PROCEEDINGS OF SCIENCE

Light meson-quark couplings and radii: ratios from one loop calculation

Fabio L. Braghin*

[a] *Instituto de Física Universidade Federal de Goiás,*
Av. Esperança, s/n, 74690-900, Goiânia, GO, Brazil

E-mail: braghin@ufg.br

In this work some results for light mesons couplings to constituent quarks obtained in a one loop calculation are compared among each other in the limit of large quark effective mass. These coupling constants are extracted from momentum dependent form factors obtained in a dynamical approach from an important term of the QCD quark-effective action. For this, exact and approximated ratios of the different light meson-constituent quark coupling constants are provided. The comparison is also done for the different light mesons and constituent quark averaged quadratic radii.

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 -17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

*Speaker

PoS(XVHadronPhysics)007

1. Introduction

The constituent quark model (CQM), as a collective set of models based in the original quark model by Gell Mann and Zweig, is one of the most successful cornerstones for the understanding of strong interactions in the Standard model. Besides its grounds in properties of Quantum Chromodynamics (QCD) it describes quite well considerable part of the hadron spectra, in particular light hadrons. Pions are almost massless being the (quasi) Goldstone bosons of the Dynamical Chiral Symmetry Breaking (ChSB) that endows constituent quarks, and hadrons, with a large effective mass that is observed. This mechanism is one of the mechanisms that contribute in the overall hadron masses when compared to the current (Lagrangian) quark masses that are often referred to measurements at high energies where partons are identified. The dressing of a gluon cloud should be the most relevant contribution responsible for the emergence of constituent quarks [1]. In models built in the 80's and 90's, a radius of the order of $0.2 - 0.3$fm has been estimated for constituent quarks [2, 3]. There are many versions of the CQM that, besides the gluon cloud, take into account a pion cloud. The Weinberg's large Nc Effective Field Theory (EFT) is an example of a CQM, with the status and power of EFT, that is fully consistent with the properties of the large Nc expansion [4]. In [5, 6] this EFT has been derived as the leading terms from a large quark and gluon effective masses expansion with leading couplings to the electromagnetic field and symmetry breaking terms. Within the constituent quark model the meson-nucleon interaction is understood in terms of meson-constituent quark coupling. For example, it has been argued that axial pion-constituent quark coupling constant should be $g_A = 3/4$ or $g_A = 1$ [2, 4]. Eventually to cope different mesons couplings and their relative role can be used to assess or to improve field theoretic schemes to help to find unambiguous parameterization of the nucleon and nuclear potentials [7].

We shall consider the non perturbative one gluon exchange quark-antiquark interaction as one of the leading terms of QCD effective action whose generating functional is given by [8, 9]:

$$Z = N \int \mathcal{D}[\bar{\psi}, \psi] \exp\, i \int_x \left[\bar{\psi} \left(i\partial\!\!\!/ - m \right) \psi - \frac{g^2}{2} \int_y j_\mu^b(x) \tilde{R}^{\mu\nu}_{bc}(x-y) j_\nu^c(y) + \bar{\psi} J + J^* \psi \right] \quad (1)$$

Where N is the normalization, J, J^* the quark sources, $\int_x$ stands for $\int d^4x$, and $a, b... = 1, ...(N_c^2 - 1)$ stands for color in the adjoint representation being $N_c = 3$. The quark gluon coupling constant is assumed to be g and the development below is akin to the Rainbow Ladder Schwinger Dyson equation (SDE). Below indices $i, j, k = 0, ...(N_f^2 - 1)$ will be used for SU(2) isospin indices and therefore $N_f = 2$. The quark current mass will be assumed to be equal for u, d quarks. The color quark currents are given by $j_a^\mu = \bar{\psi} \lambda_a \gamma^\mu \psi$, and the sums in color, flavor and Dirac indices are implicit. A Landau-type gauge will be considered for a non pertubative gluon propagator that can be written as $\tilde{R}^{\mu\nu}_{ab}(x-y) \equiv \tilde{R}^{\mu\nu}_{ab} = \delta_{ab} \left[\left(g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) R_T(x-y) + \frac{\partial^\mu \partial^\nu}{\partial^2} R_L(x-y) \right]$, where the transversal and longitudinal components are $R_T(x-y)$ and $R_L(x-y)$. This non perturbative gluon kernel therefore incorporates to some extent the gluonic non Abelian character with a corrected quark-gluon coupling such that they will provide enough strength to yield dynamical chiral symmetry breaking (DChSB). This has been found in several approaches and extensions [10–16].

In this work light meson-quark coupling constants and averaged quadratic radii will be exhibited systematically within the large quark effective mass expansion of the quark- one loop background field method as performed in Refs. [5, 17–19]. There are many theoretical calculations for the light hadrons electromagnetic and strong form factors that will not be discussed or quoted here. These coupling constants will be extracted from form factors that were obtained in a dynamical calculation with clear contact with QCD. In a one-loop quark polarization calculation, a large quark and gluon effective masses expansion is done for the model of Eq. (1). From the resulting form factors, the corresponding low energy coupling constants can be obtained (in the present work they are shown in the limit of zero meson and constituent quark momenta) and, besides that, mesons and constituent quark averaged quadratic radii can be calculated. Although the condition of zero meson momentum is not the one usually adopted for coupling constants, note that constituent quarks might be assumed to be very likely off shell, in spite of usual conditions for a CQM.

2. The quark determinant, light mesons and constituent quark currents

The method was explained in details in Refs. [5, 6, 17] and therefore it will be succintly described below. A Fierz transformation for the model (1) is performed and, by picking up the leading color singlet terms that provide the usual leading mesons couplings, it allows to investigate the flavor structure in a more complete way. Besides that, color triplet currents only contributes for higher order (and numerically smaller) being that they only survive when colorless combinations emerge. This will be shown in details elsewhere. Color singlet provides a direct relation with lightest observed states, quark-antiquark mesons. Chiral structures with combinations of bilocal currents are obtained. The Background Field Method (BFM) makes possible the calculation of a one loop effective action [20, 21]. Therefore the quark field is split into quarks, ψ_2, composing (light) quark-antiquark states that give rise to mesons and the chiral condensate, and the background quark, ψ_1 that eventually might be identified to constituent quarks. The shift of quark bilinears corresponds to performing a one loop BFM calculation and it might be written for each of the color singlet Dirac/isospin channels $m = s, p_i, v, a, a_i, v_i$ (scalar, pseudoscalar-isospin triplet, vector, axial, vector-isospin triplet, axial-isospin triplet, where the isospin singlet states were omitted) where the singlet pseudoscalar and scalar-isospin triplet will be omitted. This separation preserves chiral symmetry. The sea quark can be integrated out exactly by means of the auxiliary field method that give rise to colorless quark-antiquark states, light mesons and the chiral quark condensate. Auxiliary fields are introduced by means of the unity integrals multiplying the generating functional. The scalar degree of freedom can be eliminated by means of a chiral rotation that endows the pseudoscalar-iso-triplet ones with non linear structure and dynamics typical from the Goldstone bosons. Field renormalization constants were introduced explicitly in Refs. [22, 23]. Bilocal auxiliary fields for the different flavors can be expanded in an infinite orthogonal basis with all the excitations in the corresponding channel [8]. For the low energy regime one might pick up only the lowest energy modes, lightest $k = 0$ which corresponds to the pions in this channel, i.e. for example for the pseudoscalar isotriplet fields $P_{i,k=0} = \pi_i$, making the normalization constant of the meson to be a simple constant in the zero momentum limit. $F_k(z) = F_k(0)$. The saddle point equations for each of the remaining auxiliary fields, ϕ_q, after the integration of the sea quark, can be written from the condition: $\frac{\partial S_{eff}}{\partial \phi_q} = 0$. These equations for the NJL model and for the model (1) with Schwinger Dyson equations at the rainbow ladder level have been analyzed in many works in the vacuum or under a finite energy density. The scalar field has the only saddle point equation with non trivial solution for the quark-antiquark chiral condensate. This classical solution generates a large effective quark mass

The scalar field can be frozen by means of a chiral rotation and this produces the chiral condensate and a strongly non linear pion sector. An usual pion field definition is parametrized by the functions: $U = exp(i\vec{\pi}\cdot\vec{\sigma})$ and $U^\dagger = exp(-i\vec{\pi}\cdot\vec{\sigma})$. The corresponding Jacobian of the path integral measure will not be calculated and it might induce extra (*anomalous*) terms for the resulting form factors. By performing a Gaussian integration of the sea quark field, the resulting determinant can be written, by means of the identity $\det A = \exp\ Tr\ \ln(A)$, as:

$$\begin{aligned} S_{eff} &= -i\,Tr\ \ln\left\{-iS_f^{-1}(x-y)\right\}, && (2)\\ S_f^{-1}(x-y) &\equiv S_0^{-1}(x-y) + \Xi_s(x-y) + \Xi_v(x-y) + \sum_q a_q\Gamma_q j_q(x,y), && (3)\end{aligned}$$

where f stands for flavor in the adjoint representation, Tr stands for traces of all discrete internal indices and integration of space-time coordinates and $\Xi_s(x-y)$ and $\Xi_v(x-y)$ stand respectively for the coupling of sea quark to the scalar-pseudoscalar and vector/axial fields for the particular definition mentioned above pion field as [5, 6, 17]:

$$\begin{aligned} \Xi_s(x-y) &= F(P_R U + P_L U^\dagger)\ \delta(x-y), && (4)\\ \Xi_v(x-y) &= -\frac{\gamma^\mu}{2}\left[F_v\sigma_i\left(V_\mu^i(x) + i\gamma_5\bar{A}_\mu^i(x)\right) + F_{vs}\left(V_\mu(x) + i\gamma_5\bar{A}_\mu(x)\right)\right]\delta(x-y), && (5)\end{aligned}$$

where $F = f_\pi$ is the pion field normalization, $P_{R/L} = (1\pm\gamma_5)/2$ are the chirality right/left hand projectors and the constants F_v and F_{vs} provide the canonical field definition respectively of rho and A_1 mesons and of ω and axial f_1.

The free quark kernel can be written as $S_{0,free}^{-1}(x-y) = \left(i\partial\!\!\!/ - m\right)\delta(x-y)$, where m is so far the current quark mass. The classical solution for the scalar field, found from its gap equation, is directly incorporated into an effective quark mass $M^* = m + < s >$. The redefined quark kernel can be written as:

$$S_0^{-1}(x-y) = \left(i\partial\!\!\!/ - M^*\right)\delta(x-y). \quad (6)$$

In expression (3) the following quantity, with the usual chiral quark currents has been considered:

$$\begin{aligned} \frac{\sum_q a_q\Gamma_q j_q(x,y)}{\alpha g^2} &= 2R(x-y)\left[\bar{\psi}(y)\psi(x) + i\gamma_5\sigma_i\bar{\psi}(y)i\gamma_5\sigma_i\psi(x)\right]\\ &- \bar{R}^{\mu\nu}(x-y)\gamma_\mu\sigma_i\left[\bar{\psi}(y)\gamma_\nu\sigma_i\psi(x) + \gamma_5\bar{\psi}(y)\gamma_5\gamma_\nu\sigma_i\psi(x)\right] \quad (7)\end{aligned}$$

In this expression, $\alpha = 2/9$ from the Fierz transformation. These background quark currents are attached to components of the gluon propagator given by:

$$\begin{aligned} \bar{R}^{\mu\nu} &\equiv \bar{R}^{\mu\nu}(x-y) = g^{\mu\nu}(R_T(x-y) + R_L(x-y)) + 2\frac{\partial^\mu\partial^\nu}{\partial^2}(R_T(x-y) - R_L(x-y)), && (8)\\ R &\equiv R(x-y) = 3R_T(z-y) + R_L(x-y). && (9)\end{aligned}$$

These functions can be considered as part of a gluon cloud providing the emergence of a constituent quark current that can be dressed further by a pion cloud for the effective action shown below.

3. Light mesons couplings to constituent quarks

The complete set of leading pion and vector/axial mesons couplings to constituent quarks, with coupling constants resolved in the long wavelength limit with zero momentum exchange, were presented in Refs [5, 17–19]. In these works the estimations for numerical values of the resulting coupling constants some specific and different *ad hoc* normalization for the quark-gluon running coupling constant and gluon propagator were fixed such as to reproduce particular numerical values of a specific meson-constituent quark coupling constant of a particular channels. In the present work, numerical estimations will not be presented and only relative values will be addressed. The leading light meson-constituent quark current couplings were found to be the following:

$$\begin{aligned}
\mathcal{L}_{q-\pi} &= \frac{G_{2js}}{F}\pi_i\pi_i(\bar{\psi}\psi) + G_{ps}\pi_i\,\bar{\psi}\sigma_i i\gamma_5\psi + i\epsilon_{ijk}\frac{G_V}{F^2}\,\pi_i(\partial_\mu\pi_j)\,\bar{\psi}\gamma_\mu\sigma^j\psi, \\
&+ \frac{G_A}{F}\,(\partial^\mu\pi_i)\,(\bar{\psi}i\gamma_5\gamma_\nu\sigma^i\psi) - G^p_{ps}\frac{(\partial^2\pi_i)}{M^*F}(\bar{\psi}\sigma_i i\gamma_5\psi) - G^p_s\frac{\partial^2(\pi_i\pi_i)}{M^*F^2}(\bar{\psi}\psi), \qquad (10)\\
\mathcal{L}_{q-v} &= g_{r1}V^\mu_i j^{V,i}_\mu + (g_{r1}+g_{ax})\bar{A}^\mu_i j^{A,i}_\mu + g_{v1}V^\mu j_\mu + (g_{v1}+g_{ax})\bar{A}_\mu j^\mu_A, \qquad (11)\\
\mathcal{L}_{vja-A} &= i\epsilon^{\sigma\rho\mu\nu}F^{vja}\left[\mathcal{G}^i_{\rho\mu}(\partial_\sigma j^{V,i}_\nu) + \mathcal{G}_{\rho\mu}(\partial_\sigma j^V_\nu) + \mathcal{F}^i_{\rho\mu}(\partial_\sigma j^{A,i}_\nu) + \mathcal{F}_{\rho\mu}(\partial_\sigma j^A_\nu)\right], \qquad (12)
\end{aligned}$$

being that meson local fields were written with their canonical normalization. The last set of couplings, $\mathcal{L}_{vja-A}$, correspond to anomalous couplings [19] that emerges due to the anomalous trace of Dirac indices: $tr_D(\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho\gamma_5) = 4i\epsilon_{\mu\nu\sigma\rho}$. Although these couplings are the only ones in the eqs above that present second order dependence in external (meson and quark) momenta, there are several other non-anomalous second order derivative couplings, i.e. external momenta, which are however suppressed by a factor $1/M^*$ with respect to the first order derivative couplings presented above. In the above equations, the Abelian part of the stress tensors for the vector and axial mesons were defined for the isotriplet and isosinglet states: $\mathcal{F}^i_{\rho\mu} = \partial_\rho V^i_\mu - \partial_\mu V^i_\rho$, $\mathcal{F}_{\rho\mu} = \partial_\rho V_\mu - \partial_\mu V_\rho$, $\mathcal{G}^i_{\mu\nu} = \partial_\mu \bar{A}^i_\nu - \partial_\nu \bar{A}^i_\mu$, and $\mathcal{G}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$. The non-Abelian contributions were neglected although they can be incorporated straightforwardly.

The above coupling constants correspond to the zero external momenta limit of the corresponding form factors. In the Euclidean momentum space these form factors are written as:

$$G_{ps} = G_{2js} = 2g_{r1} = 2g_{v1} = 32d_1N_c(\alpha g^2)\,F_1(0,0), \qquad (13)$$

$$\frac{G_A}{F} = \frac{G_V}{F} = \frac{G^p_{ps}}{F} = \frac{G^p_s}{F} = 2\frac{g_{ax}}{M^*} = 8M^*F_{vja} = 32d_1N_c\,(\alpha g^2)\,F_2(0,0), \qquad (14)$$

where

$$\begin{aligned}
F_1(0,Q) &= \int_k (k\cdot(k+Q) - M^{*2})\tilde{S}_0(k)\tilde{S}_0(k+Q)R(-k), \\
F_2(0,Q) &= \int_k M^*\tilde{S}_0(k)\tilde{S}_0(k+Q)R(-k) \qquad (15)
\end{aligned}$$

where $\int_k = \int \frac{d^4k}{(2\pi)^4}$ and the following function was used: $\tilde{S}_0(k) = \frac{1}{k^2+M^{*2}}$. In the very large quark effective mass one has $F_2(0,Q) \sim F_1(0,Q)/M^*$. There are basically two types of pion coupling to quarks (and to the nucleon). The pseudoscalar and scalar momentum independent ones, are numerically large. The derivative couplings that might be suppressed due to the different form factors F_1 and F_2 and due to $1/M^*$ or F factors. G_V and G_A and also the induced scalar and pseudoscalar ones G^p_{ps}, G^p_s. For the vector and axial mesons, there are also two types. Firstly the usual Yukawa-type couplings, g_{r1}, g_{v1} with eventual corrections to the axial ones g_{ax}. Secondly, the anomalous couplings F_{vja} that is suppressed due to a numerical constant factor $1/4$ and to a factor $1/M^*$ or $1/M^{*2}$ if compared to g_{r1}. Besides that, the above definition for the anomalous coupling constant, F_{vja}, has dimension M^{-2} although it could be defined as dimensionless quantity by dividing it by M^* in Eq. (12) what might not be "natural". All the other coupling constants are dimensionless and correspond to the possible leading phenomenological couplings for mesons with cosntituent quarks/nucleon.

In Figure (1), the diagrams corresponding to the couplings (10 ,11) and (12) are drawn. The vertices with two mesons lines (dashed lines) represent two pion couplings (G_V and G_{2js}) and non-leading two-vector(axial) mesons couplings to constituent quark currents.

In the limit of very large quark effetive masses, the ratios presented in (13,14) can be put together in the following approximate form:

$$G_{ps} = G_{2js} = 2g_{r1} = 2g_{v1} \simeq G_A \frac{M^*}{F} = G_V \frac{M^*}{F} = G^p_{ps} \frac{M^*}{F} = G^p_s \frac{M^*}{F} = 2g_{ax} = 8M^{*2} F_{vja}. \quad (16)$$

Part of these relations are exact in the level of one loop calculation and part of it relies in the large quark effective mass limit. It can be seen the quark-level Goldberger-Treiman relation (CQM) [18] and similar ones. Besides that, the pion pseudoscalar coupling to quark has a coupling constant that is around twice the vector meson-constituent quark coupling constant that is of the order of the relative ratio found in the literature. The anomalous vector meson coupling constant to (axial) quark current is suppressed by $\sim 1/(4M^{*2})$ with respect to the vector meson-constituent quark vector current coupling constant in the limit of very large quark effective mass.

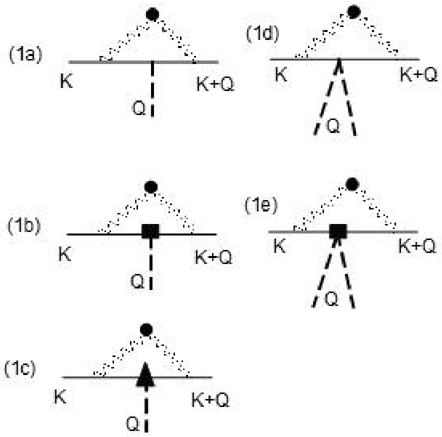

Figure 1: These diagrams correspond to the constituent quark-meson effective couplings from expressions (10 ,11) and (12). The wavy line with a full dot is a (dressed) non perturbative gluon propagator, the solid lines stand for a constituent quark (external line), and dashed lines represents mesons fields. Full square in a vertex represents a simple derivative coupling, and triangle the anomalous couplings, eg. eq. (12)

3.1 Averaged quadratic radii

From the form factors presented above, averaged quadratic radii can be defined from the different meson-constituent quark couplings and results are somewhat similar to the results obtained from the NJL model [24]. The function $F_1(K,Q)$ is known to have a non standard non-monotonic behavior with momenta K,Q mostly because of the constant effective mass approximation. A truncation of this function [17, 18] provides a more standard behavior and it reproduces more reasonably the resulting form factors that can be identified as due to a momentum-dependent effective mass. This truncation has been implemented by: $F_1(K,Q) \simeq F_1^{tr}(K,Q) \equiv M^* F_2(K,Q)$. Since the form factors are dimensionless the corresponding axial and pseudoscalar quadratic radii were defined in a standard form by:

$$< r^2 >_A = < r^2 >_V = 2\frac{F}{M^*} < r^2 >_{ax} = \frac{F}{M^*} < r^2 >^{tr}_{ps} = \frac{F}{M^*} < r^2 >^{tr}_{2js} = -6 \left. \frac{dG_A(0,Q)}{dQ^2} \right|_{Q=0} \quad (17)$$

$$< r^2 >_{ps} = < r^2 >_{2js} = 2 < r^2 >_{\rho} = 2 < r^2 >_{\omega} = -6 \left. \frac{dG_{ps}(0,Q)}{dQ^2} \right|_{Q=0} \quad (18)$$

The axial meson AQR is smaller than the vector meson AQR, within the calculation scheme above: $< r^2 >_{\bar{A}} = < r^2 >_{\rho} - < r^2 >_{ax}$, where $< r^2 >_{ax}$ is obtained from the particularly standard manipulation of the fermion determinant, otherwise, by imposing hermicity, vector and axial mesons might have the same properties. At the nucleon level, vector and axial AQR are different from each other and expected to follow: $\sqrt{< r_V^2 > / < r_A^2 >} \simeq 1.6$ [2, 24]. By changing the external momentum structure of the Feynman diagrams for light meson and constituent quarks one finds that the quantity $< r^2 >_{2js}$ stands for both, the scalar constituent quark AQR and pion AQR. Lattice calculation for the pion scalar AQR indicates its value is larger than the pion charge radius [25, 26], and it is estimated to be of the order of the strong pion (pseudoscalar) AQR. The vector AQR $< r^2 >_V$ is proportional to the pion charge AQR, $< r_\pi^2 >_{e.m.} \simeq \frac{3e}{2} < r_\pi^2 >_V \sim \frac{3e}{2}\frac{F}{M^*} < r_{2js}^2 >$ being their ratio not very different from experimental or lattice results $< r_\pi^2 >_{e.m.} \sim 0.45\text{fm}^2$ and $< r_{2js}^2 > \sim 0.68\text{fm}^2$.

The constituent quark axial and vector AQR are equal to each other and suppressed by a factor F/M^* with respect to the pseudoscalar and scalar AQR, what may signal the emergence of the pion cloud [18]. The vector mesons AQR are basically equal to the pion AQR (ps). Besides that, the electromagnetic squared radius for the rho and omega vector mesons, $< r_\rho^2 >_E$ and $< r_\omega^2 >_E$, are simply related to the strong AQR, being given by:

$$< r_\omega^2 >_{e.m.} = 3 < r_\rho^2 >_{e.m.} = \frac{3e}{2} < r_\rho^2 > = \frac{3e}{2} < r_\omega^2 > . \tag{19}$$

Finally from the anomalous vector (axial) meson coupling to the axial (vector) constituent quark current it is possible to provide estimations for axial (vector) AQR of vector (axial) meson. Those anomalous couplings corresponds to a small axial or vector component for the vector or axial mesons. For that, it is important to define a normalized dimensionless coupling function (form factor) for which there is an ambiguity. The two "natural" choices are given by:

$$(i) \quad \bar{G}^{(i)}_{vja} = \bar{K}\bar{Q} G_{vja}(K,Q), \qquad (ii) \quad \bar{G}^{(ii)}_{vja} = M^{*2} G_{vja}(K,Q), \tag{20}$$

where $\bar{K}, \bar{Q}$ may be considered to be averaged quark and meson momenta for the form factors. The advantage of this first definition is that it does not require a further normalization with the quark effective mass exclusively due to the dimension argument and all the quantities emerge naturally from the expansion of the determinant. The second definition, as just discussed, does not seem natural although it provides numerical values close to the first one, eventually slightly larger. The definition of anomalous vector or axial meson AQR will be adopted to be the following:

$$\Delta_A < r_\rho^2 > = -6 \left. \frac{d\bar{G}_{vja}}{dQ^2} \right|_{Q=0} = \frac{\bar{K}\bar{Q}}{4M^* F} < r^2 >_A . \tag{21}$$

By adopting $\bar{K}, \bar{Q} \sim 200$ MeV, $M^* \sim 360$ MeV and $F \sim 92$ MeV, it provides the following axial AQR of the rho or omega vector meson:

$$\Delta_A < r_\rho^2 > \sim 0.3 < r^2 >_A \sim 0.08 < r^2 >_\rho,$$

that is very close to the result obtained numerically in [19], i.e. around 10% of the vector meson AQR. A similar value is valid for the vector-AQR of the light axial f_1 and A_1 mesons, $\Delta_V < r^2 >_{f_1}$ and $\Delta_V < r^2 >_{\bar{A}_1}$.

Discussion We have exhibited some properties, coupling constants and averaged quadratic radii, of low energies mesons interacting with constituent quarks as developed in the last years in the framework of the constituent quark model at low energies at the one loop BFM and AFM. Although it still is missing a more complete description, with further numerical estimations in terms of the running quark mass and corresponding renormalization constants several of the relations among different channels of interactions and mesons, for example in Eqs. (16, 17, 18), should not change considerably at this one loop calculation. Results, tendencies and relative ratios, agree at least qualitatively, if not quantitatively in some cases, with experimental observation and other calculations.

Acknowledgments

The author thanks short discussions with C.D. Roberts. F.L.B. is member of INCT-FNA, Proc. 464898/2014-5 and he acknowledges partial support from CNPq-312072/2018-0, CNPq-421480/2018-1 and CNPq-312750/2021-8.

References

[1] M. Lavelle, D. McMullan, Constituent quarks from QCD, Phys. Rept. **279**, 1 (1997). E. de Rafael, The Constituent Chiral Quark Model revisited, Phys. Lett. **B 703**, 60 (2011).

[2] U. Vogl, W. Weise, The Nambu and Jona-Lasinio model: Its implications for Hadrons and Nuclei, Progr. in Part. and Nucl. Phys. **27** (1991) 195.

[3] Petronzio R., S. Simula and G. Ricco, Possible evidence of extended objects inside the proton, Phys. Rev. D 67, 094004 (2003) [Erratum-ibid. D 68, 099901 (2003)].

[4] S. Weinberg, Pions in Large N Quantum Chromodynamics, Phys. Rev. Lett. **105** (2010) 261601.

[5] F.L. Braghin, Quark and pion effective couplings from polarization effects , Eur. Phys. Journ. **A 52**, 134 (2016).

[6] F.L. Braghin, Low energy constituent quark and pion effective couplings in a weak external magnetic field, Eur. Phys. J. **A 54**, 45 (2018) . ArXiv:1705.05926.

[7] C.Downum, T.Barnes, J.R.Stone, E.S.Swanson, Nucleon-Meson Coupling Constants and Form Factors in the Quark Model, Phys.Lett. **B 638**, 455 (2006).

[8] C.D. Roberts, R.T. Cahill, J. Praschifka, The Effective Action for the Goldstone Modes in a Global Colour Symmetry Model of QCD, Ann. of Phys. **188**, 20 (1988).

[9] D. Ebert, H. Reinhardt, M.K. Volkov, Effective hadron theory of QCD, Progr. Part. Nucl. Phys. **33**, 1 (1994).

[10] D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, Phys. Lett. **B 742**, 183 (2015) and references therein.

[11] P. Maris, C. D. Roberts, Dyson-Schwinger equations: A Tool for hadron physics, Int. J. Mod. Phys. **E 12**, 297 (2003). P. Tandy, Hadron physics from the global color model of QCD, Prog.Part.Nucl.Phys. **39**, 117 (1997).

[12] K.-I. Kondo, Abelian-projected effective gauge theory of QCD with asymptotic freedom and quark confinement Phys. Rev. **D 57**, 7467 (1998) .

[13] J. M. Cornwall, Entropy, confinement, and chiral symmetry breaking, Phys. Rev. **D 83**, 076001 (2011).

[14] K. Higashijima, Dynamical chiral-symmetry breaking, Phys. Rev. D 29, 1228 (1984).

[15] B. Holdom, Approaching low-energy QCD with a gauged, nonlocal, constituent-quark model, Phys. Rev. **D 45**, 2534 (1992).

[16] Q. Wang, Y.-P. Kuang, X.-L. Wang, and M. Xiao, Phys. Rev. D 61, 054011 (2000); K. Ren, H.-F. Fu, and Q. Wang, Phys. Rev. D 95, 074012 (2017).

[17] F.L. Braghin, Light vector and axial mesons effective couplings to constituent quarks, Phys. Rev. **D 97**, 054025 (2018). F.L.B. Erratum ibid. D 101, 039901(E) (2020).

[18] F.L. Braghin, Pion constituent quarks couplings strong form factors: a dynamical approach, Phys. Rev. D 99, 014001 (2019)

[19] F.L. Braghin, Constituent quark axial current couplings to light vector mesons in the vacuum and with a weak magnetic field, Phys Rev. D 105, 054009 (2022).

[20] L.F. Abbott, Acta Phys. Pol. **B 13**, 33 (1982).

[21] S. Weinberg, *The Quantum Theory of Fields* Vol. II, Cambridge, (1996).

[22] F.L. Braghin, Weak magnetic field corrections to light vector or axial mesons mixings and vector meson dominance, J. Phys. G: Nucl. Part. Phys. 47 , 115102 (2020).

[23] A.A. Nogueira, F.L. Braghin, Spontaneous symmetry breakings in the singlet scalar Yukawa model within the auxiliary field method, arXiv:2103.05133.

[24] M. Lutz, W. Weise, Size of Hadrons, Nucl. Phys. A518, 156 (1990).

[25] J. van der Heide, J. H. Koch, E. Laermann, Pion structure from improved lattice QCD: Form factor and charge radius at low masses, Phys. Rev. D69 D 69, 094511 (2004). D. Drechsel and Th. Walcher, Hadron structure at low Q2, Rev. Mod. Phys. 80, 731 (2008).

[26] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 083C, 01 (2020). K. Nakamura et al (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.