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Electrically charged strange stars with an interacting quark matter equation of state

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The properties of electrically charged strange quark stars predicted by an interacting quark matter equation of state (EOS) based on cold and dense perturbative quantum chromodynamics (pQCD) are investigated. The stability of charged strange stars is analyzed considering that there is an electric charge distribution inside the star proportional to the third power of the radial coordinate. A comparison with the predictions derived using the MIT bag model is also presented. We show that the presence of a net electric charge inside strange stars implies in configurations with a larger maximum mass in comparison to their neutral counterparts. Moreover, we demonstrate that the pQCD EOS implies in larger values for the maximum mass of charged strange stars, with very heavy charged stars being stable systems against radial oscillations. For the considered electric charge distribution, the pQCD EOS implies unstable configurations for large values of the renormalization scale as well as for large values of the proportionality constant β, in contrast to the MIT bag model predictions.

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1. Introduction

The description of matter in the extreme conditions of density and/or temperature remains as one of the main challenges for the strong interaction theory – quantum chromodynamics (QCD) (for a recent review, see [1]). While the regime of high temperatures is constantly investigated in heavy ion collisions, for instance, at the LHC, the high baryon densities and low temperatures part of the QCD phase diagram is poorly known. This regime is crucial for a better understanding of the equation of state (EOS) of neutron stars [2], where the density is predicted to exceed nuclear saturation density and the system is expected to be described in terms of deconfined quark degrees of freedom. This conjecture is called the Bodmer-Witten hypothesis [3, 4], where strange quark matter (SQM) is the true ground state of strongly interacting matter, rather than ^{56}Fe. This hypothesis gives rise to the possibility that there is (to some extent) deconfined quark matter inside neutron stars. If SQM is present only in the star's core it is called a hybrid star, however if the star is entirely composed by SQM they are usually denoted strange quark stars (SQS) [5–7]. In this scenario, SQM is expected to be formed through nucleation processes, converting a previously hadronic star (HS) into a SQS with the liberation of a large amount of energy that generates a neutrino burst and an intense gravitational waves emission. In fact, both HSs and SQSs can exist in the Universe, this is the so-called two families scenario, where HSs are neutron stars with masses smaller than $1.5\,\mathrm{M}_{\odot}$ and those above this threshold are SQSs [8].

Robust astrophysical constraints on the EOS came from the two solar mass limit of pulsar PSR J0740+6620 [9] and from gravitational wave data from the GW170817 event [10]. In particular, the two solar mass limit challenged the description of these objects as being quark stars, which were predicted to have smaller masses by phenomenological models such as the MIT bag model. However, the main astrophysical properties of such compact objects are strongly influenced by the description of matter inside the star [7]. In this sense, the EOS defines the magnitude of the internal pressure that keeps the star from a gravitational collapse. During the last years, several phenomenological models have been proposed to describe the EOS for the deconfined quark system, considering different assumptions and approximations for the description of the interaction between quarks, as well as for the treatment of the running quark masses and coupling constant (see, e.g., Refs. [11–22]). In particular, a derivation of the of an EOS based on cold and dense perturbative QCD (pQCD) was performed in Ref. [13]. Assuming a non vanishing strange quark mass they were able to estimate the pressure at finite density at order α_S^2, where α_S is the strong coupling constant. One advantage of this EOS is that it allows the estimation of systematic uncertainties present in perturbative calculations. Furthermore, in Refs. [13, 14, 23] they have shown that SQSs can surpass the two solar mass limit constraint for large values of the re-normalization scale.

In addition to the EOS, the presence of electric charge inside the star can change its structural properties [24–27]. In general, neutrons stars, such as SQSs, are assumed to be electrically neutral. However, electrical neutrality in quark matter with finite quark masses is only achieved with the presence of leptons. In particular, the presence of electrons is necessary to achieve β equilibrium. These electrons are distributed in a layer on the star's surface, which is separated from SQM by several hundred Fermi [5]. These two systems interact through the electrostatic force, which implies in a charge distribution inside the star. In this sense, the Coulombian force acts as an addition to the pressure gradient in order to counterbalance the gravitational pull, and the charged star can remain in equilibrium under its own gravity and electric repulsion. Consequently, as shown in Refs. [28–30] using the simplest version of the MIT bag model and a non-linear EOS, respectively, the charged SQS can withstand larger masses and radii than their neutral counterpart. Moreover, charged SQSs can have a high surface electric field, which can be of order 1×10^{21} V/m [29].

In this paper, we present the predictions from pQCD for charged SQSs that was made by the authors in Ref. [25], comparing it with the known results obtained using the MIT bag model EOS. In particular, the hydro-static equilibrium and dynamical stability considering the interacting quark matter EOS derived in Refs. [13, 14] and a model for the electric charge distribution inside the star are investigated. Previous calculations for radial modes in neutral and charged SQSs were performed in Refs. [23, 28, 29] considering different models for the EOS. Our aim is to investigate the dependence on the EOS of the dynamical stability of charged SQSs against radial perturbations considering an electric charge distribution inside the star.

2. Formalism

Charged SQSs must be described by the Einstein-Maxwell field equations, with the energy-momentum tensor accounting for the energy density associated with the electric field. Due to the magnitude of the electric field on the star' surface, one has that the electric and SQM energy densities are of the same order. Consequently, this high electric field expected on the surface of a SQS affects the space-time metric and the associated energy density contributes to its own gravitational mass. Furthermore, the Coulombian interaction modifies the structure equations that describe the relativistic hydro-static equilibrium (see Ref. [24] and references therein). In order to describe a spherically symmetric static charged star, we assume a line element given by

$$\mathrm{d}s^2 = e^{2\nu(r)}\mathrm{d}t^2 - e^{2\lambda(r)}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,, \tag{1}$$

where λ and ν are known as the metric functions. From the above metric, one obtains that the stellar structure equations of the charged star composed by a perfect fluid are

$$\frac{\mathrm{d}q}{\mathrm{d}r} = 4\pi r^2 \rho_e e^{\lambda}\,, \tag{2}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \epsilon + \frac{q}{r}\frac{\mathrm{d}q}{\mathrm{d}r}\,, \tag{3}$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -(\epsilon + P)\left(4\pi r P + \frac{m}{r^2} - \frac{q^2}{r^3}\right)e^{2\lambda} + \frac{q}{4\pi r^4}\frac{\mathrm{d}q}{\mathrm{d}r}\,, \tag{4}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = -\frac{1}{\epsilon + P}\left(\frac{\mathrm{d}P}{\mathrm{d}r} - \frac{q}{4\pi r^4}\frac{\mathrm{d}q}{\mathrm{d}r}\right)\,, \tag{5}$$

where $\rho_e(r)$ is the electric charge density, $q(r)$ and $m(r)$ represent the charge and mass within radius r, respectively. The metric potential $e^{-2\lambda}$ is of the Reisser-Nordström type

$$e^{-2\lambda(r)} = 1 - \frac{2m(r)}{r} + \frac{q(r)^2}{r^2}\,. \tag{6}$$

Note that for $r > R$ we have $q(r > R) = Q$ and $m(r > R) = M$, where Q and M are the star's total electric charge and gravitational mass, respectively. For neutral stars one has that $q(r) = 0$ and the previous system of equations reduces to the usual Tolman-Oppenheimer-Volkoff equations.

Boundary conditions are necessary to solve such a system of equations. At the center of the star we have that

$$q(0) = m(0) = 0\,, \qquad \epsilon(0) = \epsilon_{\rm c}\,, \qquad \nu(0) = \nu_{\rm c}\,. \tag{7}$$

Moreover, the stellar surface ($r = R$) is reached when $p(R) = 0$, from which we determine the star' structural properties

$$m(R) = M\,, \qquad q(R) = Q\,, \qquad \nu(R) = -\lambda(R)\,. \tag{8}$$

Following the approach proposed by Chandrasekhar [31], it can be demonstrated that the star' stability can be determined from the so-called pulsation equation, which is obtained from the perturbation of fluid and space-time variables without losing spherical symmetry. In its self-adjoint form, the pulsation equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[\mathcal{P}\frac{\mathrm{d}u}{\mathrm{d}r}\right] + [\mathcal{Q} + \omega^2\mathcal{W}]u = 0 \tag{9}$$

where u is the re-normalized displacement function and for a charged star we have that [24]

$$\mathcal{P} = e^{\lambda+3\nu}r^{-2}\gamma P\,, \tag{10}$$

$$\mathcal{Q} = (\epsilon + P)r^{-2}e^{\lambda+3\nu}\left[\nu'\left(\nu' - 4r^{-1}\right) - (8\pi P + r^{-4}q^2)e^{2\lambda}\right]\,, \tag{11}$$

$$\mathcal{W} = e^{3\lambda+\nu}r^{-2}(\epsilon + P)\,, \tag{12}$$

where γ is the adiabatic index. The pulsation equation constitutes a Sturm-Liouville eigenvalue problem, which allows to obtain the eigenvalues and eigenfunctions of the radial perturbations. Defining the auxiliary variable $\eta \equiv \mathcal{P}\mathrm{d}u/\mathrm{d}r$ we can transform the above second-order differential equation into two first order differential equations given by

$$\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{\eta}{\mathcal{P}}\,, \tag{13}$$

and

$$\frac{\mathrm{d}\eta}{\mathrm{d}r} = -[\mathcal{Q} + \omega^2\mathcal{W}]u\,. \tag{14}$$

In Ref. [32], the authors demonstrated that for $\eta(0) = 1$ one has $u(0) = r^3/(3\mathcal{P}(0))$, which are the initial conditions for the integration of the equations from the origin to the surface of the star. As in Refs. [29, 32] we will use the shooting method to obtain the values of ω^2 that satisfy the boundary condition at the surface, which is given by

$$\left.\frac{\mathrm{d}u}{\mathrm{d}r}\right|_{r=R} = \eta(R) = 0\,. \tag{15}$$

The shooting method allows us to change the boundary value problem into an initial value problem, so that the structure equations can be solved using a Runge-Kutta-Cash-Karp method with adaptive step size. In this sense, we start from a trial value for ω^2, from which we obtain the values that satisfy the boundary conditions using the Newton-Raphson root finding method. These values are the eigenfrequencies of the pulsation equation. It must be emphasized that for charged SQSs the condition $\partial M/\partial \epsilon_c > 0$ is not sufficient to determine the stability of the star [29]. Therefore, it is necessary to analyze the radial perturbation modes of such object to determine its stability. Since Q is real, the next eigenfrequency is always larger than the previous one, i.e.,

$$\omega_0^2 < \omega_1^2 < \omega_2^2 \cdots < \omega_n^2 < \cdots .$$

Consequently, is is sufficient to analyze the sign of the fundamental mode to determine the star' stability, considering that unstable stars satisfy $\omega_0^2 < 0$.

In order to solve the structure equations of the charged star, one must specify an EOS and a charge distribution. Regarding the EOS, the simplest model used to describe SQM inside the quark star is the MIT bag model [33], which characterizes a degenerate Fermi gas of up, down and strange quarks. In this model, the main properties of the star only depend on the bag constant B. However, the MIT bag model is not sufficiently powerful to characterize a system with interacting quarks or more complex structures. In our analysis, we will consider the pQCD EOS calculated in Ref. [13] at order α_s^2 and a non-zero value of the strange quark mass. This description was put in a simple to use formula in Ref. [14], being given by

$$P = P_{\mathrm{SB}}(\mu_B)\left(c_1 - \frac{a(X)}{\mu_B - b(X)}\right), \tag{16}$$

where

$$P_{\mathrm{SB}}(\mu_B) = \frac{3}{4\pi^2}\left(\frac{\mu_B}{3}\right)^4 \tag{17}$$

corresponds to the pressure of a gas composed by three mass-less non-interacting quarks, also called a Stephan-Boltzmann (SB) gas, and the functions $a(X)$ and $b(X)$ are auxiliary functions (for details, see Ref. [14]). The dimensionless parameter X is proportional to the re-normalization scale parameter $\bar{\Lambda}$ that arises in the perturbative expansion and is expressed as $X = 3\bar{\Lambda}/\mu_B$. Fixing X, the energy density comes from the following thermodynamical relation

$$\epsilon = -P + \mu_B n_B , \tag{18}$$

where n_B is the baryon number density obtained from the thermodynamical relation

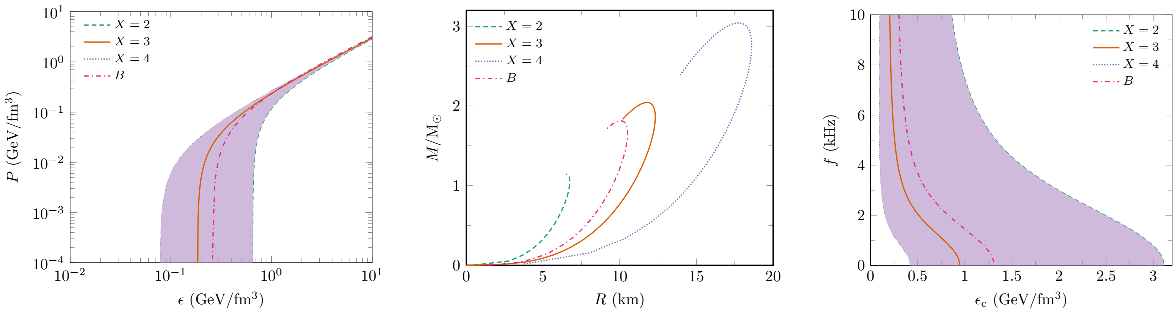
$$n_B = \frac{\partial P}{\partial \mu_B} .$$

Motivated by the studies performed in Refs. [29, 34], we will consider that the charge is proportional to the third power of the radial coordinate as follows

$$q(r) = Q\left(\frac{r}{R}\right)^3 \equiv \beta r^3 , \tag{19}$$

where $\beta \equiv Q/R^3$.

In the next section, we will present our predictions for the mass-radius profile of the SQS as well as for the fundamental mode considering the pQCD EOS and the aforementioned model for the electric charge distribution.



(a) (b) (c)

Figure 1: (a) Comparison between the MIT bag model EOS and the pQCD one considering different values of the re-normalization scale X; (b) Mass – radius profile and (c) linear fundamental frequency for a neutral SQS derived assuming the MIT bag model and pQCD EOS's.

3. Results

A comparison between the EOSs for quark matter predicted by the MIT bag model and by the cold and dense pQCD calculation performed in Ref. [13], is presented in Figure 1 (a). For this comparison, within the MIT bag model we have assumed that the bag pressure is 60 MeV fm^{-3} and that the strange quark has a mass of $m_s = 150$ MeV. For the pQCD EOS we present the predictions derived assuming different values for the re-normalization scale X. We have that the pQCD EOS is strongly dependent on the re-normalization scale, with the band representing the uncertainty associated to this scale.

For completeness, in Figure 1 (b), we show the mass-radius profile for neutral ($q(r) = 0$) SQSs, using the distinct EOSs discussed above. In the case of the pQCD EOS, the SQSs maximum masses depend strongly on the value of X, increasing with X and reaching values larger than 2 $M_\odot$ for $X \gtrsim 3$ [14]. Within our assumptions for the MIT bag model, we have verified that its predictions are similar to those obtained from the pQCD EOS for $X \approx 2.8$, where both models predict values of maximum mass smaller than 2 $M_\odot$. As already pointed out in Ref. [23], only values of X in the range between 3 and 3.2 satisfy simultaneously the GW170817 constraints of mass and radius [10].

To finish the analysis of the neutral case, we present in Figure 1 (c) the comparison between the fundamental eigenfrequencies obtained from pQCD (for different values of X) and the MIT bag model as a function of the central energy density. For convenience, we present the results for the real part of the linear frequency associated to the eigenfrequency by $f = \omega/2\pi$. In agreement with the results presented in Ref. [23], our results indicate that the configurations for the distinct values of X are stable against radial oscillations ($\omega_0^2 > 0$). We can also see that the last stable configuration (the one for which $f = 0$) happens for smaller values of the central energy density for the stiffer pQCD EOSs ($X \geq 2.8$) in comparison to the MIT bag model predictions.

In what follows, we will present our results for charged SQSs considering the pQCD EOS for $X = 3$. For comparison, we will also present the predictions derived using the massive MIT bag model EOS with $B = 60$ MeV fm^{-3} for the bag pressure. First, we present in Fig. 2 the gravitational mass as a function of the central energy density (top panels) and the radius (central panels) considering the previously mentioned charge distribution assuming different constant values

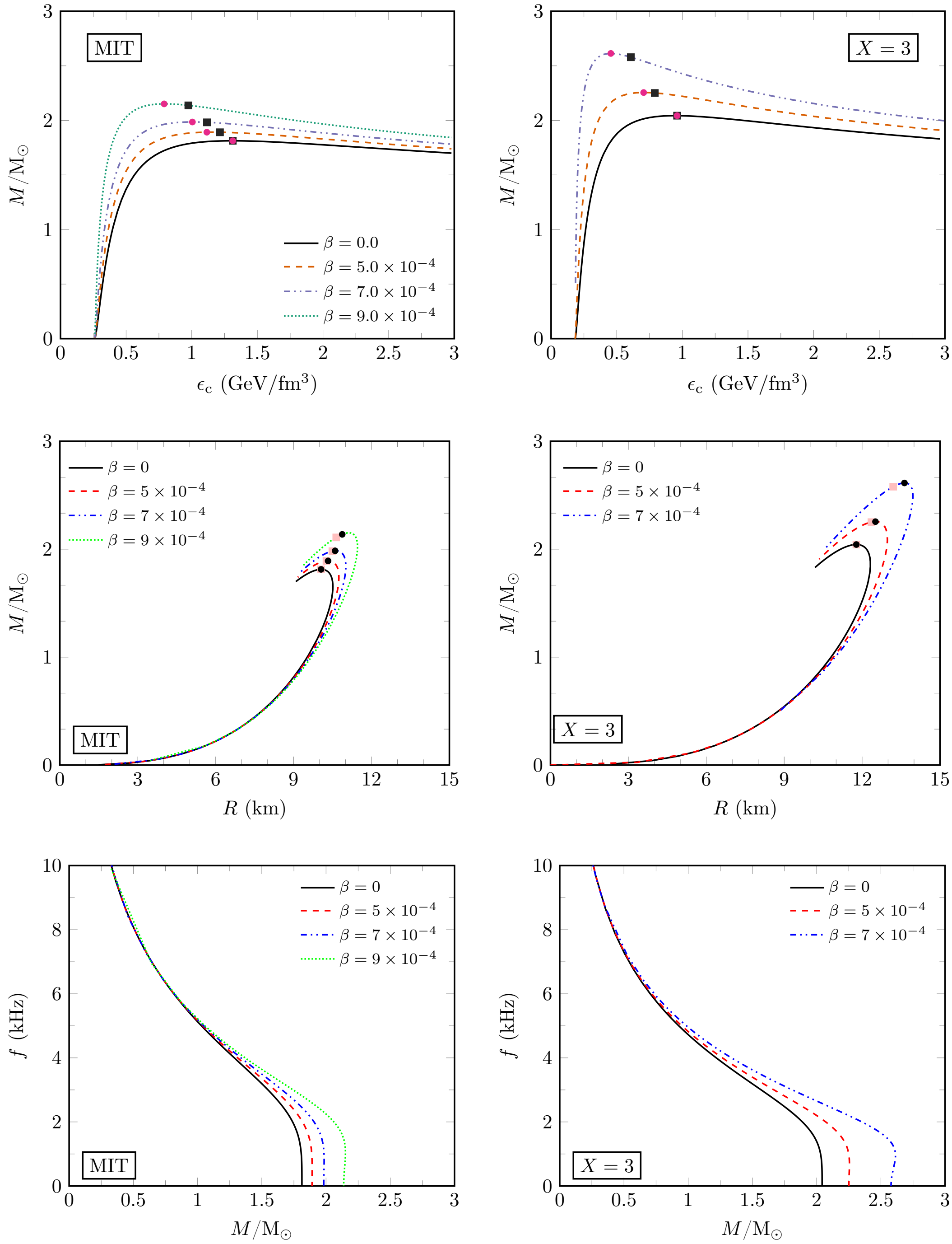


Figure 2: Predictions from the MIT bag model (left panels) and pQCD (right panels) EOSs for the gravitational mass as a function of the central energy density (top panels) and the radius (center panels), considering different values of β, where $\beta = 0$ (black solid line) corresponds to a neutral strange star. We also present the fundamental linear eigenfrequency as a function of the mass (bottom panels). The full squares represent the configurations for which $\omega_0^2 = 0$ and the full circles represent the maximum mass configurations.

of β, with $\beta = 0$ corresponding to the neutral SQS. We also present the fundamental linear eigenfrequency as a function of the mass (bottom panels). Our results show that for both EOSs the presence of charge increases the mass and radius of the stellar configurations for a given central energy density. In particular, increasing the charge diminishes the central energy density required to produce the most massive configuration (filled circle). Opposite to the neutral case, the maximum mass does not coincide with the last stable configuration (filled square), in fact, our results suggest that there are stable charged configurations beyond the maximum mass point, where $\partial M/\partial\epsilon_c < 0$, in agreement with Ref. [29]. From our results, we can see that the predictions derived using the pQCD EOS are even more sensitive to the presence of charge in the star, since the same variation of β leads to a considerable larger modification in the mass and radius of the respective stellar configurations in comparison to the MIT bag model predictions. We believe that this difference is due to the stiffness of the EOSs, being the pQCD one stiffer.

In our analysis, we have obtained that increasing β up to $9 \times 10^{-4}\,\mathrm{M_\odot km^{-3}}$ always produces stable configurations, in which $\partial M/\partial\epsilon_c > 0$ and $\omega_0^2 > 0$, for the MIT bag model EOS. In contrast, for the pQCD EOS, the same value of β gives an unstable solution for all values of ϵ_c. For this reason the associated predictions are not presented on the right panels of Fig. 2. A similar instability only occurs in the MIT bag model predictions for $\beta \gtrsim 5 \times 10^{-3}\,\mathrm{M_\odot km^{-3}}$.

4. Summary

In this paper, the equilibrium and stability of charged strange stars were investigated, considering the EOS derived from cold and dense perturbative QCD which takes into account the short range interaction between quarks. As this EOS is based on a first principle calculation, its predictions for SQSs are expected to be more realistic in comparison to those derived e.g. using the MIT bag model. We have accounted for the presence of electric charge in SQSs and have performed a detailed comparison between the predictions derived using the pQCD and MIT bag model EOSs. For both EOSs, we have verified that the presence of a net electric charge implies in SQSs with larger maximum masses in comparison to their neutral counterparts. However, the pQCD EOS leads to larger values for the maximum mass of the charged SQS, with very heavy charged stars being stable systems against radial oscillations. Finally, our results also demonstrated that the pQCD EOS produces unstable configurations for large values of the re-normalization scale X as well as for large values of the constant β, in contrast to the MIT bag model predictions.

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References

[1] R. Pasechnik and M. Sumbera, Universe **3**, no.1, 7 (2017)

[2] G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song and T. Takatsuka, Rept. Prog. Phys. **81**, no.5, 056902 (2018)

[3] A. Bodmer, Phys. Rev. D **4**, 1601 (1971).

[4] E. Witten, Phys. Rev. D **30**, 272 (1984)

[5] C. Alcock, E. Farhi and A. Olinto, Astrophys. J. **310**, 261 (1986)

[6] C. Alcock and A. Olinto, Ann. Rev. Nucl. Part. Sci. **38**, 161 (1988)

[7] N. K. Glendenning, *Compact stars* (Springer Science & Business Media, New York, 1997).

[8] A. Drago and G. Pagliara, Eur. Phys. J. A **52**, 41 (2016).

[9] H. T. Cromartie, *et al.* Nat. Astron. **4**, no.1, 72 (2019)

[10] B. Abbott *et al.* [LIGO Scientific and Virgo], Phys. Rev. Lett. **119**, no.16, 161101 (2017)

[11] G. Lugones and J. E. Horvath, Phys. Rev. D **66**, 074017 (2002)

[12] Y. Ivanov, A. Khvorostukhin, E. Kolomeitsev, V. Skokov, V. Toneev and D. N. Voskresensky, Phys. Rev. C **72**, 025804 (2005)

[13] A. Kurkela, P. Romatschke and A. Vuorinen, Phys. Rev. D **81**, 105021 (2010)

[14] E . S. Fraga, A. Kurkela and A. Vuorinen, Astrophys. J. Lett. **781**, no.2, L25 (2014)

[15] J. Torres and D. Menezes, EPL **101**, no.4, 42003 (2013)

[16] A. Zacchi, R. Stiele and J. Schaffner-Bielich, Phys. Rev. D **92**, no.4, 045022 (2015)

[17] T. Kojo, P. D. Powell, Y. Song and G. Baym, Phys. Rev. D **91**, no.4, 045003 (2015)

[18] S. Benic, D. Blaschke, D. E. Alvarez-Castillo, T. Fischer and S. Typel, Astron. Astrophys. **577**, A40 (2015)

[19] J. F. Xu, G. X. Peng, F. Liu, D. F. Hou and L. W. Chen, Phys. Rev. D **92**, no.2, 025025 (2015)

[20] H. Chen, J. B. Wei and H. J. Schulze, Eur. Phys. J. A **52**, no.9, 291 (2016)

[21] M. Dutra, O. Lourenço and D. P. Menezes, Phys. Rev. C **93**, no.2, 025806 (2016)

[22] C. J. Xia and S. G. Zhou, Nucl. Phys. B **916**, 669 (2017)

[23] J. C. Jimenez and E. S. Fraga, Phys. Rev. D **100**, no.11, 114041 (2019)

[24] A. Brillante and I. N. Mishustin, Eur. Phys. Lett. **105**, no.3, 39001 (2014)

[25] V. P. Gonçalves and L. Lazzari, Phys. Rev. D **102**, 034031 (2020)

[26] V. P. Gonçalves and L. Lazzari, Nucl. Phys. A **1004**, 122043 (2020)

[27] V. P. Gonçalves, L. Lazzari and J. Jiménez, Eur. Phys. J. C **82** 110 (2022)

[28] R. P. Negreiros, F. Weber, M. Malheiro and V. Usov, Phys. Rev. D **80**, 083006 (2009)

[29] J. D. V. Arbanil and M. Malheiro, Phys. Rev. D **92**, 084009 (2015)

[30] G. Panotopoulos and A. Rincon, Eur. Phys. J. C **79**, no.6, 524 (2019)

[31] S. Chandrasekhar, Phys. Rev. Lett. **12**, 114 (1964)

[32] K. Kokkotas and J. Ruoff, Astron. Astrophys. **366**, 565 (2001)

[33] A. Chodos, R. Jaffe, K. Johnson, C. B. Thorn and V. Weisskopf, Phys. Rev. D **9**, 3471 (1974)

[34] D. Deb, S. V. Ketov, M. Khlopov and S. Ray, JCAP **10**, 070 (2019)