS/CFT can be continued to $\Lambda > 0$ "with no obvious pathologies". This statement was based on a perturbative analysis. One of the key requirements that was tested in [4] was for the CFT partition function Z to have a *maximum* at zero sources, as opposed to a minimum or a saddle point. This translates into the Hartle-Hawking wavefunction being peaked on empty de Sitter space. Technically, the requirement is for the second derivatives of $-\ln Z$ around zero to be positive-definite, which is directly related to having positive norms & kinetic energies for all the bulk fields. The important result of [4] is that this is indeed the case, provided that the spin-statistics of the boundary CFT is reversed, as discussed in section 1.2.

However, later studies [10] have probed the dS/CFT partition function beyond the perturbative regime, with unsettling results. In particular, at large values of a constant spin-0 source, Z turns out to have *higher maxima* than the perturbative one at the origin. This implies a Hartle-Hawking wavefunction *not* peaked on empty de Sitter space: a severe challenge for the health of the model. A resolution was proposed in [11], involving a number of novel elements:

1. The boundary CFT's anti-commuting scalar fields $\phi^I(\ell)$ are replaced by *commuting* fields $Q^I(\ell)$.

2. Instead of a Hartle-Hawking wavefunction given by the CFT path integral over $\phi^I(\ell)$, a Gaussian wavefunction is postulated of the form $\Psi[Q^I(\ell)] = e^{\int d^3\ell\, Q\Box Q}$, which normally would be the path integral's *integrand*.

3. Path-integral-like functional integrals over $Q^I(\ell)$ do appear, but with a different interpretation: they arise when computing expectation values for operators, which correspond (via a dictionary involving the shadow transform) to what were (in the original picture of [4]) the CFT's *sources*.

This is a bold departure from the standard prescriptions of (A)dS/CFT. Is it a necessary one? What *should* we make of the unwanted maxima in the partition function?

2.5 What is higher-spin gravity, anyway?

Having acknowledged a potential problem with higher-spin dS/CFT, we must now confront the weakness in the foundations of HS gravity itself. The theory of HS gravity was first formulated in [5], in the Vasiliev field equations. This formulation is quite indirect, as it includes dependence of the fields on certain auxiliary coordinates (above and beyond the "expected" auxiliary coordinates, which keep track of the different fields in the HS multiplet and their spacetime derivatives). To obtain the theory's physical interactions, the equations in the auxiliary space must first be solved, which in turn requires an appropriate choice of boundary conditions. For many years, a "naive" choice of these boundary conditions was assumed, which in retrospect was the least robust component of the theory. A major challenge to this original choice arose when the cubic interactions were finally extracted from the Vasiliev equations explicitly [19], and found to be non-local. This non-locality of the cubic vertices can be repaired "by hand" by an appropriate redefinition of the physical fields [20, 21]. However, it would of course be better to arrive at the "correct" physical fields in some more direct and principled way. This was finally achieved in [22], via a better choice of boundary conditions in the auxiliary space.

But the problems don't end there! Beyond the cubic vertex, the theory is expected to have vertices of every order, and extracting these from the Vasiliev equations remains a Herculean task: even the quartic vertex is yet to be extracted in full. By itself, this would be just a technical difficulty, not a problem of principle. However, in a separate (and slightly earlier) line of work, some interaction vertices have also been extracted in a different way, by reverse-engineering from holographic correlators. At the cubic level, an elegant formula for all the vertices has been found [23]. The quartic spin-0 vertex was also studied [24], with the eventual conclusion [25] that it is badly non-local. So far, this challenge to the theory's health remains unresolved.

At the very least, the non-locality result of [25] implies that the usual language of field-theoretic Feynman/Witten diagrams does not apply well to HS gravity. This is perhaps to be expected: HS gravity is, after all, a sort of intermediate between field theory and string theory. Might its locality properties become clearer in some "more stringy" framework, just as string theory is local from the worldsheet point of view, despite being non-local in the standard field-theory sense? What is the correct language for higher-spin gravity?

We contend that de Sitter researchers are in a unique position to address this question. This is because our interest in the theory isn't "purely academical". Instead, we *desperately need it for a purpose* – of getting on hands on a working model of $\Lambda > 0$ quantum gravity. This gives us a different level of motivation to *get the theory to work*, as well as a different set of perspectives from which to view it: we want the theory to answer some specific and unusual questions, which sometimes send us in a different direction from the rest of the higher-spin community. Are we up to this task? Can we, as would-be users of HS theory, help ascertain what the theory itself is?

3. Answers

In this section, we provide some tentative and heavily-biased answers to the questions above.

3.1 Is the static-patch Hilbert space finite-dimensional?

No, which makes for a crucial difference between HS theory and GR in 4 dimensions. This answer can be justified in several ways. The first is a representation-theory argument: *the higher-spin extension of the relevant spacetime symmetry group has no non-trivial finite-dimensional representations*[2]. Indeed, the spacetime symmetry of the de Sitter static patch includes the $SO(3)$ of spatial rotations. In HS theory, this $SO(3)$ will be upgraded into its higher-spin extension $\mathfrak{hs}[SO(3)]$, generated by functions $f(y)$ of a spinor variable y^α with homogeneity degrees $2(s-1)$, where $s = 2, 4, 6, \dots$ is the spin of the gauge field associated with the symmetry (in particular, $s = 2$ corresponds to the usual $SO(3)$ rotations). Now, consider a representation of $\mathfrak{hs}[SO(3)]$, and assume that $SO(3)$ acts on it non-trivially, i.e. that it includes at least one nonzero $SO(3)$ angular momentum. This is a clearly reasonable assumption: otherwise, one can't really claim that the theory lives in 3+1d spacetime. But if $SO(3)$ acts non-trivially, then so must the generators with all spins $s > 2$, because the $SO(3)$ generators appear in their commutation relations. This in turn implies that our representation of $\mathfrak{hs}[SO(3)]$ must contain an *infinite tower* of $SO(3)$ angular momenta: any would-be largest angular momentum can always be increased by acting with a sufficiently high-spin generator. This concludes the argument: barring the trivial case of zero angular momentum, higher-spin symmetry forbids a finite-dimensional Hilbert space in the de Sitter static patch.

Let us stress the implications of this claim. The existing lore on quantum gravity, holography and black holes draws a tight link between finite horizon entropy and gravity itself. A theory without a finite horizon entropy is not a theory of fluctuating geometry, i.e. of gravity in the conventional sense. On the other hand, HS gravity *is* a theory with a spin-2 gauge field, and its description via the Vasiliev equations is diffeomorphism-invariant. Can we really claim that this is not a theory of fluctuating geometry? Surprisingly, we can.

In GR, the spin-2 gauge field and diffeomorphism symmetry are tightly linked to each other. In HS gravity, that is no longer the case: the spin-2 gauge symmetry is merely one component of the HS gauge connection Ω_μ. In particular, Ω_μ contains a component $\omega_\mu^{\alpha\dot\alpha}$, which is analogous (in the linearized theory) to GR's vielbein $e_\mu^{\alpha\dot\alpha}$, and acts as a connection for "translations". However, these, just like rotations and all the other HS gauge symmetries, act only in the internal flat tangent space, and are therefore distinct from diffeomorphisms. This raises the possibility of formulating HS theory *with all the HS gauge symmetries and their associated dynamical gauge fields intact*, but on a *fixed geometry*, with no diffeomorphism symmetry. In fact, two such formulations appear to

[2]Credit for this observation belongs to Tomonori Ugajin.

exist. The first is a close variant/generalization of the Vasiliev equations [26], which lives on a fixed pure dS or AdS geometry. The second is a newer, still unpublished [27] perturbative formulation, directly in terms of the physical (Fronsdal) fields; we will discuss this new work further in section 3.5.

Taken together, the representation-theory argument and the new fixed-geometry formulations [26, 27] all suggest that HS theory is *in some sense* non-gravitational. For those wishing to work on a realistic quantum-gravity model, this should be disappointing. There is, however, a silver lining. At our current stage in theoretical physics, $\Lambda > 0$ quantum gravity can seem like an intractable pile of conceptual and technical difficulties. The "non-gravitational" nature of HS gravity means that some of these difficulties disappear, and we can focus on overcoming the rest. In particular, if the Hilbert space isn't finite-dimensional, then we are safe from the conceptual problem with Quantum Mechanics mentioned in section 1.1. Similarly, if the geometry isn't fluctuating, then the stability of de Sitter space is no longer an issue. Instead, we are left with an "easy" problem: given an exotic and still poorly-understood bulk theory in de Sitter space, and a holographic description at its unobservable, Euclidean boundary, can we extract and study some Lorentzian physics inside the observable bulk static patch?

3.2 What's the appropriate observable in the de Sitter static patch?

What, then, is the correct thing to calculate for quantum HS gravity in the static patch of de Sitter space? Previously [17], my answer would be the static-patch Hilbert space and Hamiltonian, or, in other words, the energy spectrum in the static patch. In light of the arguments of section 3.1, this no longer seems correct. If HS theory is to be thought of as a "non-gravitational" theory on a fixed de Sitter background, then its Hilbert space will be just the standard infinite-dimensional one of QFT (times an infinite spectrum of fields). In fact, by the logic of lightfront field theory [28–31], we can expect both the Hilbert space and Hamiltonian, when evaluated *on the past or future horizon*, to be *fixed kinematically*, i.e. regardless of the theory's interactions. Let us expand on this point.

In the lightfront approach, one performs an analog of canonical quantization, but on a lightlike hypersurface. Typically, this is a lightlike hyperplane in Minkowski space, but the logic generalizes straightforwardly to its de Sitter equivalent, which is precisely our case of interest – a cosmological horizon (see e.g. [32]). Remarkably, upon quantizing in this setup, one finds a universal, interaction-independent form for the operator algebra, the Hilbert space, the Hamiltonian and the vacuum wavefunction, up to some (important) subtleties regarding zero-energy modes [33, 34]. These structures are all directly equivalent to the standard ones for in/out states on the lightlike boundary of Minkowksi space. The strong claim of lightfront quantization is that they hold just as well on a *bulk* lightlike hypersurface, even though, unlike at Minkowski infinity, the fields' interactions cannot be said to vanish. At tree level, this claim is rigorously true. With loop corrections taken into account, its status is much less clear. For the special case of super-renormalizable theories, a version of it was proved in [35]. Its broader validity, e.g. for a gauge theory like QCD, is a working assumption (not rigorously justified to our knowledge) of the lightfront QCD literature [29–31]. With gravity, the situation is more complicated still, since the geometry of a bulk lightlike hypersurface becomes itself a fluctuating degree of freedom (though the condition of being lightlike does makes it "more rigid", e.g. forbidding redefinitions of the hypersurface by transverse diffeomorphisms).

PoS(CORFU2021)355

Let us now circle back to HS theory. Viewed as a "non-gravitational" theory living on a fixed geometry, we can expect it to obey the logic of lightfront quantization on a de Sitter horizon, i.e. to have the standard kinematically-fixed (and infinite-dimensional) Hilbert space & Hamiltonian, without any concerns about the horizon's geometry being dynamical or ill-defined. Now, one may still worry, as in ordinary QFT, that the simple picture of lightfront quantization may break down due to loop corrections. We speculate that there will be no such breakdown. This is based on a general expectation that HS theory should be protected from loop corrections by its enormous HS symmetry. For example, the Maldacena-Zhiboyedov theorem [36] shows that a global HS symmetry completely fixes the boundary correlators to be those of a free CFT, as in [7]. In particular, the correlators are protected from any $1/N$ corrections, which, via the AdS/CFT dictionary, implies the vanishing of loop corrections in the bulk theory. The question of the Hilbert space & Hamiltonian on the de Sitter horizon is of course different from that of boundary correlators, but nevertheless we expect that it too should be safe from loop corrections.

Thus, we have argued that HS theory's operators and states in the de Sitter static patch, *when evaluated on its past or future horizon*, are the same as for free HS fields – a similar situation to field theory or gravity in asymptotically Minkowski space, but with de Sitter horizons playing the role of past/future lightlike infinity. This suggests that the appropriate observables for the theory are *scattering amplitudes*, i.e. the mapping coefficients between states (or field operators) on the static patch's past horizon to those on its future horizon. Note that the subject of scattering amplitudes in the de Sitter static patch can be defined and studied in its own right, quite separately from the HS model. Until recently, it was essentially unexplored terrain. Some inroads have now been made: an analysis of free massless fields of all spins [37], of the 3-point amplitude for a conformally massless scalar [38], and of all tree amplitudes in Yang-Mills theory [39] (including analogues of the Parke-Taylor formula [40] and BCFW recursion [41]). These are all bulk calculations. The ambition is to work our way up to HS gravity, and to make contact between its holographic dS/CFT description and the static-patch scattering amplitudes.

3.3 How to jump from the conformal boundary to the cosmological horizon?

This is a major challenge, and the answer is at best tentative. As discussed in [14, 42], it may be helpful to think in terms of so-called elliptic de Sitter space $dS_4/\mathbb{Z}_2$, i.e. de Sitter space with antipodal points identified [43]. However, the focus of those works was on the Hilbert space and Hamiltonian. Their relevance to the static-patch scattering problem is unclear.

The one concrete direction we can point to is a particular *choice of variables* for higher-spin dS/CFT. From its inception, HS theory has had a peculiar and fluid relationship with spacetime and locality. The Vasiliev equations themselves are written in so-called "unfolded" language, in which the fields' entire spacetime dependence is encoded *at every spacetime point*, as a tower of derivatives arranged into a "master field". A similar master-field formalism exists for the boundary CFT [44, 45], and the relationship between these bulk and boundary langauges has been discussed in [46]. For the boundary CFT and for the *linearized* bulk theory, there is also a twistor-space formalism based on the Penrose transform and its "holographic dual" [47], where the degrees of freedom are encoded in twistor functions, not tied to any bulk or boundary points at all.

This ability to switch from fields in spacetime to "master fields" at single points opens an interesting possibility for our dS/CFT conundrum, of how to "jump" from the unobservable conformal boundary onto the horizons of the static patch. Instead of "jumping" anywhere, we can just place all the degrees of freedom *at the intersection point* between the boundary and one of the horizons, i.e. at the past or future endpoint of the de Sitter observer's worldline. This idea was studied from different perspectives in [37, 42, 48]. The upshot is that one can encode the degrees of freedom in a *function of two spinor variables*, defined at the intersection point of boundary and horizon. This spinor function can be viewed equivalently as:

- The master field of CFT currents, as in [44], evaluated at the intersection point.
- The boundary limit of Vasiliev's linearized bulk master field, as in [46].
- The spinor-helicity description of initial/final states on the cosmological horizon [37].
- Matrix elements in the quantum mechanics of a free particle in the boundary CFT [42].
- Elements of HS algebra, in a basis where its associative $\star$-product is just a matrix product [49].

Thus, these variables make for a promising framework, serving as a natural encoding for both CFT and horizon data. So far, they were employed in an elegant solution of the static-patch scattering problem for *free* massless fields [37]. We expect them to be useful at the interacting level as well.

3.4 What about the unwanted maxima of the CFT partition function?

The above choice of variables for higher-spin dS/CFT turns out to be relevant [48] for the problem of the partition function's maxima at large source values, originally raised in [10]. As hinted in section 3.3, there's an entire small zoo of formalisms for HS holography in general, and for the boundary theory in particular. Within this zoo, one can distinguish between three categories: local, bilocal and spinorial. In a local formalism, we work with functions of a boundary point (sometimes with extra parameters, to encode the different spins and their polarizations); in a bilocal formalism [49–51], we work with functions of *two* boundary points; in a spinorial formalism, we work with functions of only spinor variables (which live at a *fixed* boundary point), or twistor variables (which don't refer to boundary points at all). The standard (A)dS/CFT dictionary operates in a *local* formalism, while the one advocated in section 3.3 is *spinorial*.

The surprising claim of [48] is that, while the formalisms all agree on the values of boundary correlators at separated points, they *don't* agree on the value of the partition function Z at finite sources. There is much to unpack in this statement. Naively, the value of Z can be deduced from the correlators by simply integrating them against the chosen source configuration. This naive procedure can fail due to "contact corrections", i.e. non-linear corrections to the coupling of the theory's operators to its sources, which only manifest when the spacetime positions of a correlator's arguments coincide. In our context of HS holography, such contact corrections indeed arise for all the sources with nonzero spin. In fact, for general HS sources, these corrections are not known. However, for the simple case of a spin-0 source, contact corrections are absent, so the partition function can be deduced by simply integrating the correlators. More precisely, this procedure yields the *local-formalism version* of the partition function. Now, the claim of [48] is that even in this simple case, where contact corrections don't play a role, the partition function in the *spinorial* formalism is different. In particular, for a constant spin-0 source σ on the S_3 boundary of de Sitter space, the two partition functions read [10, 48]:

$$Z_{\text{local}}(\sigma) = \exp\left(\frac{N\pi}{16}\int_1^{\sqrt{1+4\sigma}} t^2 \cot\frac{\pi t}{2}\,dt\right) ; \tag{1}$$

$$Z_{\text{spinorial}}(\sigma) = \frac{1}{(1+\pi^2\sigma^2)^{N/16}} . \tag{2}$$

These agree at the first two orders in σ, i.e. $Z = 1 - \frac{N\pi^2}{16}\sigma^2 + \dots$, but then disagree at order σ^3 and higher.

How is this possible? To understand what's happening, let us switch temporarily from Euclidean boundary signature (as in dS/CFT, or Euclidean AdS/CFT), to a Lorentzian one (as in Lorentzian AdS/CFT). Then the key observation is that the boundary CFT's fundamental fields are *on-shell* in the spinorial language, but *off-shell* in the bilocal and local ones. As a consequence, in this signature, *already the correlators at separated points* in the spinorial formalism differ from the standard ones in AdS/CFT. Specifically, the former are *Wightman functions*, constructed from on-shell propagators whose form in momentum space is $\sim \delta(p^2)$, while the latter are *time-ordered correlators*, constructed from off-shell propagators $\sim 1/p^2$. In coordinate space, these only differ by sign factors, causing the HS literature to sometimes overlook the distinction; for a careful treatment of the boundary theory taking all such distinctions into account, see [45].

Back in Euclidean signature, things take a subtle turn. On a Euclidean boundary, on-shell *states* of the CFT's particles no longer exist, unless we allow complex momenta. However, real/Hermitian *operators* on such states can still be defined, and it is these objects that describe the boundary operators and bulk fields of HS holography in the spinorial language. Nevertheless, the complexified nature of the on-shell phase space still makes itself known, by making the spinor/twistor variables of HS algebra also complex. This introduces contour ambiguities into the HS-algebra products that are used [52] to compute correlators in the spinorial language. In practice, these end up as sign ambiguities, and the spinorial-language correlators end up agreeing with the local-language ones *up to signs*, much like in the Lorentzian case. The difference is that these signs are now no longer fixed by a Lorentzian causal structure.

This is where the situation gets interesting. On one hand, the sign ambiguities in the spinorial-language correlators *can* be consistently fixed. The correlators then agree with the ones in the local formalism. Roughly speaking, they have no choice, because Euclidean signature knows only one "ordering prescription". However, this sign-fixing procedure for the correlators at separated points *can't be extended to arbitrary linear superpositions*, i.e. to arbitrary spinor/twistor functions, which encode finite boundary sources. To be convinced of this, one can notice that the sign-fixing violates certain discrete symmetries of HS algebra [48]. It is this conflict between the sign-fixing for correlators and the superposition principle (or, more poetically, between locality and HS symmetry) that allows for different partition functions as in (1)-(2).

The upshot of this winding story is as follows. The spinor variables that we advertised in section 3.3 lead to a different partition function for higher-spin dS/CFT. Unlike in the local formalism, this partition function can evaluated in closed form for arbitrary sources [48]. As a special case, the partition function for a constant scalar source on the boundary 3-sphere is given by (2), instead of the local-formalism result (1). In addition to its simplicity, the spinorial-formalism partition function (2) has a particular advantage: it clearly has a global maximum at $\sigma = 0$, without the problematic higher maxima at $\sigma \neq 0$ which were noted in [10] for the local-formalism result (1). This becomes yet another argument in favor of the spinor variables, and provides an alternative to the re-imagining of the CFT path integral proposed in [11].

3.5 What is higher-spin gravity, anyway?

We now turn to the final question: can we, as dS/CFT enthusiasts, help understand higher-spin gravity in general, and its quartic non-locality issue in particular? We claim that the answer is yes. As should be clear from these notes so far, the problem of dS/CFT forces a different perspective on HS holography. On one hand, since there's a difficult problem to solve, one becomes acutely interested in the structure of the HS partition function in different formalisms, hunting for the most efficient ways to write and manipulate it. On the other hand, one becomes less attached to the traditional local language on the boundary: since only 1 or 2 points of the conformal boundary are observable, locality on it feels rather dispensable. In sections 3.3-3.4 above, we discussed some implications of working with a spinorial formalism instead of a local one. We also mentioned in passing the third possibility: working with *boundary bilocals*. In this final section, we'll get a glimpse of the power of this third formalism. We will be brief, since the relevant results are the subject of an upcoming paper [27].

First, a quick detour into string theory. String theory can be expressed in the language of local fields (i.e. the modes of the string), but this description becomes non-local once we approach the string scale (much like HS theory, which is however nonlocal at *all* scales). In the string case, the cure for this is known: we must switch from spacetime fields to the language of the worldsheet. This has many consequences, of which we'll focus on the following:

1. There are no longer any interaction vertices, or off-shell propagators: interactions are described by the same action as the free on-shell string.

2. The fields in the string spectrum can be thought of as a perturbative expansion with respect to the string length. The string itself is then a "non-perturbative completion" of this tower of fields.

3. Similarly, in the holographic CFT picture, the string becomes a Wilson loop [53, 54] – a spatially extended "non-perturbative completion" to the spectrum of local single-trace operators.

4. At the same time, the string can also be discovered from the field language (specifically, from the supergravity limit) as a BPS black hole solution – the F1-brane [55].

Our claim is that the boundary-bilocal language in HS holography can play a role analogous to that of worldsheet language in string theory. The boundary bilocal itself is HS holography's analogue of the Wilson loop: it is the non-local object that contains and "completes" the tower of local HS currents in the boundary theory. In [47], the bulk dual of this object was described, as a certain linearized HS field solution. It was then noticed in [56] that this bulk solution is just the linearized version [57] of the Didenko-Vasiliev black hole [58] – the original "BPS solution" of HS gravity.

This leads us to propose a new formulation [27] of HS gravity, with diagrammatic rules that partly mimic those of perturbative string theory. In this formulation, there are only cubic interaction vertices. In the diagrams' internal legs, instead of general off-shell HS fields as usual, we propose a projection onto the space spanned by the linearized HS fields [47, 57] of boundary bilocals. This mimics string theory's use of "only on-shell propagators", with the meaning of "on-shell" extended to include not just the spectrum of on-shell fields, but also its "completion" into the spatially extended string.

Our claim is that this framework can overcome the non-locality problem [25] of the spin-0 quartic vertex. Having conveyed some of the excitement, we defer to [27] for details. Ask not what HS gravity can do for dS/CFT. Ask also what dS/CFT can do for HS gravity!

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