PROCEEDINGS OF SCIENCE

# To the memory of our friend
# JOHN ANDREW MADORE

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## Harald Grosse

### 1.1 Overview

Dear all,
we were all shocked, when we learnt that our friend John Madore died August last year. It is a very good idea of the organizers of this Workshop on Noncommutative Geometry to dedicate it to the memory of John. As an old friend of him, it is an honour to me, to make remarks about the live and work of John. One can divide his developments into Early Years from 1970 to 1988. A second period from 1988 to 1991: can be called From Geometry to Noncommutative Geometry, were he used Matrix Geometry and applied it to Particle Physics and studied Space-Time Geometry questions, and a third period started in 1991, which can be called Fuzzy Physics.- Some Memories will be given at the end.

### 1.2 1938-1991

John was born in 1938 in Saskatoon, Canada. He studied in Toronto (from 56-62) (at the time when Coxeter was teaching there) and went to France and Germany (63-64) (in Hamburg he interacted with Harry Lehmann). In 1969 he presented his PhD thesis on gravitational waves under the advice of Papapetrou. From 69 to 88 he went from Marseille to Brandeis and to Toronto; back in France he was employed at the Ecole Polytechnique and went finally to ORSAY. During this period he was contributing each year to a different subject as can be seen by looking to the shortened titles of publications: Gravity waves, Neutrino in GR, YM field, Monopoles, $\sigma$- model, Einstein+YM+spin3/2, Kaluza-Klein, Lax pair, Twistors, Conservation laws, Friedman universe, Cosmological model, Spin, ... all these subject somewhat related to GR, were on his agenda.-
In 1988 M. Dubois-Violette, R. Kerner and John Madore published a remarkable article on: ”Classical Bosons in Noncommutative Geometry” were they applied ”Noncommutative Differential Geometry of Matrix Algebras” to the electroweak particle model. This period ended with a seminal paper in 1991 on ”The Fuzzy Sphere”, which led to a number of developments. By chance I met John in 1991 and learnt from him about the Fuzzy Sphere. Within a few days we worked out the application of this algebra deformation to regularize the 2 dimensional Schwinger model. A long lasting friendship started.

### 1.3 Quantum Field theory - General Relativity

In extreme situations, like formation of black holes and the early universe, both mentioned subjects, Quantum Field Theory and General Relativity should merge. To cure problems of Quantum Field Theory, like UV, IR or the Landau ghost problem, respectively to cure problems of summation of the renormalized perturbation theory, one believes that inclusion of gravity effects might help. One step in this direction is the use of noncommutative geometry methods. General Relativity by itself is of course nonrenormalizable and a full quantum gravity is not yet formulated.



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### 1.4 Space-Time Geometry

From a physics point of view it is very natural to introduce a limit on the localization in space-time of the order of the Planck length. The formulation of the mentioned Matrix Geometry led John to the formulation of the Fuzzy Sphere Geometry. He used the tower of $SU(2)$ representations to get a noncommutative approximation of the algebra of functions over the sphere. When I learnt it from John, I was fascinated and we applied it to regularize the simplest QFT models. One way to formulate a QFT uses a generating function. For the action an integration over a manifold, fields and a differential calculus are needed. All this can be formulated for a noncommutative algebra resulting from a deformation of the commutative algebra of functions over a manifold. Fields become elements of a projective modul. The last step concerns the integration over "all degrees of freedom", which needs regularization and renormalization in the commutative setting. On a finite matrix algebra, of course, an interesting cut-off shows up.

### 1.5 Fuzzy Physics

A Quantum Field Theory needs first a regularization, for two Euclidean dimensions the Fuzzy Sphere of John Madore serves the purpose. One starts from three generators forming an $su(2)$ algebra, where the squares of the generators sum up to the Radius squared. Now one considers the tower of representations of $su(2)$ by $N \times N$ matrices and defines an embedding of matrix algebras respecting the definition of the sphere. In an appropriate limit one obtains the commutative algebra of functions over the sphere. It is remarkable that there exists also a differential calculus, covariant derivatives can be defined, etc.
For all manifolds, which are coadjoint orbits such a construction is possible.

### 1.6 Developments 1991-2000

John worked with many people: He has publications with Maja Burić 21, George Zoupanos 14, Jihad Mourad 11, Gaetano Fiore 9, Michel Dubois-Violette 8, Harold Steinacker 7, Harald Grosse 6, Bianca Letizia Cerchiai 6, Marco Maceda 6, Richard Kerner 5,..........
The subjects John treated in the following years are similar to the previous ones, but now a noncommutative version, extension, deformation is given. Typical subjects were: 91 noncommutative Schwinger model, 92 noncommutative extension of electrodynamics, 93 almost commutative geometry, 94 noncommutative extension of gravity, 95 linear connection and curvature on quantum plane, 95 definition of differential calculus, 96 noncommutative Kaluza-Klein, 96 classical gravity + fuzzy space-time, 96 noncommutative geometry and gravity, 97 noncommutative Riemannian geometry, 98 Poisson structure and curvature, 98 $q$-deformed phase-space, 99 finite field theory, 99 hidden symmetry of deformed space-time, 2000 gauge theory on noncommutative space...
In 2000 the second edition of John´s book appeared, which summarizes his work in a remarkable simple but nevertheless mathematical rigorous style. As for the years after 2000 Maja Burić will give more details.



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### 1.7 Memories

I remember meeting John at so many places, of course, quite often in Orsay but also quite often in Munich (with Julius Wess). John liked CORFU (with George Zoupanos), he regularly attended Bayrisch-Zell, and of course, we met often in Vienna too, ... And looking to my interactions with John, I always enjoyed them, we all enjoyed his humor; there are jokes, which we remember very well,...
In summary, we lost an eminent mathematical physicist and we lost a dear friend! We miss you, John!

## Maja Burić

### 2.1 Overview

I met John ten years later than Harald, in 2001, when much of the subject of noncommutative geometry was layed out. When I heard his talk at the Sokobanja conference I was very excited about the prospect to define noncommutative gravity as in GR – through geometry. From there our association gradually evolved. John was very enthusiastic and very easy to collaborate with – he would drag you immediately to calculations while general and physics discussions were done along.

Looking at his INSPIRE record and from coversations and work with him, I figure that he was one of rare physicists of his generation with strong backgrounds both in general relativity and particle physics. He worked on various aspects of Einstein's and other, alternative theories on gravity as well as on chiral symmetry and $\sigma$-models. Very important and unifying mahematical aspect of his work is differential geometry, applied in particular to gauge theories and general relativity. His view on the subject was summarized in Physics Reports paper *"Geometric Methods in Classical Field Theory"* (1981).

John published many papers (two of them 500+ cited) and *"An introduction to noncommutative differential geometry and its physical applications"* (CUP, first ed. 1995, second ed. 1999): in this book his proposal for noncommutative differential geometry, the noncommutative frame formalism, was developed. To the list that Harald discussed I would add two less known papers, important to his ideas and views on noncommutative geometry. One is *"Kaluza-Klein aspects of noncommutative geometry"*, talk given at the Chester conference (1988), where he contributed to observation (very important to the noncommutativity-community at the time) that one can use external noncommutative dimensions generated by finite matrices to blow up classical singularities and build models of the Kaluza-Klein type (see J. Madore's introductory text in this volume). Another paper, *"Conservation Laws and Integrability Conditions for Gravitational and Yang-Mills Field Equations",* written with M. Dubois-Violette (1987), was in part in basis of his belief that Jacobi constraints in the frame formalism, that is integrability conditions, can replace the usual equations of motion.

In Orsay in early 90's John developed and studied mathematical aspects of differential geometry (linear connections, curvature) on noncommutative spaces. Due to Julius Wess, Munich became in the 90's a big research center for noncommutativity: John was its important member. His often visits to LMU directed to some extent his work to more concrete physical questions. Julius with John created a fruitful environment for discussion, work and development of noncommutative field theory and noncommutative gravity. Many papers were done together and separately, and many lines of investigation have been initiated.

### 2.2 After 2000

In years after 2000, John's work was done in two main directions: gauge theories and particle physics models on noncommutative spaces and noncommutative gravity. Among results of the first group one should certainly mention paper *"Gauge theory on noncommutative spaces"* (2000), written with S. Schraml, P. Schupp and J. Wess, where covariant coordinates (or covariant momenta, as John would have it) were introduced. Other results address classical field theories and their quantization on fuzzy spaces (with Grosse, Steinacker) as well as particle-physics models defined on fuzzy coset spaces or spaces with fuzzy extra dimensions (Aschieri, Steinacker, Zoupanos).

In exploration of noncommutative geometry the main results were on the $q$-deformed Heisenberg and Lobachevsky planes and on quantum Euclidean spaces (with Cherchiai, Fiore). Finally, applications of the noncommutative frame formalism to gravity included some general results about the resolution of space-time singularities, on the semiclassical expansion, on Poisson-structure energy-momentum as well as results on gravitational waves and on cosmological – Kasner, FLRW, de Sitter fuzzy models (with Maceda, Zoupanos, Burić). One of John's unfinished projects (done in part with me) was to find noncommutative extensions of spherically symmetric gravity configurations in four dimensions, in particular, the fuzzy Schwarzschild black hole. John believed that noncommutative frame formalism had enough constraints to make solution to this problem unique or almost so; furthermore as mentioned, he hoped that Jacobi identities, that is associativity, will be an equivalent of the equations of motion. Both issues are still unsolved.

### 2.3 Noncommutative frames

John's most important achievement, in his own opinion, was the noncommutative frame formalism. It goes a long way, from the basic definitions of differential and differential forms to the full geometry including the metric and curvature and description of fields. The main idea of the formalism is simple and strong: a noncommutative space, that is an algebra $\mathcal{A}$, has two basic structures:

∘ the algebraic structure which defines the position algebra, that is commutation relations between coordinates $x^\mu$, $[x^\mu, x^\nu] = i\bar{k} J^{\mu\nu}(x)$ , and

∘ the geometric structure given by the momentum algebra,
$[p_\alpha, p_\beta] = K_{\alpha\beta} + p_\gamma F^\gamma{}_{\alpha\beta} - 2 p_\gamma p_\delta Q^{\gamma\delta}{}_{\alpha\beta}$

∘ two structures are related through gravity, that is, by a generalized version of the correspondence principle, $[p_\alpha, x^\mu] = e^\mu_\alpha(x)$ ;

∘ compatibility of the structures constrains possible, that is, consistently defined noncommutative spaces.

The formalism has numerous advantages: it is defined generally and not example by example (as many noncommutative differential calculi are); it includes as particular cases commutative manifolds, matrix spaces and operator algebras; it gives the accepted geometric description of important spaces like the fuzzy sphere and the Heisenberg algebra. Furthermore, since it is defined in analogy with the moving frame formalism of Cartan, it is adjusted to describe gravity. But whether it is perhaps too constrained to cover all physically relevant configurations of the gravitational field, is a question that only future research will answer.

### 2.4 Memories

John was very nice, open and sincere person. He made good friends with many of his collaborators not only through work but also in discussions on history, philosophy, language & literature; not to mention his sense of humor and numerous jokes. On one of his visits to Athens he wanted to do a dedication to his old friend George Zoupanos, and started his seminar with the limerick:

*There was an old Prof from Corfu*
*Who used strings to tie up his shoe*
*'Till a nut from Orsay*
*Told him that was OK*
*But noncommutativity would do.*

I was there present and thought that John deserved a limerick too, so this was my attempt:

*There was an old Prof from Orsay*
*Complained about star products all day*
*Yet a scheme made he*
*To frame gravity*
*In a noncommutative way!*