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Cosmological implications of the QCD phase transition in the Early Universe

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A QCD phase transition is predicted to occur during the early stages of the dynamical evolution of the Universe, with the order and the observational implications of this phase transition being themes of debate. In this contribution we investigate the impact on the cosmological parameters of different treatments for the phase transition and distinct models for the equations of states (EoSs) of the partonic and hadronic phases. In particular, we consider that the phase transition can be of first order or a crossover and estimate the Hubble, deceleration and jerk parameters. Our results demonstrate that these parameters are sensitive to the modelling of the QCD phase transition.

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1. Introduction

The fact that the Universe is expanding in accelerated way is well known since the end of twenty century [1–3]. The mathematical scope to describe this comes from the formalism of General Relativity through the field equations that has been proposed by A. Einstein [4]. Recent observations provide evidence that at large scale (>100 Mpc) the Universe can be take as being homogeneous and isotropic, the called Cosmological Principle [5, 6]. Taking into account a metric that describe the Cosmological Principle one can derive the Friedmann's equation [7, 8] which derscribe the expansion of the Universe. An investigation of the Friedmann's equation regarding diferent compositions and the fact that the Universe is almost flat [9, 10] provide that the history of the Universe is characterized by three distinct periods [11]: radiation–, matter– and Λ–dominated periods[1]. In what follows, we will focus on the period in which the Universe is dominated by radiation, which occurs when the age of the Universe is $<$ 23 000 yr and its temperature is greater than 70 MeV. During this period, two distinct phase transition are predicted to occur: the electroweak and strong (QCD) phase transitions. In our analysis, we will restrict our discussion to the QCD phase transition. There are two important conerstones that support the existence of this phase transition, they are confinement [13] and asymptotic freedom [14–17]. The confinement is a feature of Quantum Chromodynamics (QCD) which ensure that at low energies (temperatures) quarks and gluons are confined inside the hadrons. On the other hand, the asymptotic freedom, implies that at high energies (temperatures) these quarks and gluons behaves like free particles. As a consequence, for the very high temperatures predicted to be present in early phase of the Universe, the quark and gluons are expected to form a quark - gluon plasma (QGP). Moreover, with the decreasing of the temperature due to the evolution of the Universe, the phase transition from the partonic to the hadronic phase is expected to be present.

The nature and description of the QCD phase transition are still under debate. In this contribution we investigate the impact of the modelling of the QCD phase transition on the cosmological parameters, which are related to time derivatives of scale factor which describes how scales evolves in an expanding homogeneous and isotropic Universe. We consider distinct models for the equations of state (EoS) of the partonic and hadronic phases and estimate the Hubble, deceleration and jerk parameters assuming that the phase transition is of first order or a crossover. As we will demonstrate in what follows, our results indicate that these parameters are sensitive to the description of the QCD phase transition.

This contribution is organized as follows. In section 2 the formalism needed to describe the evolution of the Universe is briefly reviewed. In section 3 the EoS for hadronic and partonic phases are introduced as well as the interpolation to be use in some cases. Our results are presented in section 4 and our main conclusions are summarized in section 5.

2. Formalism

The expansion of the Universe is described by the Friedmann's equation [7, 8] which can be derived from General Relativity [4] making use of the Robertson–Walker metric [18–21]. Due to

[1] The Λ designate the Cosmological Constant that accounts the contribution of Dark Energy (see *e.g.* [12] and references therein) whose nature is theme of intense debate.

the fact that we are focused in radiation–dominant period the Friedmann's equation reads

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\varepsilon} \,, \tag{1}$$

in natural units. In Eq. (1) a is the scale factor, G is the gravitational constant and ε is the energy density. In our notation, the dot above some quantity means time derivative. From the stress–energy tensor one can derive the fluid equation, which is given by

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}\left(\varepsilon + P\right) = 0 \,, \tag{2}$$

where P is the pressure. Combining the above equations we can write

$$\dot{\varepsilon} = -3\sqrt{\frac{8\pi G}{3}\varepsilon}\left(\varepsilon + P\right) \ . \tag{3}$$

In order to solve the Eq. (1) and Eq. (3), the equation of state (EoS) should be specified. In section 3 we will introduce some of those EoS.

The cosmological parameters can be defined considering the expansion of the scale factor around an arbitrary time $\tilde{t}$, such that

$$\begin{aligned} a(t) &= \sum_{n=0}^{\infty} \frac{a(\tilde{t})^{(n)}}{n!}(t-\tilde{t})^n \\ &= a(\tilde{t})\left[1 + \frac{\dot{a}(\tilde{t})}{a(\tilde{t})}(t-\tilde{t}) + \frac{\ddot{a}(\tilde{t})}{2!a(\tilde{t})}(t-\tilde{t})^2 + \frac{\dddot{a}(\tilde{t})}{3!a(\tilde{t})}(t-\tilde{t})^3 + \cdots\right] \ . \end{aligned} \tag{4}$$

One has that the Hubble, deceleration and jerk parameters will be given by [22–24]

$$H \equiv \frac{\dot{a}}{a} \,, \quad q \equiv -\frac{\ddot{a}}{aH^2} \quad \text{and} \quad j \equiv \frac{\dddot{a}}{aH^3} \,, \tag{5}$$

respectively. In what follows, the solutions from Eq. (1) and Eq. (3) will used as input to estimate these cosmological parameters.

3. Equations of State

In this Section we will present a brief review of the equations of state for partonic and hadronic phases used in our analysis. Moreover, the interpolation procedure will also be discussed.

3.1 Hadronic Phase

3.1.1 Hadronic Model I: Pion Gas

The simplest model to describe the hadronic phase is that this phase is characterized by a gas of massless non–interactive pions (PG). The associated EoS will be given by

$$\varepsilon(T) = g_H \frac{\pi^2}{30} T^4 \,, \tag{6}$$

$$P(T) = g_H \frac{\pi^2}{90} T^4 \,, \tag{7}$$

where $g_H = 3$ are the degrees of freedom for the pion gas formed by the three possible states π^-, π^0 and π^+.

a_1	a_2	a_3	a_4
4654 GeV^{-1}	−879 GeV^{-3}	8081 GeV^{-4}	−7 039 000 GeV^{-10}

Table 1: Parameters for Eq. (8) [25].

3.1.2 Hadronic Model II: Hadron Resonance Gas

A more realistic model to describe the hadronic phase is the Hadron Resonance Gas model (HRG), which considers the presence of a larger number of hadronic states. In our analysis, we will consider that the EoS is given by the parametrization proposed in Ref. [25], which is expressed as follows

$$\frac{\varepsilon - 3P}{T^4} = a_1 T + a_2 T^3 + a_3 T^4 + a_4 T^{10} \ , \tag{8}$$

whose parameters can be found in Table 1.

3.2 Partonic Phase

3.2.1 Partonic Model I: MIT Bag Model

The simplest model for the partonic phase in the early Universe is the MIT Bag Model [26], which assume that quarks and gluons are confined in a finite region commonly called bag in which they can be treat perturbatively [27]. The EoS for the MIT Bag Model is given by

$$\varepsilon(T) = g_{QGP}\frac{\pi^2}{30}T^4 + B \ , \tag{9}$$

$$P(T) = g_{QGP}\frac{\pi^2}{90}T^4 - B \ , \tag{10}$$

where g_{QGP} determines the degrees of freedom present in QGP. In our analysis, we have taken into account three quark flavours, which implies

$$g_{QGP} = g_{\mathrm{g}} + \frac{7}{8}(g_{\mathrm{q}} + g_{\bar{\mathrm{q}}}) = 2 \times 8 + \frac{7}{8}2(2 \times 3 \times 3) = 47.5 \ , \tag{11}$$

with $7/8$ being associated to the Fermi–Dirac statistics. In addition, B is the bag pressure (or bag parameter) that ensures the confinement. Here we consider $B^{1/4} = 200$ MeV.

3.2.2 Partonic Model II: C-Bag Model

In Ref. [28, 29], the authors have proposed a modification of MIT Bag Model, denoted hereafter C Bag Model (C–BM), which takes into account of the interation between the partons and generates predictions similar to those obtained in Lattice QCD [30, 31]. The associated EoS is given by

$$\varepsilon(T) = \sigma T^4 - CT^2 + B \ , \tag{12}$$

$$P(T) = \frac{\sigma}{3}T^4 - CT^2 - B \ , \tag{13}$$

whose parameters are [32]

$$\sigma = 13.01 \ , \qquad C = 6.06T_i^2 \ , \qquad B = -2.34T_i^4 \ , \tag{14}$$

where T_i is the inflection temperature.

a_n	b_n	c_n	d_n
−8.7704	3.9200	0	0.3419
a_d	b_d	c_d	d_d
−1.2600	0.8425	0	−0.0475
c_t	$\tilde{t}$	t_0	P_{id}
3.8706	T/T_C	0.9761	$95\pi^2/180$

Table 2: Parameters for Eq. (15). Extracted from Ref. [33].

3.2.3 Partonic Model III: Lattice Ansatz

In recent years, several groups have study finite temperature Lattice QCD in order to derive the QCD EoS and have obtained very promising results. However, such subject is still in development and the results obtained by distinct groups differ, mainly in the description of the system at small temperatures and different number of flavours and finite values for the quark masses. In our analysis, we will consider two different parameterizations for the lattice QCD results. One of them is the parametrization proposed by the HotQCD collaboration, denoted lattice ansatz - 1 hereafter, which is given by [33]

$$\frac{P}{T^4} = \frac{1}{2}\left\{1 + \tanh\left[c_t\left(\tilde{t} - t_0\right)\right]\right\} \frac{P_{id} + a_n/\tilde{t} + b_n/\tilde{t}^2 + c_n/\tilde{t}^3 + d_n/\tilde{t}^4}{1 + a_d/\tilde{t} + b_d/\tilde{t}^2 + c_d/\tilde{t}^3 + d_d/\tilde{t}^4} \,. \tag{15}$$

The parameters in Eq. (15) can be found in Table 2. It is important to point out that $T_C = 154$ MeV is a QCD transition temperature suitably chosen and P_{id} is the P/T^4 value in the ideal gas limit [33].

In addition, we also will consider the parameterization proposed by the Wuppertal–Budapest collaboration, in which the trace anomaly is parametrized as [30]

$$\frac{\epsilon - 3P}{T^4} = \exp\left(-\frac{h_1}{t} - \frac{h_2}{t^2}\right)\left\{h_0 + \frac{f_0\left[\tanh\left(f_1 t + f_2\right) + 1\right]}{1 + g_1 t + g_2 t^2}\right\} \,, \tag{16}$$

whose parameters are presented in Table 3. Henceforth the associated results will be denoted by lattice ansatz-2. It is important to stress that this parametrization, according to Ref. [30], can be extrapolated for small values of temperature, with its predictions in this region being similar to those derived assuming a hadronic phase.

3.3 Interpolation

In our analysis we will consider the following combinations of partonic and hadronic EoSs: MIT Bag Model with Pion Gas (MIT+PG), MIT Bag Model with Hadron Resonance Gas Model (MIT+HRG), C–BM with Hadron Resonance Gas Model (C–BM+HRG) and lattice ansatz-1 with Hadron Resonance Gas Model (LA1+HRG). For C–BM+HRG and LA1+HRG models, an interpolation is needed to connect the partonic and hadronic EoSs. Following Ref. [24] we will assume

h_0	h_1	h_2
0.1396	−0.1800	0.0350
f_0	f_1	f_2
2.76	6.79	−5.29
t	g_1	g_2
$T/200\,\mathrm{MeV}$	−0.47	1.04

Table 3: Parameters for Eq. (16). From Ref. [30].

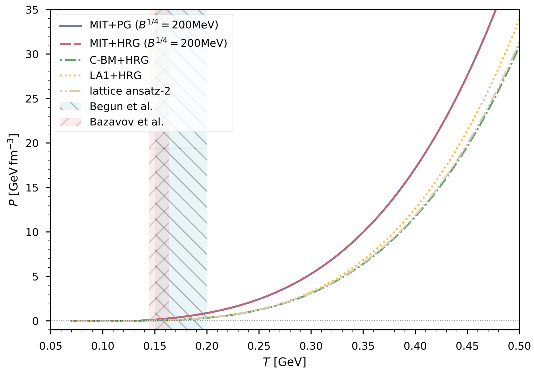


Figure 1: Behaviour of the pressure as a function of temperature predicted by the MIT+PG (continuous), MIT+HRG (dashed) with $B^{1/4} = 200\,\mathrm{MeV}$ and C–BM+HRG (dash–dot), LA1+HRG (dotted) and lattice ansatz-2 (dash–dot–dot) models. The shaded regions delimitate de region where the crossover is expected to occur as predicted in Ref. [32] (pattern "\") and Ref. [33] (pattern "/").

that it is given by [24]

$$P_{HRG}(T) = P_{mod}(T_l)\left(\frac{T}{T_l}\right)^4 + g(T)\,, \tag{17}$$

where $g(T)$ reads

$$g(T) = T^4\left[a_1\,(T - T_l) + \frac{a_2}{3}\left(T^3 - T_l^3\right) + \frac{a_3}{4}\left(T^4 - T_l^4\right) + \frac{a_4}{10}\left(T^{10} - T_l^{10}\right)\right]\,, \tag{18}$$

where P_{mod} and T_l are the pressure for partonic model and the interpolation temperature, respectively. The parameters can be found in Table 1.

4. Results

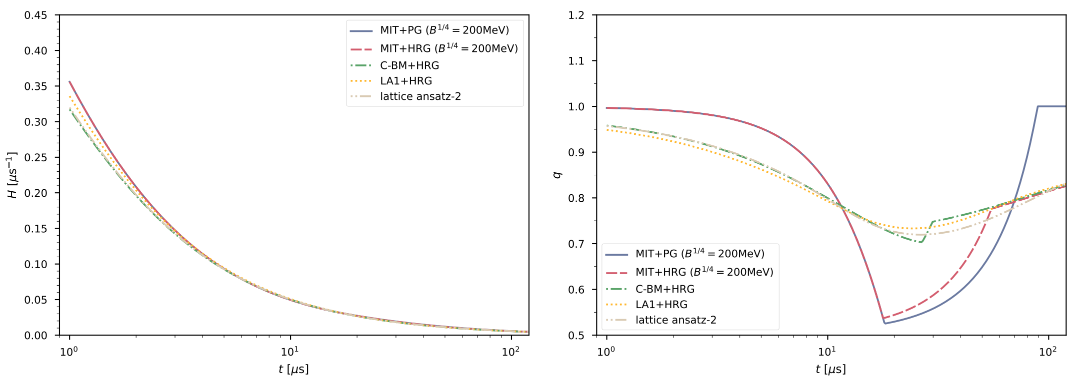
Initially, in Figure 1 we present a comparison between the EoS considered in this contribution. One has that all the results for pressure is continuous at transition as expected. The MIT+PG and

MIT+HRG models predict larger values for the pressure at large temperatures, while those based on the Lattice results predict smaller values. The main difference between the predictions occurs in the transition region.

In Figure 2a we present our predictions for the Hubble parameter H. One has that for the hadronic phase, *i.e.*, for later times, the predictions of the distinct models are very similar. One the other hand, for early time, we can notice that the LA1+HRG, lattice ansatz 2 and and C–BM + HRG models predict smaller values of H than those predicted by the MIT+PG and MIT+HRG models.

The predictions for the deceleration parameter q are presented in Figure 2b. In this case, one has that MIT+PG and MIT+HRG models predict a strong decrease of the q parameter during the phase transition. Such behaviour is directly associated to the first order phase transition considered by these models. In contrast, for a crossover transition, as assumed in the LA1+HRG and lattice ansatz 2 models, such decreasing is smaller and the behaviour of q is smooth. For the C–BM+HRG model, a small dip is predicted, which is associated to the fact that it is based on the MIT bag model. Such results demonstrate that the behavior of q in the transition region is sensitive to the EoS considered.

Finally, the results for the jerk parameter j are presented in Figure 3. One has that for a first order transition, as predicted by the MIT+PG and MIT+HRG models, j is constant during the transition, strongly decreasing (increasing) at the beggining (end) of the transition, with the behaviour at large t being sensitive to the hadronic model considered. On the other, for a crossover, a smooth behaviour for the time dependence of j is predicted, with the LA1+HRG and lattice ansatz-2 predictions being similar. One also has that the C–BM+HRG results are similar to these predictions, but differ by the presence of a rapid modification during the transition, which is reminescent of the MIT bag model used as input in its construction. The results presented in Figure 3 indicate that the jerk parameter, as well the deceleration one, is sensitive to the modelling of the phase transition.



(a) Hubble parameter

(b) Deceleration parameter

Figure 2: Behaviour of the (a) Hubble H and (b) deceleration q parameters as a function of time predicted by the MIT+PG (continuous), MIT+HRG (dashed) with $B^{1/4} = 200\,\text{MeV}$ and C–BM+HRG (dash–dot), LA1+HRG (dotted) and lattice ansatz-2 (dash–dot–dot) models.

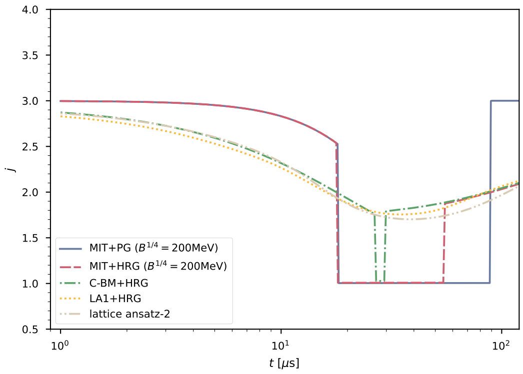


Figure 3: Behaviour of the jerk parameter j as a function of time predicted by the MIT+PG (continuous), MIT+HRG (dashed) with $B^{1/4} = 200\,\text{MeV}$ and C–BM+HRG (dash–dot), LA1+HRG (dotted) and lattice ansatz-2 (dash–dot–dot) models.

5. Summary

In this work we have studied the impact of the modelling of the strong phase transition on the cosmological parameters H, q and j, which are defined in terms of time derivatives of the scale factor. We have considered two distinct alternatives for the phase transition (first order or crossover) and distinct models for the partonic and hadronic EoS. Our results have demonstrated that the normalization of H at early times is sensitive to the model considered. However, all models predict a smooth time dependence for this cosmological parameter. In constrast, our results indicated that the behaviour of q and j in the transition regime is strongly dependent of the modelling of the phase transition. While for a crossover transition the behaviour of these cosmological parameters is smooth, one has that if the transition is of first order, these quantities are strongly reduced during the phase transition. Such distinct behaviours can be useful to constrain the modelling of the EoS and improve our understanding about the QCD phase transition.

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