PROCEEDINGS OF SCIENCE

Kaon, Nucleon and Δ^* Resonances with Hidden Charm

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In this presentation we discuss the generation of hadrons with an exotic quark content and hidden charm emerging from three-body interactions. To be more specific, we have studied the $KD\bar{D}^*/K\bar{D}D^*$ and $ND\bar{D}^*/N\bar{D}D^*$ systems and predicted states generated from the respective three-body dynamics. In the case of the $KD\bar{D}^*/K\bar{D}D^*$ system we predict the formation of a meson state with mass around 4307 MeV and quantum numbers $I(J^P) = 1/2(1^-)$, $K^*(4307)$, and from the study of the $ND\bar{D}^*/N\bar{D}D^*$ system we found N^* states with masses in the range $4400 \sim 4600$ MeV, width of $2 \sim 20$ MeV and positive parity.

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 -17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

[*]Speaker

PoS(XVHadronPhysics)029

1. Introduction

In the past decades a large number of exotic states were observed experimentally due to the access available to higher energy regions at different facilities around the world, drawing a lot of attention to the subject. As a consequence there exist families of exotic states know as X, Y and Z (see, e.g., Refs. [1–5]).

Exotic mesons, usually, can be understood as tetraquarks or as states originated from two-meson dynamics. Meanwhile, exotic baryons are usually described as pentaquarks or as states generated from baryon-meson dynamics. Most of the exotic states claimed recently have hidden or explicit heavy flavors (charm and bottom).

In this talk we discuss the results obtained for two different three-body systems whose interaction generate exotic states: $KD\bar{D}^*/K\bar{D}D^*$ and $ND\bar{D}^*/N\bar{D}D^*$, both of them with hidden charm arising from the $D\bar{D}^*/\bar{D}D^*$ subsystem. The main motivation to investigate the $KD\bar{D}^*/K\bar{D}D^*$ system came from the fact that the subsystems $D\bar{D}^*$, KD, KD^* have attractive interactions in s-wave from which the states $X(3872)/Z_c(3900)$, $D_{s0}(2317)$ and $D_{s1}(2460)$ arise, respectively [6–13]. Also, if a state was to be found, the quantum numbers of the generated state (considering all interactions in s-wave) were expected to be compatible with a K^* with a mass in the charmonium sector. Such a state would be a prediction that could encourage experimental searches for kaons at energies around $\sim 4-5$ GeV. Similarly, the study of the $ND\bar{D}^*$ system was inspired by the fact that the ND and ND^* subsystems are attractive and give rise, among others, to the state $\Lambda_c(2595)$ [14–19]. Such attractive interactions, together with the attraction in the $D\bar{D}^*/\bar{D}D^*$ subsystem, lead us to contemplate that the formation of N^* states with a three-body nature as a consequence of the $ND\bar{D}^*$ dynamics is quite probable. Besides, the recent announcement of the LHCb collaboration claiming the existence of possible hidden charm pentaquarks with non-zero strangeness, masses around 4459 MeV [20] and quantum numbers yet to be determined further motivated us to carry out the calculations, since the masses found are close to the threshold of the $ND\bar{D}^*$ system.

2. Formalism

The systems we are interested in are composed of three hadrons. To investigate their interactions we must obtain the corresponding T-matrix and this can be done by solving the Faddeev equation

$$T = T^1 + T^2 + T^3,$$
$$T^1 = t_1 + t_1 G[T^2 + T^3], \qquad T^2 = t_2 + t_2 G[T^1 + T^3], \qquad T^3 = t_3 + t_3 G[T^1 + T^2], \tag{1}$$

with G being a three-body loop function and t_i, $i = 1, 2, 3$, the two-body t-matrices describing the (jk) subsystems $[j, k = 1, 2, 3,\ j \neq k \neq i]$.

Solving the Faddeev equations, usually, is a cumbersome task. However, the three-body systems we are intending to study ($ND\bar{D}^*$ and $KD\bar{D}^*$) have an interesting characteristic that allows us to simplify the formalism for determining the T-matrix. When a three-body system, composed by the particles P_1, P_2 and P_3, has P_3 lighter than the other two particles and P_1P_2 cluster as a bound state/resonance, we can treat their interactions as those of a particle with fixed scattering centers. As a consequence, Eq. (1) can be written as [21–24]

$$T = T_{31} + T_{32}, \quad \text{with} \quad T_{31} = t_{31} + t_{31} G_3 T_{32}, \qquad T_{32} = t_{32} + t_{32} G_3 T_{31}, \tag{2}$$

where t_{31} (t_{32}) is the two-body t-matrix related to the subsystem P_1P_3 (P_2P_3) and G_3 the propagator of P_3 in the cluster.

Considering $K(N)$ as P_3, D as P_1 and $\bar{D}^*$ as P_2 for the $KD\bar{D}^*(ND\bar{D}^*)$ system, with $D\bar{D}^*$ clustering as the states $X(3872)$ or $Z_c(3900)$ (with isospin 0 or 1, respectively), both our systems fulfil the the aforementioned criteria.

In this manner, to determine the three-body T-matrices for the $ND\bar{D}^*$ and $KD\bar{D}^*$ systems we must obtain T_{31} and T_{32}, given in Eq. (2), for each system. In the following we are going to succinctly present the formalism to do this. More details can be found in Refs. [25, 26].

The elements T_{31} and T_{32} in Eq. (2) depend on the two-body t-matrices t_{31} and t_{32} as well as the loop function, G_3, of P_3 in the cluster. The latter propagator of P_3, with P_3 being K (N) for the system $KD\bar{D}^*(ND\bar{D}^*)$, is given by

$$G_K = \frac{1}{2M_a}\int \frac{d^3q}{(2\pi)^3}\frac{F_a(\mathbf{q})}{q_0^2 - \mathbf{q}^2 - m_K^2 + i\epsilon}, \quad G_N = \frac{1}{2M_a}\int \frac{d^3q}{(2\pi)^3}\frac{m_N}{\omega_N(\mathbf{q})}\frac{F_a(\mathbf{q})}{q_0 - \omega(\mathbf{q}) + i\epsilon}. \tag{3}$$

The term F_a in Eq. (3) is a form factor related to the molecular nature of the cluster ($D\bar{D}^*$) and can be written as [27–29]

$$\begin{aligned} F_a(\mathbf{q}) &= \frac{1}{N}\int_{|\mathbf{p}|,|\mathbf{p}-\mathbf{q}|<\Lambda} d^3\mathbf{p}\;\; f_a(\mathbf{p})f_a(\mathbf{p}-\mathbf{q}), \\ f_a(\mathbf{p}) &= \frac{1}{\omega_{a1}(\mathbf{p})\omega_{a2}(\mathbf{p})}\cdot\frac{1}{M_a - \omega_{a1}(\mathbf{p}) - \omega_{a2}(\mathbf{p})}, \end{aligned} \tag{4}$$

with M_a being the mass of the cluster, $N = F_a(\mathbf{q}=0)$ is a normalization constant, Λ represents a cut-off ~700 MeV and $\omega_{ai} = \sqrt{m_{ai}^2 + \mathbf{p}^2}$.

Next we need to calculate t_{31} and t_{32}. The determination of t_{31} and t_{32} for the $KD\bar{D}^*$ and $ND\bar{D}^*$ systems is similar. Hence, to illustrate the procedure we consider, for example, the $KD\bar{D}^*$ system. First, let's determine the expression for t_{31}.

The $K(D\bar{D}^*)$ system, considering $D\bar{D}^*$ as $X(3872)$ or $Z_c(3900)$, has three possible isospin configurations: $|KX, 1/2, 1/2\rangle$, $|KZ_c, 1/2, 1/2\rangle$ and $|KZ_c, 3/2, 3/2\rangle$ (where we use the notation $|A, I, I_3\rangle$, with A indicating the system, I is the total isospin of the system and I_3 the corresponding isospin third component). Considering, for instance, the state $|KZ_c, 1/2, 1/2\rangle$, the amplitude t_{31} for $KZ_c \to KZ_c$ in isospin 1/2 is given by

$$\langle KZ_c, 1/2, 1/2|\, t_{31}\, |KZ_c, 1/2, 1/2\rangle\,. \tag{5}$$

Remembering that t_{31} is related to the KD subsystem, to determine the matrix element in Eq. (5) it is convenient to express the $|KZ_c\rangle$ state in terms of the isospin of the KD subsystem. To do this we use the Clebsch-Gordan coefficients

$$\left|KZ_c, \frac{1}{2}, \frac{1}{2}\right\rangle = \frac{1}{2}\left[|KD,1,1\rangle \otimes \left|\bar{D}^*, \frac{1}{2}, -\frac{1}{2}\right\rangle + \frac{1}{\sqrt{2}}\left(|KD,1,0\rangle + |KD,0,0\rangle\right) \otimes \left|\bar{D}^*, \frac{1}{2}, \frac{1}{2}\right\rangle\right]. \tag{6}$$

Using Eq. (6) together with Eq. (5) we get that

$$\langle KZ_c, I=1/2, I_3=1/2|\, t_{31}\, |KZ_c, I=1/2, I_3=1/2\rangle \equiv t_{1(22)} = \frac{1}{4}(t_{KD}^{I=1} + 3t_{KD}^{I=0}), \tag{7}$$

where the $t^{I=a}_{KD}$ is the two-body t-matrix for the KD subsystem with isospin $I = a$, $a = 0, 1$, and the subscript $'(22)'$ in Eq. (7) stands for the transition $KZ_c \to KZ_c$, an element of the coupled channel t-matrix. Since the system KX can also have total isospin $1/2$ it can couple to KZ_c, such that the transitions $KX \to KX$ ($t_{31(11)}$) and $KX \leftrightarrow KZ_c$ ($t_{31(12)}$ or $t_{31(21)}$) have to be considered in order to obtain t_{31}. The determination of the isospin weights for the transitions $KX \to KX$ and $KX \to KZ_c$ requires the exact same procedure as that for the case of $KZ_c \to KZ_c$, and the results are summarized in Table 1.

	KX	KZ_c
KX	$\frac{1}{4}\left(3t^{I=1}_{KD} + t^{I=0}_{KD}\right)$	$\frac{\sqrt{3}}{4}\left(t^{I=1}_{KD} - t^{I=0}_{KD}\right)$
KZ_c	$\frac{\sqrt{3}}{4}\left(t^{I=1}_{KD} - t^{I=0}_{KD}\right)$	$\frac{1}{4}\left(t^{I=1}_{KD} + 3t^{I=0}_{KD}\right)$

Table 1: t_{31} amplitudes for the $KD\bar{D}^*$ system in terms of the two-body t-matrices for the KD subsystem.

The computation of t_{32} needs an analogous analysis to determine the contributions from interactions in different isospins and the results happen to coincide with those shown in Table 1, except for a global minus sign to be considered for the non-diagonal terms and changing $D \to \bar{D}^*$.

Since N has the same isospin as K the results on the isospin weights obtained for $KD\bar{D}^*$ in Table 1 changing $K \to N$. The $t_{(32)}$ matrix for the $ND\bar{D}^*$ system is also analogous to the one obtained for the $KD\bar{D}^*$ system.

So, to determine the t_{31} and t_{32} matrices we need to obtain the two-body t-matrices for the KD, $K\bar{D}$, $K\bar{D}^*$, KD^*, ND, $N\bar{D}$, $N\bar{D}^*$ and ND^* subsystems in different isospin configurations. These amplitudes can be obtained by solving the Bethe-Salpeter equation

$$t_{AB} = V_{AB} + V_{AC}G_{CC}t_{CB}, \tag{8}$$

where V_{AB} is the kernel for a channel made of two hadrons and can be obtained with an appropriate effective Lagrangian, and G_{CC} is a two body loop function. The loop function G is divergent and has to be regularized either with a cut-off or with dimensional regularization.

To obtain the two-body t-matrices t_{DN} and $t_{\bar{D}^*N}$, we consider two distinct models: one based on the $SU(8)$ spin flavor symmetry [30] and other based on the $SU(4)$ and heavy-quark spin symmetries [31, 32].

In the case of the $KD/K\bar{D}^*$ subsystems, to determine the t_{KD} and $t_{K\bar{D}^*}$ we solve Eq. (8) using as kernel the amplitude obtained from an effective Lagrangian based on heavy-quark spin symmetry [33–35].

Notice that the systems ND and ND^*, unlike KD KD^*, can be coupled in s-wave because of their quantum numbers [in s-wave the state ND (ND^*) has spin-parity $J^P = 1/2^-$ ($J^P = 1/2^-, 3/2^-$)]. To consider the coupling between ND and ND^*, in the $SU(4)$ model [32] a transition amplitude $DN \to D^*N$ is obtained through box diagrams (more details can be found in Ref. [32]). In the $SU(8)$ model the $DN \to D^*N$ transition is described by a Weinberg-Tomozawa amplitude Ref. [30].

3. Results

In figure 1 we show the results obtained for $|T|^2$ as a function of the energy of the $KD\bar{D}^*$ system considering the configurations KX and KZ_c. As we can see, in both processes, $KX \to KX$ and $KZ_c \to KZ_c$ [considering KX and KZ_c as coupled channels when solving Eq. (2)], a peak around 4300 MeV shows up. The width of the Z_c state was included in the formalism by implementing the transformation $M \to M - i\Gamma/2$, with $\Gamma \sim 28$ MeV, in the corresponding form factor of Eq. (4). When solving Eq. (4) we have varied the cut-off from 700 to 750 MeV, but no significant difference in the results were obtained from this variation (see Fig. 1).

It is interesting to note that the results for the $KX \to KX$ transition presented in Fig. 1 show a second peak around 4375 MeV which is related to the threshold of the three-body system.

When considering the $KZ_c \to KZ_c$ transition with isospin $3/2$ no signal of a bound state is found in $|T|^2$. Studies of further properties of $K^*(4307)$ can be found in Refs. [36, 37].

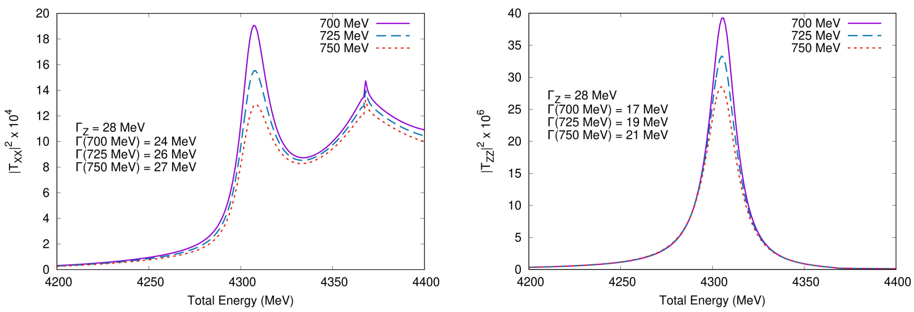

Figure 1: Modulus squared of the three-body T-matrix for the transitions $KX \to KX$ (left) and $KZ_c \to KZ_c$ (right) considering the coupling between the KX and KZ_c channels.

Figure 2 shows the results for the $NX \to NX$ and $NZ_c \to NZ_c$ transitions with $I(J^P) = 1/2\,(1/2^+)$ and Fig. 3 shows the corresponding results in case of $I(J^P) = 1/2\,(3/2^+)$. In both cases we have considered the inputs of Refs. [31, 32] and NX and NZ_c as coupled channels. As can be seem, the plots show that there are two peaks close to 4400 MeV and 4550 MeV for all four transitions.

The cut-offs used when calculating Eq. (4) vary from 700 MeV to 770 MeV and this variation causes a shift of $3 - 5$ MeV on the masses of the states obtained.

The results for the $ND\bar{D}^*/N\bar{D}D^*$ system, obtained by considering different models to determine the two-body t-matrices of the subsystems ($SU(4)$ or $SU(8)$), are very similar and the small difference found from the differences in the two-body models, as well as the different cut-off used to solve Eq. (4), provide us uncertainties in the masses and widths of the states found. The results for the masses and widths of the N^*'s obtained together with the corresponding uncertainties are summarized in Table 2.

Spin-parity	Mass(MeV)	Width (MeV)	Spin-parity	Mass(MeV)	Width (MeV)
$1/2^+$	$4404 - 4410$	2	$3/2^+$	$4467 - 4513$	$\sim 3 - 6$
$1/2^+$	$4556 - 4560$	$\sim 4 - 20$	$3/2^+$	$4558 - 4565$	$\sim 5 - 14$

Table 2: Masses and widths of the three-body N^*'s states found in the study of the $ND\bar{D}^*$ system.

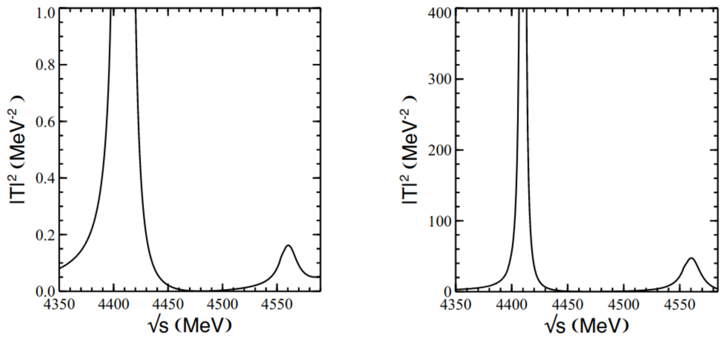

Figure 2: $|T|^2$ for the transitions $NX \to NX$ (left) and $NZ_c \to NZ_c$ (right) with $I(J^P) = 1/2\,(1/2^+)$ as functions of $\sqrt{s}$.

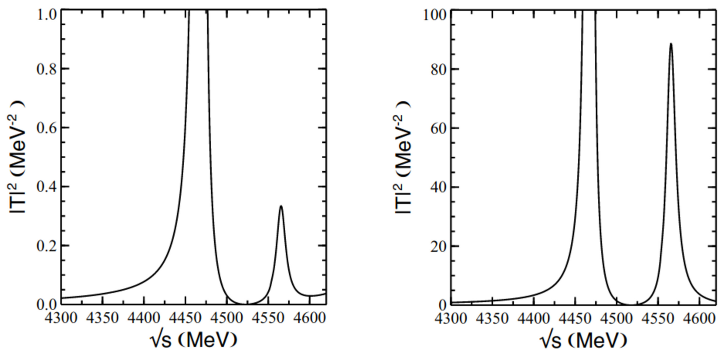

Figure 3: $|T|^2$ for the transitions $NX \to NX$ (left) and $NZ_c \to NZ_c$ (right) with $I(J^P) = 1/2(3/2^+)$ as functions of $\sqrt{s}$.

In case of $I = 3/2$, the $ND\bar{D}^*/N\bar{D}D^*$ interaction is capable of generating Δ^* states with hidden charm and masses around 4359 MeV with $\Gamma \sim 1.5$ MeV and 4512 MeV with $\Gamma \sim 4$ MeV. For more details we refer the reader to Ref. [38]

4. Conclusions and acknowledgments

We conclude from this study that considering $D\bar{D}^*/\bar{D}D^*$ as a cluster and adding a nucleon or a kaon to it generates states with hidden charm and three-body molecular nature, i.e., states that are described in terms of hadrons interacting with each other while keeping their identities.

The results obtained for the $KD\bar{D}^*/K\bar{D}D^*$ system show that a K^* meson around 4307 MeV should be observed in experimental investigation, while for the $ND\bar{D}^*/N\bar{D}D^*$ system N^* states with $I(J^P) = 1/2(1/2^+, 3/2^+)$ and masses around $4400 - 4600$ MeV as well as Δ^*'s with masses $4359 - 4512$ MeV and $I(J^P) = 3/2(1/2^+)$ are predicted.

This work is supported by the Conselho Nacional Científico e Tecnológico (CNPq), grant number 305526/2019-7 and 303945/2019-2, and by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), processes numbers 2019/17149-3, 2019/16924-3 and 2020/00676-8.

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