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Chiral symmetry breaking in Curci-Ferrari model

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Lattice simulations show that the running coupling constants obtained through QCD vertices differ in the infrared even though their bare values are the same. For instance, at low momenta the strength of the quark-gluon coupling is about twice the ghost-gluon coupling. In particular, none of them diverge at a finite non-zero infrared momentum scale contrarily to what is predicted by standard perturbation theory. Moreover, the ghost-gluon coupling remains moderate even in the infrared. This observation motivates the use of perturbation theory in the ghost-gluon sector in the frame of a massive deformation of QCD Lagrangian in Landau gauge. However, perturbation theory in the quark sector within this massive Lagrangian does not bring as good results as in the pure Yang-Mills case, which is consistent with the larger value of the quark-gluon coupling constant. We propose, then, a controlled systematic expansion in full QCD based in two small parameters: first the Yang-Mills sector couplings and second the inverse of the number of colors (large-Nc limit). This systematic expansion allows us to properly introduced the use of the renormalization group for the rainbow resummation. At leading order, this double expansion leads to the well-known rainbow approximation for the quark propagator whose solution shows spontaneous chiral symmetry breaking for sufficiently large quark-gluon coupling constant. We present here some of the results discussed in [1, 2] and work in progress.

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1. Introduction

Dynamical chiral symmetry breaking is the emergent infrared phenomena of QCD responsible for the sudden increment in the mass of quarks as the momentum scale decreases. Lot of studies has shown to recover this effect by different truncation in the gap equation for fermions (see e.g. [3–6]).

Another infrared phenomena is the emergent gluon mass observed by lattice simulations This behavior is observed in the region that is usually called the non-perturbative region of QCD since standard perturbation theory within Faddeev-Popov Lagrangian is not valid there. Therefore, non-perturbative techniques have been developed in order to study the "nonpertubative regime" of QCD. Some of them are Dyson-Schwinger equations (DSE), the functional renormalization group or the variational Hamiltonian approach. Lattice simulations also observe that the coupling constants remains finite in the infrared and moreover, specially in the ghost-gluon sector, their values are moderate. This fact supports the idea that some kind of perturbation theory should be possible or at least not as far from reality as within the Faddeev-Popov Lagrangian.

As a fully justified gauge-fixed Lagrangian in the infrared is still not known due to the Gribov ambiguity [7], we proposed to use a Lagrangian motivated by phenomenological observations. Concretely, we propose to add a gluon mass term in Faddeev-Popov Lagrangian in Landau gauge [8, 9]. This renormalizable Lagrangian is a particular case of Curci-Ferrari Lagrangians in Landau gauge [10]. A compendium of results obtained within this model can be found in [11]. In particular, one-loop results within this model give very accurate results for Yang-Mills two and three- point correlation functions when comparing with lattice data (see [9, 12]). Moreover, two-loop corrections in the ghost-gluon sector [13, 14] show that perturbation theory using the Curci-Ferrari model in Landau gauge is well under control. These results support the idea that the ghost-gluon sector within the Curci-Ferrari model is perturbative and therefore the expansion parameter related to Yang-Mills coupling constant is small even in the infrared. In [1, 2] we propose an expansion using two small parameters in the infrared QCD: the Yang-Mills coupling (g_g) and the inverse of the number of colors (N_c). For the large-N_c limit, we use the t'Hooft counting and therefore the coupling constants can be written as $g_g \sim \lambda_g/\sqrt{N_c}$ and, similarly, $g_q \sim \lambda_q/\sqrt{N_c}$ for the quark-gluon coupling constant. In this approximation λ_g can be treated as a small parameter whereas λ_q is not small. Both approximations can be merged together in what we call rainbow-improved (RI) loop expansion [1]. We present here the main ideas of this double expansion as well as some of the results from [2] computed at next to leading order in the RI-loop expansion including the flow of the masses and coupling in a self consistent form. Finally, we also discuss some work in progress.

2. Massive Landau-gauge QCD and the RI expansion

We study the standard Landau gauge-fixing actions in Euclidean space supplemented with a gluon mass term

$$S = \int d^d x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i h^a \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2} m^2_\Lambda (A^a_\mu)^2 + \sum_{i=1}^{N_f} \bar{\psi}_i (\not{D} + M_\Lambda) \psi_i \right]. \tag{1}$$

The covariant derivatives applied to fields in the adjoint (X) and fundamental (ψ) representations read respectively

$$(D_\mu X)^a = \partial_\mu X^a + g_\Lambda f^{abc} A_\mu^b X^c,$$
$$D_\mu \psi = \partial_\mu \psi - i g_\Lambda A_\mu^a t^a \psi,$$

with f^{abc} the structure constants of the gauge group and t^a the generators of the algebra in the fundamental representation. The Euclidean Dirac matrices γ satisfy $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\not{D} = \gamma_\mu D_\mu$ and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_\Lambda f^{abc} A_\mu^a A_\nu^b$ is the field-strength tensor. Finally, the parameters g_Λ, M_Λ and m_Λ are respectively the bare coupling constant, quark mass and gluon mass, defined at some ultraviolet scale Λ. For simplicity, we only consider degenerate quark masses, but the generalization to a more realistic case is trivial. The bare gluon propagator reads

$$G_{0,\mu\nu}^{ab}(p) = \delta^{ab} G(p) \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right). \tag{2}$$

where $G(p) = \frac{1}{p^2 + m_\Lambda^2}$. The bare ghost propagator is

$$G_0^{ab}(p) = \frac{\delta^{ab}}{p^2} = \delta^{ab} G_0(p). \tag{3}$$

The bare quark propagator $S_0(p)$ is:

$$S_0(p) = \left[-i\not{p} + M_\Lambda \right]^{-1}. \tag{4}$$

while the full quark propagator $S(p)$, can be parametrized as:

$$S(p) = \left[-iA(p)\not{p} + B(p) \right]^{-1} = i\tilde{A}(p)\not{p} + \tilde{B}(p), \tag{5}$$

where

$$\tilde{A}(p) = \frac{A(p)}{A^2(p)p^2 + B^2(p)}, \tag{6}$$
$$\tilde{B}(p) = \frac{B(p)}{A^2(p)p^2 + B^2(p)}, \tag{7}$$

so that the tree-level propagator corresponds to $A = 1$ and $B = M_\Lambda$.

In order to renormalize the theory we introduce the renormalization factors, defined as:

$$A_{\mu,\Lambda}^a = \sqrt{Z_A} A_\mu^a \mathrm{C}_\Lambda^\mathrm{a} = \sqrt{\mathrm{Z_C}} \mathrm{C}^\mathrm{a} \quad \text{and} \quad \psi_\Lambda = \sqrt{Z_\psi} \psi, \tag{8}$$

and renormalized masses and coupling constant

$$m_\Lambda^2 = Z_{m^2} m^2, \quad M_\Lambda = Z_M M, \quad \text{and} \quad g_\Lambda = Z_{g_g} g_g. \tag{9}$$

The presence of the mass term makes the theory infrared safe, as a consequence we can define a renormalization scheme without reaching a Landau pole. In this infrared-safe (IS) scheme we extend the non-renormalization theorem for the gluon mass: $Z_{m^2} Z_A Z_C = 1$ also to the finite parts of the renormalization factors. In addition we use the non-renormalization Taylor's theorem for the definition of the ghost-gluon coupling $Z_{g_g}\sqrt{Z_A}Z_C = 1$. The renormalization factors of the quark sector can be fixed by the prescription

$$S^{-1}(p=\mu_0,\mu_0) = -i\not{\mu}_0 + M(\mu_0), \tag{10}$$

where, for short, we use the same notation μ_0 for the RG scale and for an Euclidean vector of norm μ_0.

This renormalization scheme gives a renormalized coupling constant that is finite. Moreover, g_g vanishes at zero momentum and remains moderate at all scales. Therefore a pertubative expansion in this coupling can be can be safely applied. Several quantities have been computed within this model and successfully compared with lattice data (see [11] for an exhaustive discussion). The good agreements could could suggest that the main effect of Gribov copies can be encoded in a gluon mass term. The successful one-loop results within this massive model for Yang-Mills theory convinced us that even in the infrared the ghost-gluon sector can be studied treating the ghost-gluon coupling constant (g_g) as a small parameter. Therefore, this model could allow us to interpret and separate the features which are truly non-pertubative from those that can be understood using perturbation theory.

For this purpose, it is important to mention that one loop results for quark correlators are not as accurate as Yang-Mills ones [15, 16]. We can interpret this since the coupling constant defined from the quark-gluon vertex is two or three times larger than the ghost-gluon coupling. Therefore the expansion parameter, $g^2/(4\pi)N$, is almost nine-times larger in the quark sector becoming of order one. Furthermore, emergent phenomema such as the chiral symmetry breaking can not be reproduced with a finite number of diagrams. In order to know which diagrams we need to take into account it is important to estimate their contribution. We proposed in [1] to organize the diagrams in what we called the Rainbow-improved (RI) expansion. The RI-expansion uses λ_g (defined as $g_g = \lambda_g/N_c$) and $1/N_c$ as small parameters. To be more specific, to study ℓ-loop order in the RI-expansion of any correlation function, we need to write all diagrams of standard perturbation theory with up to ℓ loops, then we count the powers of Yang-Mills coupling (λ_g) and $1/N_c$ that appear in those diagrams and add all diagrams (with possibly more loops) of the same order in λ_g and $1/N_c$. It is important to note, that we are treating on different footing the coupling constant obtained through Yang-Mills vertices (g_g) with respect to the coupling constant defined from the quark-gluon vertex (g_q). Both couplings are related to the same bare value and all UV properties are well preserved.

In [1, 2] several examples of the RI-one-loop approximation are shown. In particular, at RI-leading order ghost and gluon propagators are in its tree-level form but rainbow-ladder equation depicted in Fig. 1 is obtained for the quark propagator. At this order, we can not deduce the running of the coupling so in [1] we study this approximation by using a toy model that gives the standard one-loop flow for the running coupling constant in the UV and saturates in the infrared. This Rainbow equation has been long known to reproduce chiral symmetry breaking, but, the RI-expansion gives a fully consistent way of understand its validity. Moreover, the presence of small parameters allows us to study the corresponding corrections of Rainbow equation arriving in RI-expansion. In particular, the next to leading order in the RI-expansion studied in [2] allows us to consistently compute the running for the masses and coupling constants.

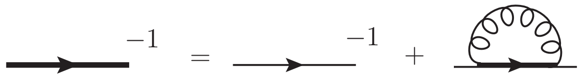

Figure 1: Rainbow-ladder equation for the quark propagator.

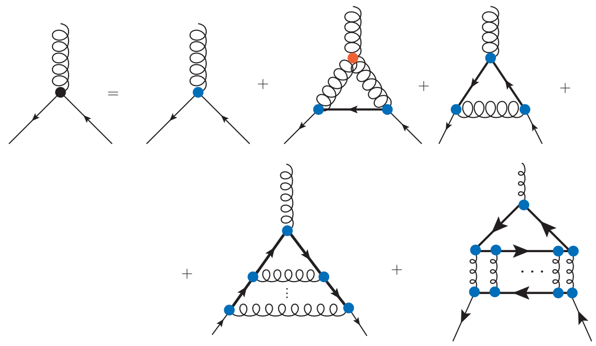

Figure 2: Quark-gluon vertex at RI-one-loop approximation. The blue dots represent the quark-gluon coupling constant while red dots represent Yang-Mills one.

At next to leading order the quark propagator remains with its rainbow-ladder form while the ghost and gluon propagator only include one-loop corrections with rainbow-ladder quark-propagators. This allows one to compute the anomalous dimension associated with the ghost and gluon propagators. Once the anomalous dimension of the ghost and gluons fields are computed through the corresponding propagators, the non-renormalization theorems give the $\beta-$functions for the running of the gluon mass and the Yang-Mills coupling constant. Both flows are coupled to the $\beta-$function of the coupling constant related to the quark-gluon vertex. The latter, can be obtained from RI-one-loop quark gluon vertex depicted in Fig. 2. However, in order to obtained the $\beta-$function at the same RI-one-loop order is not necessary to include all the diagrams needed for the description of the vertex. To obtain a consistent $\beta-$function is suffices to compute only the contribution arriving from the diagram with a three-gluon vertex which encodes one-loop divergences. A detailed explanation is given in [2].

For these reasons, the flows are computed by solving the coupled system:

$$
\begin{aligned}
\mu\frac{dg_q}{d\mu}\Big|_{g_\Lambda,m_\Lambda} &= g_q(\gamma_\psi + \frac{1}{2}\gamma_A) + g_q\mu\frac{d{\lambda_1}^\Lambda}{d\mu}\\
\mu\frac{dg_g}{d\mu}\Big|_{g_\Lambda,m_\Lambda} &= g_g(\gamma_C + \frac{1}{2}\gamma_A)\\
\mu\frac{dm^2}{d\mu}\Big|_{g_\Lambda,m_\Lambda} &= m^2(\gamma_C + \gamma_A)
\end{aligned}
$$

where γ_χ represents the anomalous dimension related to the different fields $\chi = \{A, c, \psi\}$ which is defined as

$$
\gamma_\chi = \mu\frac{d\log Z_\chi}{d\mu}\Big|_{g_\Lambda,m_\Lambda},
$$

and λ_1^Λ the scalar function of the quark-gluon vertex that contains the tree-level contribution. It is important to note that λ_1, γ_A and γ_ψ are also coupled with Rainbow-ladder quark propagator.

We solve the system of coupled equations as a function of the two free parameters g_g and m at 10GeV. In particular, the initial condition for the quark-gluon coupling constant is fixed to the Yang-Mills coupling constant at large momentum scale using standard perturbation theory. If the quark-gluon vertex is computed in the kinematical configuration where both quarks are orthogonal and with equal norm, the relation between the couplings is the following:

$$g_q(\mu) = g_g(\mu)\left(1 + \frac{g_g^2 N}{64\pi^2}(5 - 3\log(2))\right)$$

The advantages of having a self-consistent running is that we can study the range of the gluon mass parameters that can reproduce lattice data for the quark and gluon propagator.

3. Results

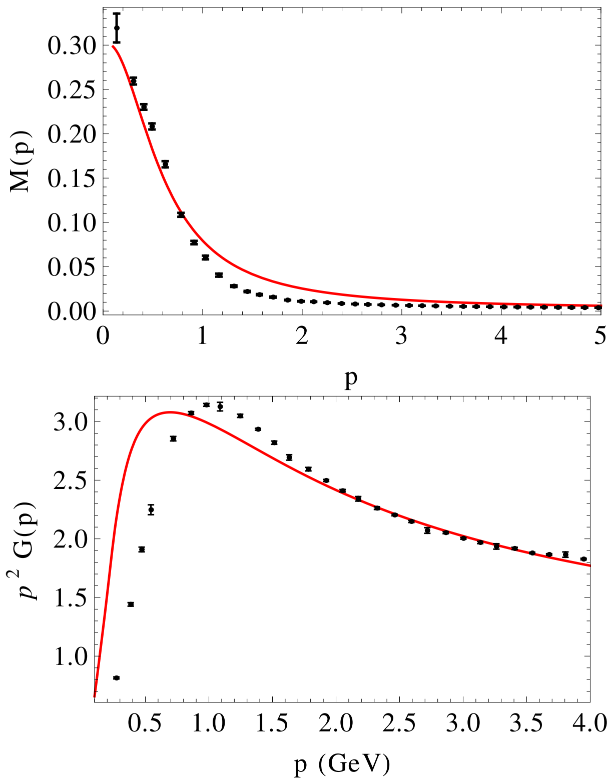

Figure 3: Quark mass function $M(p)$ (top) and gluon dressing function $p^2G(p)$ (bottom) compared with lattice data from [17] and [18], respectively, using $M_0 = 3$ MeV, $m_0 = 0.15$ GeV and $g_0 = 1.94$.

In [2] we solve the renormalized RG-improved Rainbow-equation of Fig. 1 taking into account the flow of the running couplings and the gluon mass at RI-one-loop order. In Fig. 3 we show the comparison of RI-one-loop results of the quark-mass function and the gluon propagator with the corresponding lattice data. This shows that there exists a set of parameters which are capable of reproducing lattice data. Moreover, if only one of these quantities is fitted the agreement obtained is almost perfect. Results in [2] show that a massless gluon is not compatible with the solution of the rainbow equation for the quark mass function.

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The advantage of this analysis is that we can reproduce at this order in the RI-expansion in a self consistent way the correct properties of infrared correlation functions even near the chiral limit. This study opens the door to analyse directly measurable properties of hadrons.

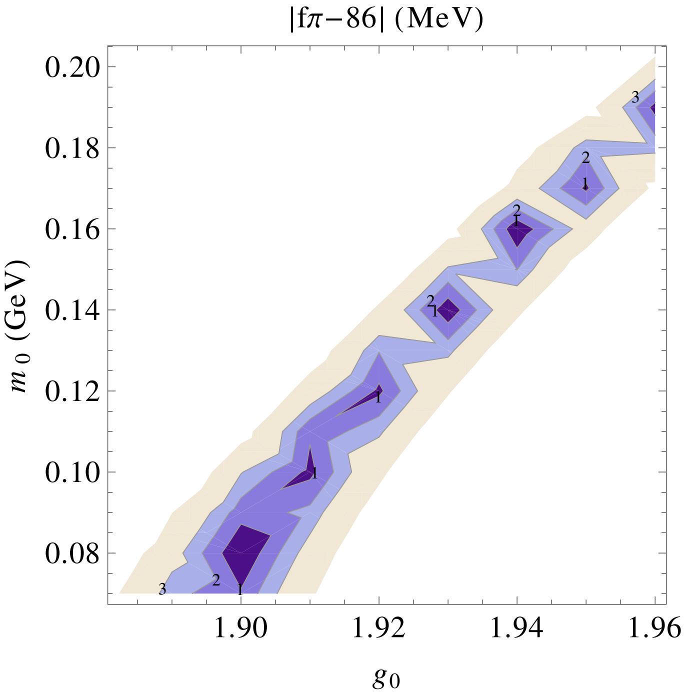

Figure 4: Relative difference of f_π with 86 MeV (the extrapolated chiral value). The contour correspond to $|f_\pi - 86|$ =1,2 and 3 from darker to lighter respectively. f_π was computed using $M(10GeV) = 0.003$ GeV.

We use the results obtained for the quark propagator and flows in this scheme to compute the pion decay constant f_π in the chiral limit. Using the same set of parameter as in Fig. 3 we obtain $f_\pi = 87.9$ MeV which is really close of the expected chiral value 86 MeV. It is important to stress that this value of f_π is a pure prediction of the model since no extra fitting is done. The study can be extended to a larger region of parameters. Preliminary results are shown in Fig. 4 where we study the error for f_π obtained within this model when comparing with the expected chiral value as a function of the free parameters. We can see in Fig. 5 that the region of parameters that give the correct value for f_π matches with the region of parameters that reproduces correctly the quark propagator. In this way, even though our approach gives a well estimated value for f_π, fitting this quantity is not enough to determine the value of the gluon mass.

4. Conclusions

The existence of two small parameters in the infrared QCD with massive gluons gives a way of arranging the different contribution of Feynman diagrams. The next to leading order of the expansion proposed in [1, 2] reproduces chiral symmetry breaking and infrared correlation functions. The running of the masses and coupling is obtained self-consistently within the same order of the double expansion. The latter gives a better understanding of the relevant contributions and allows the computation of measurable quantities such as the pion decay constant.

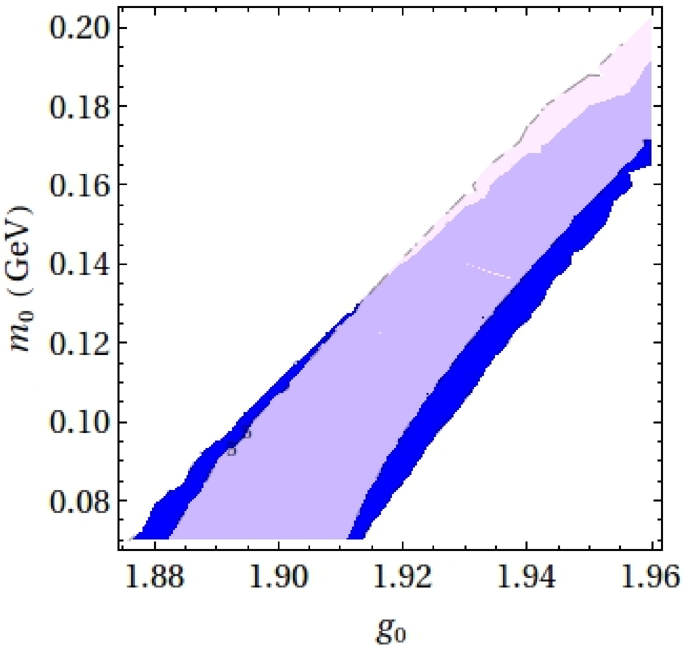

Figure 5: Comparison between region of 5% of error from 4 (light-magenta) with the region of 15% error in the quark mass (blue)

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