PROCEEDINGS OF SCIENCE

Renormalization group improved QCD thermodynamics

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We use the renormalization group optimized perturbation theory (RGOPT) to evaluate the quark contribution, P_q, to the QCD pressure at NLO (two loop level). In this application the complete QCD pressure is then obtained simply by adding the perturbative NLO contribution from massless gluons to the resummed P_q. At the central scale $M = 2\pi T$ our complete QCD pressure, $P = P_q + P_g$, shows a remarkable agreement with lattice predictions for $0.25 \lesssim T \lesssim 1\,\text{GeV}$. As expected, the RG properties native to the RGOPT resummation significantly reduce the embarrassing scale dependence that plagues popular analytical methods such as standard thermal perturbative QCD and hard thermal loop perturbation theory (HTLpt).

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 -17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

[*] Speaker

PoS(XVHadronPhysics)040

1. Introduction

The description of the phase transitions occurring within hot and dense hadronic matter represents one of the major theoretical challenges in contemporary hadronic physics, despite the enormous progress achieved by lattice QCD (LQCD) numerical simulations [1]. This is due to the notorious sign problem [2], which arises when finite chemical potential (μ) values are considered preventing LQCD to be reliably applied at finite baryonic densities[1]. A possible analytical alternative to LQCD is the so called *hard thermal loop perturbation theory* (HTLpt) [3] which represents a powerful resummation method. When applying this technique one deforms the original Lagrangian by a Gaussian mass term to be treated as an interaction, defining a modified perturbative expansion which then leads to a sequence of (variationally improved) approximations at successive orders. At NNLO the HTLpt predictions in Refs.[4, 5] are remarkably close to the lattice results for temperatures down to $T \gtrsim 2\,T_{pc}$ for the commonly chosen "central" renormalization scale choice, $M = 2\pi\sqrt{T^2 + \mu^2/\pi^2}$. Unfortunately, even a moderate scale variation of a factor 2 dramatically affects the pressure and related thermodynamical quantities by relative variations of order 1 or more. More recently, an alternative dubbed *renormalization group optimized perturbation theory* (RGOPT) [6, 7], which combines the standard *optimized perturbation theory* (OPT) [8] framework with renormalization group (RG) properties has been proposed. One of RGOPT main advantages is that it restores RG invariance at all stages of the calculation, in particular when fixing the arbitrary variational mass parameter. At vanishing temperatures it has been used in QCD up to high (three and four-loop) orders to estimate the basic coupling α_s [7], predicting values very compatible with the world averages [9]. Also accurate values of the (vacuum) quark condensate were obtained at four-loop [10] and five-loop [11] orders. Regarding QCD at extreme conditions the method has been initially used to evaluate the quark contribution to the pressure at two-loop (NLO) at finite densities and vanishing temperatures [12]. The results obtained in that work show how the method improves over perturbative QCD (pQCD). More recently, the work of Ref. [12] has been extended in order to describe the finite temperature domain producing results which are remarkably close to LQCD even when the "central" scale varies by a factor of 2 [13, 14]. In this case the RGOPT predictions were compared to those furnished by pQCD and HTLpt showing a substantial improvement as far as scale dependence is concerned. In this contribution we review some of the main results obtained in Refs. [13, 14]. The work is organized as follows. In the next section we present the analytical results for the QCD pressure at the two loop level. Our numerical results and conclusions are presented in Sec. 3.

[1]Except at low densities where the problem may be circumvented, e.g., by performing a Taylor expansion around vanishing chemical potential results.

2. QCD pressure

In the case of a massive theory the two loop perturbative quark contribution to the QCD pressure can be obtained by combining results from Refs. [10, 15]

$$\begin{aligned}\frac{P_q^{PT}}{N_f N_c} &= -\frac{m^4}{8\pi^2}\left(\frac{3}{4}-L_m\right)+2T^4 J_1 - 3g\frac{m^4}{2(2\pi)^4}C_F\left(L_m^2-\frac{4}{3}L_m+\frac{3}{4}\right)\\ &- gC_F\left\{\left[\frac{m^2}{4\pi^2}(2-3L_m)+\frac{T^2}{6}\right]T^2 J_2+\frac{T^4}{2}J_2^2+m^2T^2J_3\right\},\end{aligned} \tag{1}$$

where $L_m = \ln(m/M)$, $g \equiv 4\pi\alpha_s(M)$, M is the arbitrary renormalization scale in the $\overline{\text{MS}}$-scheme, $C_F = (N_c^2-1)/(2N_c)$, $N_c = 3$, and $N_f = 3$. The thermal and in medium integrals, J_i, can be found in Refs. [13, 14]. The RGOPT pressure can be obtained by following the prescription described in Refs. [13, 14] so that the quark contribution to the pressure becomes

$$\begin{aligned}\frac{P_q^{RGOPT}}{N_f N_c} &= \frac{P_q^{PT}}{N_f N_c}+\frac{m^4}{(2\pi)^2}\left(\frac{\gamma_0}{b_0}\right)\left(\frac{1}{2}-L_m\right)+m^2\left(\frac{\gamma_0}{b_0}\right)T^2 J_2\\ &+ \frac{m^4}{(4\pi)^2 b_0}\left\{\frac{1}{g}\left(1-\frac{\gamma_0}{b_0}\right)+\left[(b_1-2\gamma_1)\pi^2-\frac{(b_0-2\gamma_0)}{3}\right]\right\},\end{aligned} \tag{2}$$

where the additional terms, with respect to Eq.(1), arise from subtracting a finite zero point contribution of the form $(m^4/g)\sum_k s_k g^k$, performing the substitutions $m \to m(1-\delta)^{\gamma_0/(2b_0)}$ and $g \to \delta g$, reexpanding to order-δ and finally setting $\delta = 1$. The explicit form of the RG coefficients γ_0, b_0 as well as the relevant s_k can be found in Refs.[13, 14]. To fix the variational mass, m, one may proceed in two different ways [12–14]: using either a standard stationarity criterion [16], the mass optimization prescription (MOP):

$$\left.\frac{\partial P_q^{RGOPT}}{\partial m}\right|_{\overline{m}} \equiv 0\,, \tag{3}$$

or, alternatively, the *reduced* RG equation

$$\left[M\partial_M + \beta(g)\partial_g\right] P_q^{RGOPT} = 0\,, \tag{4}$$

which is the criterion to be considered in the present work [2] The coupling $g(M)$ is determined from standard PT two-loop running, with the renormalization scale M chosen as a multiple of πT as usual. Unfortunately, when applied to Eq.(2), both Eq.(3) and Eq.(4) yield a complex dressed mass $\overline{m}(g,T,\mu)$ for a substantial part of the physically relevant T,μ range. Nevertheless, this issue can be cured in an RG consistent manner by performing a renormalization scheme change (RSC)[7, 12, 14]. With this aim we define a RSC acting only on the variational mass in our framework, $m \to m'(1+B_2 g^2)$, where a single B_2 parametrizes a NLO RSC from the original $\overline{\text{MS}}$-scheme inducing an extra term $-4gm^4 s_0 B_2$ in Eq.(1). Within this RSC B_2 may be considered an extra variational parameter to be fixed by a sensible prescription.

Considering specifically the RG Eq.(4), it can be conveniently written as an equation for $\ln(m^2/M^2)$,

$$-\ln\frac{m^2}{M^2}+B_{rg}\mp\frac{8\pi^2}{g}\sqrt{\frac{2}{3}D_{rg}(B_2)}=0\,, \tag{5}$$

[2]The reader interested in the MOP prescription is referred to Refs. [13, 14].

with $B_{rg}, D_{rg}(B_2)$ given respectively in Eqs. (4.10) and (4.11) of Ref.[14]. To yield real solutions the arbitrary RSC parameter B_2 is fixed by partly fully cancelling D_{rg}. In particular for the RG prescription one may use $D_{rg}(B_2) = 0$ to uniquely fix B_2. Once the prescription to fix the RGOPT parameters has been established we can finally consider the full QCD pressure by simply adding to Eq.(2) the NLO glue contributions [17]

$$P_g^{PT} = \frac{8\pi^2}{45} T^4 \left[1 - \frac{15}{(4\pi)^2}\, g\right] . \tag{6}$$

3. Numerical results and conclusions

Let us now compare the NLO RGOPT full QCD results with those from HTLpt [5, 18] and (massless) pQCD [19], as well as with available LQCD data from Refs. [20–22] at $\mu_B = 0$. For the numerical evaluations of NLO quantities we take the exact two-loop running coupling (see, e.g., Ref. [12]) and vary the scale within the range $\pi T \le M \le 4\pi T$.

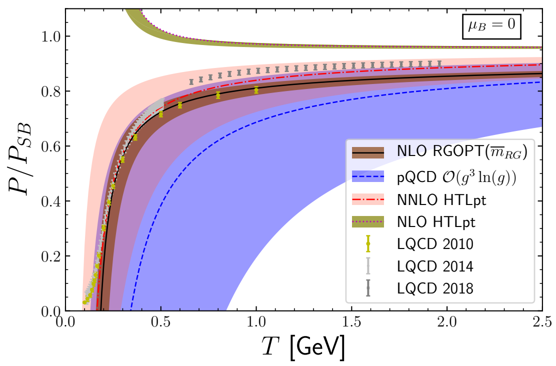

Figure 1: NLO RGOPT (RG prescription) plus NLO P_g^{PT} pressure (brown band) compared to N^{3}LO $g^3 \ln g$ pQCD (light blue band), NLO HTLpt (light green band) and NNLO HTLpt (light red band), with scale dependence $\pi T \le M \le 4\pi T$, and to lattice data [20–22] at $\mu_B = 0$.

Let us choose $\Lambda_{\overline{\rm MS}} = 335$ MeV for $N_f = 3$ which is very close to the latest world average value [9]. Notice that, for consistency, the NNLO HTLpt [5] and $O(g^3 \ln g)$ pQCD [19] numerical results reproduced here have been obtained rather with a three-loop order running coupling.
The results displayed in Fig.1 show that the RGOPT NLO predictions for the central scale, $M = 2\pi T$ compare very well with lattice results for temperatures starting at $T \simeq 0.25$ GeV which lies within the relatively strong coupling regime ($\alpha_s \simeq 0.3$). This agreement persists up to $T = 1$ GeV, the highest value for the LQCD data in Ref. [20]. Comparing our NLO results with those from NNLO HTLpt one also observes that the RGOPT successfully attenuates the scale dependence issue. Due to technical difficulties in applying our approach to the gluon sector we have only considered the purely perturbative NLO contribution on top of the variationally resummed quark contributions.

Acknowledgments

M.B.P. and T.E.R. are partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil) and by Coordenação de Aperfeiçoamento de Pessoal de Nível

Superior-(CAPES-Brazil)-Finance Code 001. This work was also financed in part by INCT-FNA (Process No. 464898/2014-5).

References

[1] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature **443**, 675 (2006).

[2] P. de Forcrand, PoS LAT **2009**, 010 (2009).

[3] J. O. Andersen, E. Braaten and M. Strickland, Phys. Rev. Lett. **83**, 2139 (1999);

[4] J. O. Andersen, L. E. Leganger, M. Strickland and N. Su, JHEP **1108**, 053 (2011); S. Mogliacci, J. O. Andersen, M. Strickland, N. Su and A. Vuorinen, JHEP **1312**, 055 (2013); N. Haque, J. O. Andersen, M. G. Mustafa, M. Strickland and N. Su, Phys. Rev. D **89**, 061701 (2014).

[5] N. Haque, A. Bandyopadhyay, J. O. Andersen, M. G. Mustafa, M. Strickland and N. Su, JHEP **1405**, 027 (2014).

[6] J. L. Kneur and A. Neveu, Phys. Rev. D **81**, 125012 (2010).

[7] J. L. Kneur and A. Neveu, Phys. Rev. D **88**, 074025 (2013).

[8] A. Okopinska, Phys. Rev. D **35**, 1835 (1987).

[9] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, 030001 (2018).

[10] J. L. Kneur and A. Neveu, Phys. Rev. D **92**, 074027 (2015).

[11] J. L. Kneur and A. Neveu, Phys. Rev. D **101**, 074009 (2020).

[12] J. L. Kneur, M. B. Pinto and T. E. Restrepo, Phys. Rev. D **100**, 114006 (2019).

[13] J. L. Kneur, M. B. Pinto and T. E. Restrepo, Phys. Rev. D **104**, L031502 (2021).

[14] J. L. Kneur, M. B. Pinto and T. E. Restrepo, Phys. Rev. D **104**, 034003 (2021).

[15] J. I. Kapusta and C. Gale, *"Finite-temperature field theory: Principles and applications"* (Cambridge University Press, 2006).

[16] P.M. Stevenson, Phys. Rev. D **23**, 2916 (1981); Nucl. Phys. B **203**, 472 (1982).

[17] E. V. Shuryak, Sov. Phys. JETP **47**, 212 (1978); S. A. Chin, Phys. Lett. B **78**, 552 (1978).

[18] N. Haque, M. G. Mustafa and M. Strickland, Phys. Rev. D **87**, 105007 (2013).

[19] K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, Phys. Rev. D **67**, 105008 (2003).

[20] S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, JHEP **11**, 077 (2010).

[21] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B **730**, 99-104 (2014).

[22] A. Bazavov, P. Petreczky and J. H. Weber, Phys. Rev. D **97**, 014510 (2018).

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