

A comparative analysis for the form factors and coupling constant of the $D_s DK^*$ meson vertex

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In this proceeding, we made an critical comparison between our method used in the calculation of form factor and coupling constant using the QCD Sum rules method and other one method using also QCDSR (Eur.Phys.J.C 78 (2018) 7, 606).

The vertex that we are comparing is the $D_s DK^*$, that is important to compute cross section of the process $B \rightarrow K^* \pi$ within effective theories.

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In this paper, we report the result of our study of the coupling constant for the vertex $D_s DK^*$ using QCD Sum Rules (QCDSR) [1]. This vertex were also studied in previous works using different techniques such as SU(4), Light Cone Sum Rules (LCSR) [2], and a previous study also using QCD Sum Rules(QCDSR) [3]. The numerical value of the coupling constant obtained in these studies are significantly different from each other. Regarding the SU(4) and the LCRS methods, the procedures and techniques used to obtain the coupling constant are very different from the ones used in the QCDSR technique. As for the QCDSR, different procedures and constraints also can lead to discrepancies in the results.

The QCDSR that were developed by Shifman, et all. [4] is the technique used in our studies of hadronic vertices, that were reviewed in Ref. [5]. In the QCDSR formalism, to obtain a numerical result of the coupling constant, it is necessary to calculate the three point correlation function, in two ways: the QCD side and the Phenomenological side. After the match of both sides of the sum rule, this calculation give us the form factor of the vertex, that is a function $g(Q^2)$, where Q is the momentum of the of the particle off-shell, in the process. The form factor is the inner information of the interaction of all particles evolved in the process. There is currently no direct experimental information about any form factors of hadronic vertices. The coupling constant is defined as a numerical value when the form factor, $g(Q^2)$, assume the unique value obtained by $Q^2 = -m^2$. As the QCDSR is valid in the momentum space where $Q^2 > 1 \text{ GeV}^2$, it is necessary to extrapolate the results of the SR to achieve the $Q^2 = -m^2$, which means that the particle is on-shell. This extrapolation is the source of one of the more delicate approximations that causes uncertainties of the method.

Therefore in a vertex with three different mesons we can obtain three different form factors. When each particle can be a projectile of the process, this projectile hits the other two particles that are the targets of the process, and each projectile "sees" the other two in a different way and it brings up three form factors. But since we have an unique coupling constant, the calculation of the three form factors gives only one coupling constant, thus minimizing the uncertainties of the technique.

As we are working for a long time in this subject, we developed a technique to improve the method with less uncertainties, that no only includes the calculation of the three form factors, but also the pole-continuum contributions, the OPE convergence, the good window of Borel mass and momentum space in the SR and the better functions used to extrapolate the result of the sum rule.

In this work we like to comment the difference that exists within the QCDSR calculation of this vertex, between our technique and the one presented in Ref. [3]. We also compare our results with the ones from LCSR and SU(4).

In our QCDSR study of the vertex $D_s DK^*$ we calculate the form factors for the three possible cases of off-shell mesons for this vertex: $M = D_s, D, K^*$. The details of the calculation of this sum rule can be seen in [1].

The numerical analysis of the sum rule were performed considering the constraints imposed by the QCDSR: pole over continuum dominance, OPE convergence, stability of Borel mass and Q^2 windows. All these constraints are important to obtain reliable results within the region of validity of the sum rules, in the Euclidean region $Q^2 > 1 \text{ GeV}^2$.

The form factors were fitted to the numerical data computed as either a monopolar function, or an exponential function, as shown in Fig. (1). The form factors are chosen according to the best fit of each form factor to the numerical data.

The coupling constant is then obtained by extrapolating the form factors outside of the Euclidean region $Q^2 < 0$, and then computing the value of the form factor at the off-shell meson pole:

$$g_{D_s DK^*}^{(M)} = g_{D_s DK^*}^{(M)}(Q^2 = -m_M^2). \quad (1)$$

In Table 1, we show for the three form factors, the function that fits the numerical data to achieve the better value of the coupling constant, the cut-off parameters, and the value of coupling constant achieved with each SR.

Form factor	Function	\mathcal{A}	\mathcal{B}	$g_{D_s DK^*}^{(M)}$
$g_{D_s DK^*}^{(D)}(Q^2)$	$\frac{\mathcal{A}}{\mathcal{B}+Q^2}$	109.52	45.140	$2.63_{-0.23}^{+0.23}$
$g_{D_s DK^*}^{(K^*)}(Q^2)$	$\mathcal{A} e^{-\mathcal{B}Q^2}$	1.488	0.560	$1.91_{-0.21}^{+0.20}$
$g_{D_s DK^*}^{(D_s)}(Q^2)$	$\frac{\mathcal{A}}{\mathcal{B}+Q^2}$	50.431	30.332	$2.32_{-0.55}^{+0.62}$

Table 1: Numerical results for the parametrization of the form factors and corresponding coupling constants.

The final result for the coupling constant of the vertex is taken from the mean value of the coupling constants presented in Table 1:

$$g_{D_s DK^*} = 2.29_{-0.41}^{+0.65}. \quad (2)$$

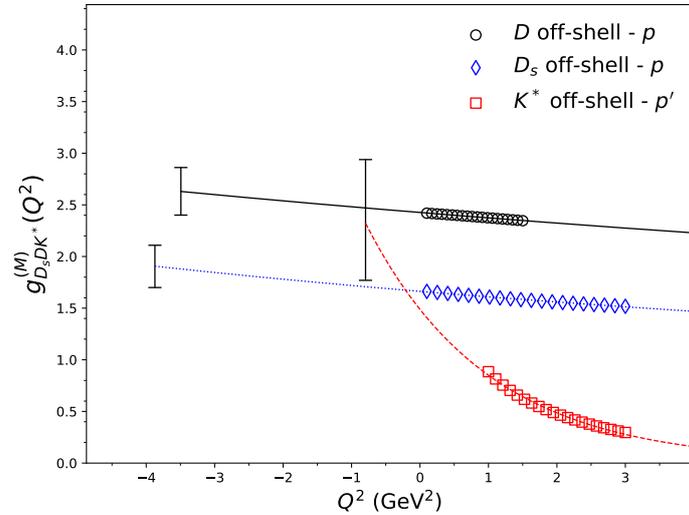


Figure 1: Form factors of the vertex $D_s DK^*$, and their respective extrapolated functions. The values of the form factor at the off-shell meson pole and the error bars are also indicated.

In the Fig. (1), we can see the SR, for the cases of D and D_s off-shell, the SR was extrapolated using a monopolar function, giving a similar result for the coupling constant. Instead of this, in the case with K^* off-shell, the monopolar function could take a coupling constant away of the other two fits. In this case we extrapolate using the exponential function. In others works, we observe that the monopolar fit is better for vertices containing only heavy particles, and when light mesons are present, the exponential fit is more suitable, see in Ref. [5].

Also, in the vertex $D^* D \pi$, that is the only one that have experimental value of the coupling constant, the exponential parametrization was assumed for π off-shell, and a resulting coupling constant really closer to the experimental value [6]. If instead we have used a Gaussian function, the experimental the result would be very far from the experimental one.

In the other QCDSR study of this vertex [3], the fit is made by a gaussian function, and it is our opinion that this is the main reason that the coupling constant obtained therein is 30% larger than ours. But this is not the only difference between both studies; the other ones are discussed in Ref. [1].

Finally, in the Table 2 we show the results for the coupling constant obtained by different methods in previous studies.

Reference	$g_{D_s DK^*}$
Our QCDSR	$2.29^{+0.65}_{-0.41}$
SU(4) [7]	5
LCSR [2]	1.61 ± 0.62
QCDSR [3]	3.26 ± 0.43

Table 2: Comparison of the coupling constant of the vertex $g_{D_s DK^*}$ that were obtained in different works.

We notice large discrepancies between the results. The larger discrepancy is for the $SU(4)$ relations that could be greater in this vertex due to the mass of the s quark, that is comparable to the masses of the u, d quarks. Studies of processes that depend on this coupling, such as the $B \rightarrow K^* \pi$ decay, can provide some insight in which method provides the better result.

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