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Exotic heavy hadrons with a three-body nature

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In this talk we present a summary of our latest results on the investigation of three-body systems with explicit/hidden charm and with explicit/hidden strangeness. To be more concrete, in case of systems with explicit strangeness, we pay attention to: (1) KDD and (2) $KD\bar{D}^*$. In the first system, a charm +2, isospin $1/2$ and strangeness +1 state is obtained with a mass around 4140 MeV, while in the second one, a K^* state, with hidden charm, and a mass close to 4307 MeV is found. We have also studied a system of mesons with hidden strangeness, $DK\bar{K}$, and a system with a baryon and hidden charm, zero strangeness, $ND\bar{D}^*$. In the former case a D-meson with mass around 2900 MeV is found to be generated, while in the latter one several N^* and Δ^* states with hidden charm are obtained with masses around 4600 MeV. All these states constitute predictions of our model and future experimental confirmation of them would be relevant to elucidate the properties of the strong interactions in the presence of heavy quarks.

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1. Introduction

Within the present theory of Quantum Chromodynamics, nature allows the existence of exotic mesons and baryons, i.e., those whose quantum numbers and properties cannot be described with a $q\bar{q}$ seed for mesons and qqq for baryons. Indeed, the formation of such hadrons has been confirmed experimentally, with some of the recent findings coming from the LHCb and BESIII collaborations regarding the formation of multi-quark states with hidden charm, with and without strangeness [1–3]. Within the category of exotic hadrons, particularly interesting are those with several units of charm and bottom quantum numbers. Such research is definitely crucial for shedding light on the working of the strong interactions in the presence of heavy quarks. However, it is still not clear whether the properties of the exotic mesons and baryons found can be understood in terms of compact tetraquarks/pentaquarks or as molecules of hadrons. And a lot of theoretical and experimental efforts are being presently put to clarify this issue.

In this talk we summarize the results we found for several three-body systems involving heavy mesons, possessing explicit/hidden charm as well as explicit/hidden strangeness. In particular, we report our findings for the KDD, $KD\bar{D}^*$, $DK\bar{K}$ and $ND\bar{D}^*$ systems.

2. Formalism and Results

The solution of the Faddeev equations [4] to study three-body systems turns to be a quite useful method to investigate the formation of exotic mesons and baryons with a molecular nature (for a recent review, see Ref. [5]). Considering as kernel for the Faddeev equations the two-body t-matrices obtained from the resolution of the Bethe-Salpeter equation in its on-shell factorization form [6], several three-body systems have been studied in the last years and generation of mesons and baryons with a molecular nature has been claimed [7–9].

The advantage of our formalism is that not only the Bethe-Salpeter equation

$$t = v + vgt, \tag{1}$$

where v is the kernel and g is a loop function of two hadrons, becomes an algebraic equation, but also the Faddeev equations

$$\begin{aligned} T^1 &= t^1 + t^1 G[T^2 + T^3], \\ T^2 &= t^2 + t^2 G[T^1 + T^2], \\ T^3 &= t^3 + t^3 G[T^1 + T^2], \end{aligned} \tag{2}$$

where t^i, $i = 1, 2, 3$, is the two-body t-matrix describing the interaction of particles (jk), $j \neq k \neq i$ and G is a three-body loop function, become simplified due to a cancellation of three-body forces (briefly discussed below) arising in the formalism [7–9]. In this way, once the kernel v in Eq. (1) is obtained, the numerical calculations of the three-body T-matrix for the system, $T = T^1 + T^2 + T^3$, can be handled, even if a large number of coupled channels are required to be considered.

In our approach, the kernel v in Eq. (1) is determined by using effective Lagrangians based on the relevant symmetries for the problem under investigation, like the chiral and heavy quark symmetries of the strong interaction [10, 11]. The solution of Eq. (1) within coupled channels reveals the generation of two-hadron molecular states. Some of the well known examples of two-hadron molecular states (found within different models, even in lattice calculations) are: (1) the KD, ηD_s interactions form the state $D^*_{s0}(2317)$ [12–15]. (2) The $D\bar{D}^*$ system in isospin 0 generates $X(3872)$ while in isospin 1 the $Z_c(3900)$ is formed [16–20]. (3) The $K\bar{K}$ interaction give rise to $f_0(980)$ [6], while the ND/ND^* system originates $\Lambda_c(2595)$ [21–23].

Considering these ingredients, the solution of Eq. (2), for example, for the DDK and coupled channels shows the generation of a state with isospin 1/2, charm +2, strangeness +1 and mass around 4140 MeV [24]. Let us discuss this case in some detail. The three-body coupled systems considered are: $D^+D^0K^+$, $D^+D^+K^0$, $D^+D^+_s\pi^0$, $D^+D^+_s\eta$, $D^0D^+K^+$, $D^0D^+_s\pi^+$. We start by calculating the t-matrices for the different two-body subsystems. We obtain the DK, $D_s\eta$ and $D\pi$ amplitudes from the Lagrangian based on chiral and heavy quark symmetries

$$\mathcal{L}_{HP} = \frac{1}{4f^2}\left\{\partial^\mu H[P,\partial_\mu P]H^\dagger - H[P,\partial_\mu P]\partial^\mu H^\dagger\right\}, \tag{3}$$

where H and P stand for matrices of heavy and pseudoscalar mesons

$$H = \begin{pmatrix} D^0 & D^+ & D^+_s \end{pmatrix} \tag{4}$$
$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

The kernels obtained from the Lagrangian in Eq. (3) are projected on s-wave and used as an input to solve the Bethe-Salpeter equation in a coupled channel approach, within the on-shell approximation. In such an approach, the loop function is required to be regularized. Using the scale $\mu = 1000$ MeV and subtraction constant $a(\mu) = -1.846$, following Ref. [13], one obtains a pole which well describes $D^*_{s0}(2317)$. In case of the interaction of two heavy mesons, we determine the amplitudes through exchange of a vector meson. We show in Fig. 1, as an example, the diagrams for the $D^+D^0 \to D^+D^0$ process. Similar diagrams are considered for other heavy meson interactions. Such diagrams are

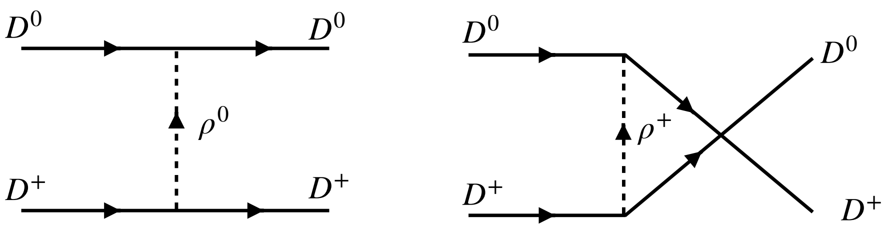

Figure 1: Diagrams contributing to the $D^+D^0 \to D^+D^0$ amplitude.

calculated using a Lagrangian compatible with chiral and heavy quark symmetries

$$\mathcal{L}_{VPP} = -ig\langle V^\mu[\tilde{P},\partial_\mu\tilde{P}]\rangle, \quad g = \frac{M_V}{2f}, \tag{5}$$

where M_V is the mass of the exchanged vector meson, f is the decay constant of D, with value 165 MeV, and

$$\tilde{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}, \tag{6}$$

$$V^\mu = \begin{pmatrix} \frac{\omega}{\sqrt{2}} + \frac{\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega}{\sqrt{2}} - \frac{\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & K_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}. \tag{7}$$

Explicit expressions for the different amplitudes can be found in Ref. [24]. In this case too, the potential is projected on s-wave and the Bethe-Salpeter equation is solved regularizing the loop function using dimensional regularization. The subtraction constant is varied to test the stability of the results.

At this point we find it useful and important to discuss a very interesting finding in our works. We find that for the different systems we have studied, which are always made of two or more mesons, there appears a cancellation between different sources of three-body forces. Coincidentally, the off-shell parts of the two-body amplitudes also give rise to contributions to three-body forces. The reason of such a finding is that the off-shell parts of the amplitudes for two-body systems involving mesons are proportional to the inverse of a propagator. An example of such a three-body force is depicted in form of diagrams in Fig. 2. Typically, in case of systems made of mesons, one can

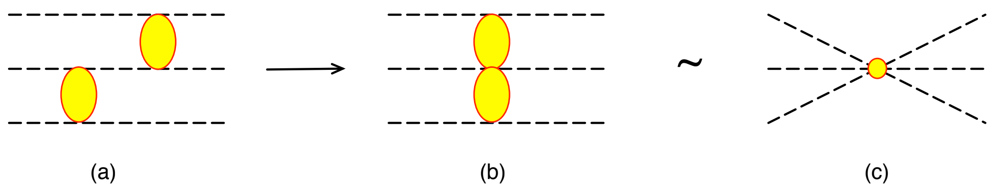

Figure 2: (a) A lowest order term of the Faddeev partitions. (b) Effective contribution of the same diagram, due to the cancellation of the off-shell parts of the two-body amplitudes with a propagator in the three-body diagram. As a consequence of the cancellation, diagram (b) can be interpreted as a three-body contact term shown in (c).

also get a three-body contact term directly from the Lagrangian. For example, one can expand the terms in the chiral Lagrangian to account for any number of pseudoscalar fields. Thus one can get a six field term, which is a three-body contact term. We found that the sum of all such terms giving rise to three-body contact interactions cancel [7–9]. Such a cancellation is found to be exact, analytically, in certain limit, such as the chiral limit. Deviations from such a limit leads to a negligible contribution from the sum of all three-body contact terms, with the on-shell parts of the two-body t-matrices remaining to be considered. In case of the DDK system, we evaluated the sum of all the sources of three-body forces in two different ways. We considered the Lagrangian based on SU(4) as well as heavy quark symmetries for the heavy-light meson interactions. In the former case, we obtained a six field term directly from the Lagrangian but not in the latter case. Still the sum of the all the contact terms was found to be exactly the same in both cases, which tends to zero in the chiral limit. Such cancellations lead to an important simplification in solving Faddeev equations and seem to always appear in systems made of mesons.

With the three-body amplitude at hand, it is required to make an isospin projection, in order to interpret the results. There are several ways to make such isospin projections. For instance, one can choose to define a basis in which the three-body states are labeled in terms of the total isospin I and the isospin of one of the two-body subsystems, say, I_{23}, the isospin of one of the DK subsystems in case of DDK. In this way, the transition amplitude $\langle I, I_{23} \mid T_R \mid I, I_{23}\rangle$ can be evaluated. The isospin of the DK system can be 0 or 1, thus, giving rise to a total isospin of 1/2 or 3/2. We plot the isospin projected, squared three-body amplitudes as a function of the total energy and the invariant mass of one of the subsystems. We find that a state arises at energy 4140 MeV, when $D^*_{s0}(2317)$ is formed in one of the DK subsystems. As a consequence of its nature, the three-body state found in the DDK system can decay to two-body channels like D_sD, D_sD^* and DD^*_s through triangular loops, producing a small width. These decay mechanisms were investigated in Ref. [25], resulting in a total width of 2-3 MeV for the state. We also estimated the size of our state to be around 1.0-1.4 fm, in Ref. [24], which indicates the underlying molecular nature of the state. Very similar results have also been found within a model based on one kaon exchange [26] and the Gaussian expansion method [27]. Theoretical predictions of such a double charm state is already calling the attention of experimental groups. However, only a preliminary search has been conducted, so far, by the Belle collaboration [28], which did not show a clear signal of a state in the DD^*_s invariant mass spectrum. Availability of better statistics data, in future, can be useful to find a DDK quasi bound state.

The other system we studied, with charm content, is $KD\bar{D}^*$ system. In this case, we considered that $D\bar{D}^*$ system forms a bound state, $X(3872)$, and which remains unperturbed during the scattering with a kaon. Such conditions allow us to solve the Faddeev equations in a fixed center approximation, where one needs to solve two coupled partitions, instead of the three standard ones (for more details on applicability of the fixed center approximation see Refs. [29–32]). In our study, we consider $KX(3872)$ to couple with $KZ_c(3900)$ and find that a K^* meson with mass ~ 4307 MeV arises from the coupled three-body dynamics. We show the squared amplitude for the KZ channel in Fig. 3, for total isospins 1/2 and 3/2. As can be seen from Fig. 3, we do not find any state with isospin 3/2. The figure also shows the uncertainties in the results coming from an unknown parameter in the formalism, which is a three-momentum cut-off needed to evaluate the propagation of the kaon through the cluster. It can be seen that our results depend weakly on this parameter.

Since in our formalism $Z_c(3900)$ can be considered as a state generated from the $D\bar{D}^*$ and $J/\psi\pi$ coupled channels, it can decay to $J/\psi\pi$ [33], producing a width for $Z_c(3900)$ of around 30 MeV. When implementing this width into our three-body calculation, the $K^*(4307)$ state attains a width of around 18 MeV. In view of the nature found for this three-body state, $K^*(4307)$ can decay to two-body channels too, like $J/\psi K^*(892)$, $\bar{D}D_s$, $\bar{D}D^*_s$ and $\bar{D}^*D^*_s$, through triangular loops. These decay widths were determined in Ref. [34], finding $\sim$ 7 MeV for $J/\psi K^*(892)$, $\sim$ 0.5 for $\bar{D}D^*_s$,

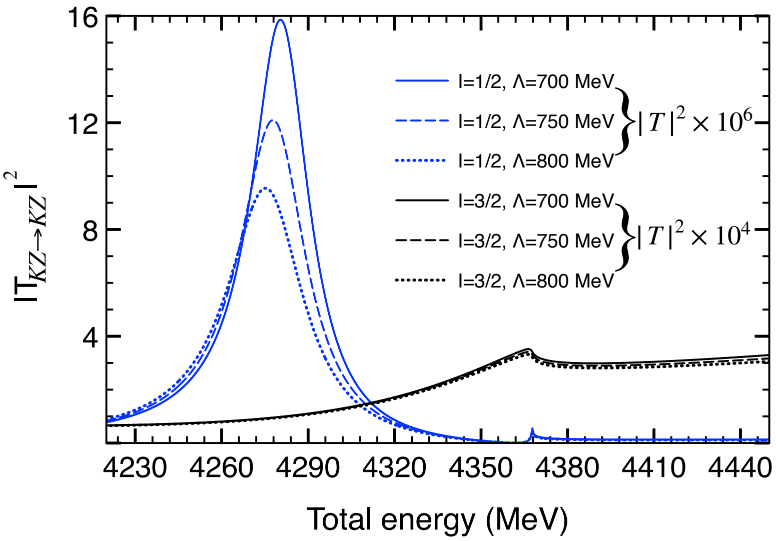

Figure 3: Modulus squared amplitudes for KZ system in isospin 1/2 and 3/2.

$\bar{D}^*D_s^*$ and ~ 1 MeV for $\bar{D}D_s$. The fact of having an important $KZ_c(3900)$ molecular component in the wave function of $K^*(4307)$ indicates that the state can also easily decay to a channel like $J/\psi\pi K$. In this way, the reconstruction on the $J/\psi\pi K$ invariant mass in processes involving these particles in their final states can be a promising way of observing this $K^*(4307)$ (as proposed in Ref. [35]). In fact data on the processes $B^+ \to J/\psi\pi^+\pi^0K^0$ and $B^+ \to J/\psi\pi^+\pi^-K^+$ already exist [36] and the $J/\psi\pi\pi$ mass spectra have been reconstructed to study $X(3872)$. However, the $J/\psi\pi K^*(892)$ mass spectrum has not been reconstructed and we would like to strongly encourage the investigation of this part of the data.

Let us now discuss a system with explicit charm but hidden strangeness, $DK\bar{K}$. We studied this system sometime ago [37] by solving the Faddeev equations for the following coupled channels: $D^0K^+K^-$, $D^0K^0\bar{K}^0$, $D^0\pi^+\pi^-$, $D^0\pi^-\pi^+$, $D^0\pi^0\pi^0$, $D^0\pi^0\eta$, $D^+K^0K^-$, $D^+\pi^-\pi^0$, $D^+\pi^-\eta$, $D^+\pi^0\pi^-$. In Ref. [37], we studied two effective configurations of $DK\bar{K}$ using QCD sum rules also. In both formalism, we found a D meson with mass around 2900 MeV and width of 55 MeV [37], when the $K\bar{K}$ system generates $f_0(980)$. The interest in this state got recently revived by increased efforts being put in, by the LHCb collaboration, in searching for open charm mesons with mass around 3000 MeV [39]. These latter experimental investigations focussed on systems like $D^*\pi$ and $D\pi$ and at least two states are needed to fit the data. The quantum numbers of these states are not yet known and the data have relatively poor statistics [39]. It is, thus, difficult to analyze if any of those structures can be associated with the $D(2900)$ predicted in our works [37, 38]. With the hope for better statistics data to be determined in near future, we found it timely to study the decay of our $DK\bar{K}$ to two body channels. The internal structure of our state allows it decay to the following two-body channels: $D^*\pi$, $D^*\bar{K}$, $D_{s0}^*\bar{K}$. We studied these decays in detail in Ref. [38] and found that the largest decay width comes from $D(2900) \to D_{s0}^*(2317)\bar{K}$, indicating that $D_{s0}^*(2317)\bar{K}$ can be a more encouraging channel to find $D(2900)$.

Finally, we would like to report the results of our work on a system with hidden charm and null strangeness, which is $ND\bar{D}^*$. Our study was motivated by the finding of pentaquarks by

LHCb [41, 42], whose masses lie close to the threshold of systems like $NX(3872)$, $NZ_c(3900)$. The findings of LHCb have lead to an intense, running, debate in the community and a series of theoretical descriptions for the states found by LHCb have been brought forward (for some recent reviews covering this topic, see [43, 44]). Most of the descriptions proposed for the pentaquarks of Refs. [41, 42] attribute negative parities to them. However, states formed by interactions of systems like $NX(3872)$, $NZ_c(3900)$ would have positive parities and such states should have masses in the energy region scanned by LHCb. Even if the pentaquarks found in Refs. [41, 42] are indeed negative parity states, then there is no reason for their positive parity partners to not exist. Keeping this in mind and the fact that the interaction of nucleon with D/D^* can lead to formation of states, like $\Lambda_c(2595)$, we studied $ND\bar{D}^*$ in Ref. [40]. The system indeed reveals formation of several narrow N^* and Δ^* states with masses around $4400-4600$ MeV and a positive parity. Such states arise when the $D\bar{D}^*$ system clusters as $X(3872)$ and $Z_c(3900)$ and the ND/ND^* system forms $\Lambda_c(2595)$ [40]. The next step of our study is to investigate if such positive parity states can be seen in the invariant mass spectrum of NJ/ψ or if other systems show better signals. For this purpose we should study the decay of our states to two-body channels like NJ/ψ and $\bar{D}^{(*)}\Sigma_c$. Such channels are expected to be important due to the nature found for these three-body states. Such studies should be done in the near future.

3. Conclussions

In this talk we have presented the results of our latest studies on the generation of exotic mesons and baryons as a consequence of the three-body dynamics involved in several systems constituted by heavy mesons, like KDD, $KD\bar{D}^*$, $DK\bar{K}$ and $ND\bar{D}^*$. In particular, in the KDD system we find a state around 4140 MeV with charm +2 and strangeness +1. Similar results have been obtained by studies of the same system with other distinct formalisms. A study of the decays of such a system to two-body channels has also been done. We have discussed that theoretical findings are motivating interest of experimental groups. The study of the $KD\bar{D}^*$ and $ND\bar{D}^*$ systems reveals the formation of Kaon and Nucleon/Delta resonances with hidden charm, respectively, with masses in the region of 4300-4600 MeV. In case of the K^* arising from $KD\bar{D}^*$ interactions, we have studied its two-body decays. In order to encourage the reconstruction of the $J/\psi\pi K$ mass spectrum, in data coming from weak decays of the B-meson, we have made a theoretical investigation of the process and shown that the $J/\psi\pi K$ mass spectra can be useful to confirm the existence of our K^*-state. We have also provided an outlook on our study of N^* states with hidden strangeness. In the hidden strangeness sector, a D-meson with mass around 2900 MeV is obtained as a consequence of the dynamics involved in the $DK\bar{K}$ system. We have recently studied further properties of this $D(2900)$, with the intention to facilitate its identification in forthcoming experimental data.

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